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## Seepage forces, important factors in the formation of horizontal hydraulic fractures and bedding-parallel fibrous veins ('beef' and 'cone-in-cone')

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1    **Seepage forces, important factors in the formation of horizontal hydraulic**  
2    **fractures and bedding-parallel fibrous veins (“beef” and “cone-in-cone”)**

3

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9

10   **Suggested running title:** Seepage forces and “beef”

11

12    **Abstract**

13

14    Bedding-parallel fibrous veins (“beef” and “cone-in-cone”) are common to a number of  
15    sedimentary basins, especially those containing black shale. The type locality is SW England.

16    The commonest mineral in the fibres is calcite. The fibres indicate vertical opening, against  
17    the force of gravity. In the past, this has been attributed to fluid overpressure. However, a

18    simple analysis, based on Von Terzaghi’s concepts, leads to the conclusion that, for the

19    fractures to be horizontal, either the rock must be anisotropic, or it must be subject to

20    horizontal compression. By means of a more complete analysis, supported by physical

21    models, we show that horizontal fractures are to be expected, even if the rock is isotropic and

22    there are no tectonic stresses. Upward fluid flow, in response to an overpressure gradient,

23    imparts seepage forces to all elements of the solid framework. The seepage forces counteract

24    the weight of the rock, and even surpass it, generating a tensile effective stress. The process

25    may lead, either to tensile hydraulic fracturing, or to dilatant shear failure. We suggest that

26 these two failure modes, and the availability of suitable solutes, explain the frequent  
27 occurrence of “beef” and “cone-in-cone”, respectively.

28

## 29 **Introduction: bedding-parallel veins (“beef” and “cone-in-cone”)**

30

31 Bedding-parallel veins of fibrous calcite or other minerals are common to a number of  
32 sedimentary basins, especially those containing black shale. Good examples are the veins of  
33 fibrous calcite in Jurassic strata along the coastal cliffs of SW England (Fig. 1). In the 1800s,  
34 quarrymen called this material “beef”, because of its resemblance to fibrous steak. So  
35 common is this material that it has given its name to a stratigraphical unit, the “Shales-with-  
36 Beef” of Liassic age (Lang et al., 1923). On average, the veins appear to have grown in a  
37 horizontal attitude and the fibres appear to have grown vertically. Less common in the area  
38 are “cone-in-cone” structures: multiply nested cones of fibrous calcite, in which the fibres  
39 have also grown vertically on average.

40 In the early days of geological investigation, beef and cone-in-cone attracted attention  
41 and were subjects of debate (Sorby, 1860). Were the structures “displacive”, in the sense of  
42 the walls moving apart (the modern term would be “dilatant”)? What was the nature of the  
43 forces involved? Did the fibres passively infill fracture space, or did they actively push the  
44 host rock apart by force of crystallization? These questions found no immediate answers and  
45 they are still relevant today (Hilgers and Urai, 2005; Wiltchko and Morse, 2001).

46 From studies of fibrous veins, especially those containing oblique fibres, there is a  
47 consensus that the fibres grow incrementally, partly or totally tracking the history of relative  
48 displacement of the walls (Taber, 1918; Durney and Ramsay, 1973). In some examples,  
49 opening and infilling seem to have occurred episodically, by a crack-seal mechanism

50 (Ramsay, 1980). In other examples, there may have been no loss of continuity within the  
51 fibres, as they grew.

52 Although calcite is perhaps the commonest fibrous mineral in dilatant veins, other  
53 infilling minerals may also adopt a fibrous habit. Examples are gypsum (Shearman et al.,  
54 1972; Hilgers and Urai, 2005), quartz (Ramsay, 1980; Hilgers and Urai, 2005), pyrobitumen  
55 or carbon (Parnell, 1999), dolomite, halite, strontianite, celestite, various borates such as  
56 ulexite, barite, the various kinds of asbestos, albite, emerald, and even precious metals, such  
57 as gold and silver. Arguably, almost any mineral may adopt a fibrous habit, if it grows during  
58 progressive or episodic opening of a vein, especially within porous rock. However, some  
59 species are undoubtedly more common than others. Intuitively, cryptocrystalline materials,  
60 such as pyrobitumen and some metals, are less likely to be truly fibrous. We suggest that the  
61 name “beef” should not be restricted to veins of calcite, but should encompass all fibrous  
62 bedding-parallel veins, regardless of their composition. In what follows, we will use the term  
63 in this sense.

64 Our current understanding is that fibrous minerals grow by precipitation, mainly from  
65 supersaturated aqueous solutions, as a result of chemical reactions, or changes in physical  
66 conditions, especially temperature and pressure. Several experimenters have succeeded in  
67 reproducing fibrous textures by precipitation and growth of crystals from supersaturated  
68 solutions, either into open space (Taber, 1918), or into a crack (Means and Li, 2001; Hilgers  
69 and Urai, 2002; Nollet et al., 2006). However none of these experiments have provided  
70 insights into the mechanics of fracturing, because the cracks were artificial and the  
71 specimens were too thin to allow adequate monitoring of stresses.

72 One possibility is that fracturing is due to relatively high pressures of fluid circulating  
73 through the pores.

74

## 75 **Fluid overpressure and fluid flow**

76

77 High values of pore fluid pressure are common in sedimentary basins, especially at depth  
78 (Swarbrick et al., 2002). Commonly, the fluid pressure increases with depth, but there may  
79 be significant deviations from such a simple trend. If the pressure is greater than that  
80 predicted by the weight of an equivalent column of water, it is said to be an overpressure.

81 The frequent occurrence of fluid overpressure in sedimentary basins has led to an intense  
82 debate concerning its origin. Amongst the many possible causes, the most popular would  
83 seem to be mechanical compaction, hydrocarbon generation, or a combination of both  
84 (Swarbrick et al., 2002). As shale compacts readily and may also generate hydrocarbons  
85 upon burial, it is perhaps no accident that beef is common in such rock. In this context, it is  
86 intriguing that some beef contains solid hydrocarbons between calcite fibres, or liquid  
87 hydrocarbons within them. Examples have been described from Jurassic source rocks in the  
88 Neuquén Basin of Argentina (Parnell and Carey, 1995; Parnell et al., 2000). Horizontal  
89 fractures infilled with bitumen alone have been described from several areas, including the  
90 Neuquén Basin (Parnell and Carey, 1995) and New York State (Lash and Engelder, 2005).  
91 Fractures that carry hydrocarbons, including beef, may provide valuable evidence for  
92 hydrocarbon generation as a contributing cause of overpressure.

93 Darcy's Law of steady fluid flow in a permeable medium provides an explanation for  
94 pressure profiles of simple form (Fig. 2; Appendix). The law states that the rate of flow in a  
95 given direction is proportional to the permeability and to the overpressure gradient. Consider  
96 three examples.

97 1. A linear pressure profile is an indication that permeability is invariant with depth (Fig.  
98 2A).

- 99 2. A local pressure ramp will occur across a sealing layer of relatively small permeability  
100 (Fig. 2B). The fluid pressure then reaches a maximum at the base of the sealing layer.  
101 3. A parabolic hump is an indication that fluid is being generated within a source layer of  
102 small permeability (Fig. 2C). The fluid pressure then reaches a maximum about half way  
103 up the layer, instead of at its base.

104

105 In the last two examples, the fluid pressure may locally attain or even surpass lithostatic  
106 values.

107

## 108 **Hydraulic fracturing**

109

110 A major impetus for understanding the macroscopic mechanisms of beef formation came  
111 from the petroleum industry, when hydraulic fracturing was found to be the reason for  
112 borehole break-outs (Hubbert and Willis, 1957). A dilatant fracture propagates readily  
113 through brittle rock, if it contains a mobile fluid under high pressure, such as drilling mud.  
114 Break-outs occur if the fluid pressure exceeds the sum of (1) the compressive stress in the  
115 host rock, acting normal to the fracture, and (2) the tensile strength of the rock.

116 It is useful to distinguish between two mechanisms of hydraulic fracturing (Mandl and  
117 Harkness, 1987). In external hydraulic fracturing, a fluid coming from outside penetrates an  
118 impermeable rock. This mechanism is generally held to be responsible for magmatic dykes  
119 and sills. In internal hydraulic fracturing, an overpressured fluid migrates through pore space,  
120 generating a fracture at an internal point of weakness. This mechanism is held to be  
121 responsible for mineral-filled veins.

122 Hydraulic fracturing due to overpressure has been seen as a potential mechanism for  
123 opening bedding-parallel veins (Hillier and Cosgrove, 2002; Shearman et al., 1972; Stoneley,  
124 1983).

125

## 126 **Effective stresses and seepage forces**

127

128 On considering a permeable material, it is useful to distinguish between (1) the total  
129 stress, acting on both the solid framework and the pore fluid, and (2) the effective stress,  
130 acting on the solid framework alone. These concepts originated in soil mechanics (Von  
131 Terzaghi, 1923). To our knowledge, they were first used in geology to explain ready sliding  
132 on a flat-lying thrust, as a result of fluid overpressure (Hubbert and Rubey, 1959). On  
133 applying the concepts to an isotropic rock, one finds that a horizontal fracture can form, only  
134 if the least effective stress is vertical, and the fluid overpressure exceeds the weight of the  
135 overburden by an amount equal to the tensile strength of the solid framework. However, in a  
136 purely lithostatic situation, where a sedimentary basin is subject to no forces except those of  
137 gravity, the greatest effective stress should be vertical and the least effective stress should be  
138 horizontal (Sibson, 2003). On this basis, beef should not form in lithostatic basins containing  
139 isotropic rock. On the contrary, necessary conditions for the formation of beef would seem to  
140 be, either (1) a context of horizontal compression, due to additional tectonic forces, or (2) a  
141 high susceptibility to fracturing along bedding, in other words, an anisotropy of tensile  
142 strength (Cosgrove, 1995, 2001; Lash and Engelder, 2005).

143 Recently it has been argued (and demonstrated by physical modelling) that although von  
144 Terzaghi's concept would appear to be correct, it is dangerous to calculate the effective stress  
145 at a point, by first considering the total stress without regard for fluid flow, and then  
146 subtracting the fluid overpressure (Cobbold et al., 2001; Mourgues and Cobbold, 2003). This

147 is because overpressure tends to vary in space, so that a pore fluid migrates in response to its  
148 gradient, according to Darcy's Law. Microscopically, the migrating fluid imparts a seepage  
149 force to each element of the solid framework and this seepage force modifies the balance of  
150 forces acting on the element. Macroscopically, the seepage force **per unit volume** is equal to  
151 the overpressure gradient (see Mourgues and Cobbold, 2003, for a more detailed discussion  
152 of this topic). A complete analysis should therefore take seepage forces into account. On  
153 doing so, one finds that, although the greatest effective stress in most sedimentary basins  
154 should be vertical, as indeed it is according to measurements on many wells (Hillis, 2003), in  
155 extreme situations the greatest effective stress may become horizontal, as a result of seepage  
156 forces, and this will be the subject of our analysis.

157

## 158 **Horizontal hydraulic fractures in physical models**

159

160 Horizontal hydraulic fractures have been reported in a number of experiments on physical  
161 models.

162 For example, in experiments on fluid escape structures, in which overpressured water  
163 flowed upward through horizontal layers of fine-grained glass spheres and silicon carbide,  
164 horizontal water-filled fractures appeared beneath the least permeable layers (Nichols et al.,  
165 1994, their fig. 3).

166 In other experiments, the tensile strengths of homogeneous cohesive powders (organic  
167 polymers) have been measured experimentally, by forcing gas upward through columns of  
168 particles (Valverde et al., 1998; Watson et al., 2001). The experimenters reported that  
169 horizontal cracks formed, when the fluid overpressure exceeded the weight of the  
170 overburden.



171 We have done similar tests, to determine the physical properties of silica powder, for a  
172 grain size between 75 and 125  $\mu\text{m}$ . Such a powder typically is cohesive and has a small  
173 tensile strength, enabling it to form open fractures (Galland et al., 2006). The apparatus was a  
174 cylindrical shear box, 11 cm in diameter, similar to the one of Mourgues and Cobbold  
175 (2003). After having been poured into the apparatus, the density of the powder was  
176 approximately  $1.4 \text{ g cm}^{-3}$ . The pore fluid was compressed air. The fluid pressure was  
177 measured at a number of regularly spaced points within the powder, by means of hypodermic  
178 needles, connected to U-tubes filled with water. The vertical fluid pressure profile was very  
179 close to linear, according to Darcy's Law, and the calculated permeability was 2.8 darcy.  
180 When the air flowed upwards, the apparent weight of the powder decreased, presumably as a  
181 result of seepage forces. The highest pressure that the powder was able to sustain was slightly  
182 larger than its weight, but we lack precision on this critical value of pressure, because of  
183 difficulties in correcting for sidewall friction (see Mourgues and Cobbold, 2003).

184 As the fluid pressure increased, internal structures appeared in the following four stages.

- 185 1. Small horizontal cracks formed within the powder. The cracks connected to small gas  
186 chimneys near the upper surface (Fig. 3).
- 187 2. The cracks coalesced, to form large jagged horizontal fractures. These fractures then  
188 widened, filling with air, and the overburden lifted.
- 189 3. The fluid pressure dropped suddenly and the fractures became unstable, migrating  
190 upward.
- 191 4. When the fractures reached the surface, the powder started to bubble.

192 Further tests will be necessary to explain this sequence of events. For the purposes of this  
193 paper, we note that horizontal hydraulic fractures formed when the fluid pressure exceeded  
194 the weight of overburden. The experiments therefore provide a physical basis for the analysis  
195 that follows.

196

197

**198 Mechanical analysis**

199

200 In this section, we develop the equations for elastic deformation and brittle failure of a  
201 sedimentary column, which is subject to gravity and seepage forces (Fig. 4). We assume a  
202 lithostatic context, in which the column is laterally confined, so that there are no horizontal  
203 strains. The effect of gravity (weight of overburden) is to generate a compressive stress that  
204 increases with depth. The effect of vertical seepage forces (overpressure gradient) is to  
205 counteract the weight, or even to overcome it. The aim of the analysis is to determine the  
206 conditions under which a horizontal dilatant fracture may develop.

207

*208 Equations of linear elasticity*

209

210 During progressive loading, most rocks undergo a first phase of small but recoverable  
211 elastic deformation, before reaching a yield point (Jaeger and Cook, 1979). Subsequent  
212 deformation is mainly non-elastic and irrecoverable. We consider each of these phases in  
213 turn.

214 For the elastic phase, the relationship between applied stress and infinitesimal strain is  
215 approximately linear. For simplicity, let us therefore assume linear isotropic elastic  
216 behaviour, as expressed by the following equations in three dimensions (Jaeger and Cook,  
217 1979, page 110):

218

$$\begin{aligned}
& \sigma_1 = (\lambda + 2G)\varepsilon_1 + \lambda\varepsilon_2 + \lambda\varepsilon_3 \\
219 \quad & \sigma_2 = \lambda\varepsilon_1 + (\lambda + 2G)\varepsilon_2 + \lambda\varepsilon_3 \\
& \sigma_3 = \lambda\varepsilon_1 + \lambda\varepsilon_2 + (\lambda + 2G)\varepsilon_3
\end{aligned} \tag{1}$$

220

221 Here  $\sigma_1, \sigma_2$  and  $\sigma_3$  are principal stresses (positive, when compressive);  $\varepsilon_1, \varepsilon_2$  and  $\varepsilon_3$  are  
222 principal infinitesimal strains (positive, when shortening); and  $\lambda$  and  $G$  are elastic constants  
223 (Lamé parameters). Notice that the composite factor  $(\lambda + 2G)$  relates stress and strain in one  
224 and the same direction, whereas  $\lambda$  relates stress and strain in two perpendicular directions.

225

226 *Lithostatic setting*

227

228 In a sedimentary basin that is not subject to tectonic loading, internal stresses are mainly  
229 due to gravity. Consider such a lithostatic setting. Assume that there are no horizontal strains,  
230 only a vertical compaction ( $\varepsilon_1 \neq 0, \varepsilon_2 = \varepsilon_3 = 0$ ). The greatest stress is vertical. The least  
231 stresses are horizontal and equal in all directions, so that  $\sigma_2 = \sigma_3$ . Under these conditions of  
232 axial symmetry, equations (1) simplify to (Jaeger and Cook, 1979, page 113):

233

$$\begin{aligned}
& \sigma_1 = (\lambda + 2G)\varepsilon_1 \\
234 \quad & \sigma_2 = \lambda\varepsilon_1 \\
& \sigma_3 = \lambda\varepsilon_1
\end{aligned} \tag{2}$$

235

236 Hence:

$$237 \quad \sigma_2 = \sigma_3 = \left( \frac{\nu}{1-\nu} \right) \sigma_1 \tag{3}$$

238 where  $\nu = \frac{\lambda}{2(\lambda + G)}$  is Poisson's ratio.

239 Equation (3) is a linear relationship between the greatest and least principal stresses, of  
 240 the form  $\sigma_3 = k_e \sigma_1$ , where  $k_e$  is an elastic constant of proportionality. For most rocks,  $\nu$  is  
 241 between 1/4 and 1/3 (Jaeger and Cook, 1979), so that  $k_e$  is between 0.33 and 0.5.

242 If the rock yields, the equations of linear elasticity no longer hold. Instead, the stresses  
 243 satisfy a yield criterion. Simplest is the linear relationship of Coulomb for a frictional but  
 244 non-cohesive material:

245

$$246 \quad \tau = \mu \sigma_n \quad (4)$$

247

248 Here  $\tau$  and  $\sigma_n$  are the shear stress and normal stress acting on a plane, and  $\mu$  is a constant,  
 249 the coefficient of internal friction. In a space of shear stress versus normal stress (Mohr  
 250 space, Fig. 5), equation (4) plots as a straight line through the origin (a linear Mohr envelope)  
 251 and the state of stress plots as a circle (the Mohr circle). Shear failure (Coulomb slip) occurs  
 252 if the circle touches the straight line.

253 By simple trigonometry (Fig. 4),

254

$$255 \quad \sin \phi = (\sigma_1 - \sigma_3) / (\sigma_1 + \sigma_3) \quad (5)$$

256

257 Hence:

258

$$259 \quad \sigma_3 = \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right) \sigma_1 \quad (6)$$

260

261 Here  $\phi$  is the angle of internal friction of the material, so that  $\mu = \tan \phi$ . Equation (6) is  
 262 another linear relationship between greatest and least stresses, of the form  $\sigma_3 = k_y \sigma_1$ , this

263 time for yield. For most granular materials and brittle rock,  $\phi$  is between  $30^\circ$  (Byerlee, 1978)  
 264 and  $50^\circ$  (Mourgues and Cobbold, 2003). Therefore  $k_y$  is between 0.13 and 0.33.

265 On comparing the elastic and brittle behaviours, we see that for most rocks,  $k_e \leq k_y$ . This  
 266 means that the failure envelope is steeper than the elastic envelope (Fig. 6). Hence the rock  
 267 does not yield under lithostatic conditions, except at the origin, where all stresses vanish, this  
 268 being a trivial solution.

269

270 *Fluid flow through a non-cohesive material in a lithostatic setting*

271

272 Now consider that a pore fluid is flowing upward, as a result of a vertical gradient in  
 273 pressure that is greater than hydrostatic, in other words, an overpressure gradient. According  
 274 to the principle of Von Terzaghi (1923), the overpressure takes up part of the load, reducing  
 275 the effective stress in the solid framework, so that:

276

$$277 \quad \sigma'_1 = \sigma_1 - P$$

$$278 \quad \sigma'_3 = \sigma_3 - P \tag{7}$$

279

280 Here  $P$  is the overpressure and  $\sigma'_1$  and  $\sigma'_3$  are the greatest and least values of effective stress  
 281 in the solid framework. If we assume that fluid flow has no effect other than this, the Mohr  
 282 circle displaces toward the origin by an amount equal to  $P$ , without any change in size (Fig.  
 283 7). This simple result can be found in many well-known geological publications, including  
 284 articles (Hubbert and Rubey, 1959; Sibson, 2003) and textbooks (e.g. Jaeger and Cook, 1979;  
 285 Price and Cosgrove, 1990). However, it may be dangerously **oversimplified**, for situations in  
 286 which there is fluid flow (Mourgues and Cobbold, 2003). Darcy's Law for fluid flow implies  
 287 that seepage forces act on each solid element of the framework. The seepage forces modify

288 the general balance of forces on the element. For upward flow, the seepage forces act  
 289 vertically, not horizontally. Hence the vertically integrated effective stress will be reduced by  
 290 an amount equal to the vertically integrated overpressure:

291

$$292 \quad \sigma'_1 = \sigma_1 - P \quad (8)$$

293

294 However, the same is not true for the horizontal direction, in which there are no seepage  
 295 forces. Instead, for elastic deformation in non-cohesive material under lithostatic conditions,  
 296 the horizontal effective stress should be directly proportional to the vertical effective stress  
 297 (Hillis, 2003):

298

$$299 \quad \sigma'_3 = k_e \sigma'_1 = k_e (\sigma_1 - P) \quad (9)$$

300

301 It is convenient to express the overpressure  $P$  as a fraction of the total vertical stress:

302

$$303 \quad P = \lambda \sigma_1 \quad (10)$$

304

305 Substituting (10) in (9):

306

$$307 \quad \sigma'_3 = k_e (1 - \lambda) \sigma_1 \quad (11)$$

308

309 This relationship provides a good measure of the way in which horizontal stresses vary with  
 310 depth in many overpressured basins (Hillis, 2003). As the overpressure increases, the Mohr  
 311 circle not only shifts to the left in Mohr space, but also decreases in size (Fig. 8; Hillis, 2003;  
 312 Mourgues and Cobbold, 2003). Eventually, at the origin, the Mohr circle reduces to a point.

313 Only at that point does the circle touch the failure envelope. All other situations are stable.  
314 Thus our analysis cannot account for the formation of bedding-parallel veins in a lithostatic  
315 setting, even if there is fluid flow. Some other factor is required. In any case, for a material to  
316 undergo tensile failure, it must have a tensile strength. Let us therefore incorporate this  
317 factor.

318

319 *Fluid flow through cohesive material in a lithostatic setting*

320

321 For rocks also that have tensile strength, the failure envelope in Mohr space enters the  
322 field of tensile stress, and in general it curves as it does so (Fig. 9). The intercept of the  
323 failure envelope on the vertical axis is the cohesion; and the intercept on the horizontal axis is  
324 the tensile strength.

325 In such a cohesive material, the fluid overpressure may become greater than the weight of  
326 overburden. The seepage force provides a lift. The weight compensates for part of it. The  
327 remainder causes a tensile stress and consequent elastic stretching. As the overpressure  
328 increases, the Mohr circles for effective stress become progressively larger, but this time in  
329 the tensile field. The least stress is now in absolute tension and it acts vertically. It is  
330 proportional to the greatest stress, according to the relationship  $-\sigma'_3 = -k_y \sigma'_1$ . Thus the  
331 greatest stress is also tensile, but it acts horizontally.

332 Eventually, the Mohr circle may touch the failure envelope. If it does so at a point on the  
333 horizontal axis, the rock will fail in tension, producing a horizontal open fracture. For this to  
334 happen, the tensile strength should be smaller than the cohesion. Under these conditions, the  
335 fluid overpressure is equal to the overburden weight plus the tensile strength. This provides a  
336 simple and rather general explanation for the origin of beef.

337       Alternatively, if the Mohr circle touches the failure envelope at points above or below the  
338 horizontal axis, the rock will fail in mixed mode (shear and dilation), producing a conical  
339 fracture. For this to happen, the cohesion should be smaller than the tensile strength. This  
340 result may explain the origin of some cone-in-cone structures.

341

342

### 343 **Discussion**

344

345       According to the above analysis, large vertical seepage forces may cause the principal  
346 effective stresses to become tensile. Because the least effective stress is vertical, fractures  
347 may form in horizontal orientations. This result is simple and quite general, in the sense that  
348 it does not appeal to any particular tectonic setting. On the contrary, it assumes that stresses  
349 are lithostatic and that vertical forces are due to gravity and fluid flow. According to this  
350 analysis, it should be no surprise to find beef in sedimentary basins, even if the rock is  
351 isotropic and tectonic stresses are negligible.

352       If tectonic stresses do come into play, then their effects **will add to** those of seepage  
353 forces. **Thus** one would expect to find beef in a compressional setting. In contrast, in an  
354 extensional or strike-slip setting, where the least effective stress is horizontal, one would  
355 expect to find vertical veins.

356       The beef in the Liassic shale of SW England is parallel to bedding. If it formed in a  
357 horizontal attitude during early burial, when the tectonic context of the Wessex Basin was  
358 extensional (Underhill and Stoneley, 2003), this could mean that seepage forces were more  
359 significant than tectonic forces. Alternatively, the beef may have formed in a later context of  
360 Cretaceous or Tertiary compression, when the Wessex Basin became inverted. Clearly it



361 would be of great interest to be able to date the formation of this beef – and of other beef in  
362 general.

363 To summarize this section, strong overpressure is capable of generating seepage forces as  
364 large as, or larger than, the weight of overburden. Thus seepage forces should not be  
365 neglected in mechanical analyses of failure.

366

367

## 368 **Conclusions**

369

370 By considering elastic behaviour, followed by yield, we have shown that bedding-parallel  
371 veins of fibrous calcite or other minerals (beef and cone-in-cone) may form in an isotropic  
372 rock, under lithostatic conditions, if there is enough overpressure. This result cannot be  
373 obtained from Von Terzaghi's concept of effective stress in its simplest form, as carried by  
374 most geological textbooks and articles. Instead, it is necessary to invoke seepage forces,  
375 which inevitably exist during Darcy flow. The vertical seepage forces that accompany  
376 upward flow, in response to an overpressure gradient, counteract the weight of overburden  
377 and may even surpass it, generating a vertical tensile stress in the solid framework. If  
378 horizontal seepage forces are negligible, the value of the horizontal effective stress results  
379 only from the elastic proportionality between principal stresses. Thus the horizontal stress is  
380 also tensile. However, it is smaller in magnitude than the vertical stress. If the rock fails in  
381 tension, the fractures will therefore be horizontal. This provides a simple but universal  
382 explanation for the occurrence of beef in tectonically inactive settings. If the rock fails in  
383 shear, the fractures will be conical and dilatant. This may provide an explanation for cone-in-  
384 cone structures.

385 Experiments on cohesive granular materials, under lithostatic conditions, where the only  
386 forces are due to gravity and overpressure, bear out the theoretical predictions.

387 The above results may change if there are additional stresses of tectonic origin. Beef is to  
388 be expected in a compressional context, less so in an extensional or strike-slip context. Beef  
389 is also more likely to occur if the rock is mechanically anisotropic, or has internal surfaces of  
390 weakness in tension.

391

392

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394

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397 Southampton Univeristy for showing us some of the best exposures of beef in SW England.

398

399

### 400 **Appendix. Steady vertical flow of a pore fluid**

401

402 In this Appendix, we derive the equations for pressure profiles due to steady Darcy flow  
403 of a pore fluid in a vertical direction, including a source term.

404 In one (vertical) dimension, Darcy’s Law for steady fluid flow through a porous medium  
405 takes the form:

406

$$407 \quad q = - (k/\mu) (dp/dz - \rho g) \quad (A1)$$

408

409 Here  $q$  is the macroscopic rate of vertical flow or Darcy velocity (positive downwards),  $k$  is  
 410 the intrinsic permeability,  $\mu$  is the viscosity of the pore fluid,  $p$  is the fluid pressure,  $z$  is the  
 411 vertical distance (positive downwards),  $\rho$  is the density of the pore fluid, and  $g$  is the  
 412 acceleration due to gravity. Equation A1 is analogous to the steady-state equation for one-  
 413 dimensional heat conduction (Carslaw and Jaeger, 1959), pressure taking the place of  
 414 temperature.

415 If the Darcy velocity, density, viscosity, and permeability are all invariant with depth,  
 416 equation A1 integrates to:

417

$$418 \quad p = \rho g z - (q\mu/k)z \quad (A2)$$

419

420 This is the equation of a straight line (Fig. 2A). It is analogous to a linear thermal gradient in  
 421 the Earth's crust (Carslaw and Jaeger, 1959). In equation A2, the first term on the right  
 422 represents a hydrostatic gradient, and the second term represents an overpressure gradient.  
 423 For example, in equation A1, take  $(dp/dz - \rho g) = 2 \times 10^3 \text{ Pa m}^{-1}$ . If the fluid viscosity is  $\mu =$   
 424  $10^{-3} \text{ Pa s}$  (the viscosity of water) and the intrinsic permeability is  $k = 10^{-17} \text{ m}^2$  ( $10^{-5}$  darcy),  
 425 the Darcy velocity will be  $q = 2 \times 10^{-11} \text{ m s}^{-1}$  (about 0.6 mm/a).

426 If there is a jump in permeability, for example from layer A to layer B, the pressure  
 427 gradient also jumps. However, the Darcy velocity must be identical in both layers. Therefore  
 428 from equation A1 the ratio of pressure gradients in layers A and B is equal to the inverse  
 429 ratio of their permeability values:

430

$$431 \quad (dp/dz)_A / (dp/dz)_B = k_B / k_A \quad (A3)$$

432

433 This implies that a pressure ramp will occur in a layer of small permeability. For example,  
 434 take the same values as before for the overpressure gradient, the fluid viscosity, the intrinsic  
 435 permeability of an overburden, and the Darcy velocity, but insert a sealing layer, 13 times  
 436 less permeable (Fig. 2B). The pressure then ramps to a lithostatic value at the base of the  
 437 sealing layer.

438 Now consider the additional possibility that some fluid generates (or vanishes) within  
 439 the material. The vertical velocity therefore varies with depth:

440

$$441 \quad dq/dz = - Q \quad (A4)$$

442

443 Here  $Q$  represents a source (or sink). It measures a rate of increase (or decrease) in volume,  
 444 per unit initial volume.

445 On deriving equation A1 with respect to  $z$  and substituting equation A4, we obtain:

446

$$447 \quad d^2p/dz^2 = Q\mu/k \quad (A5)$$

448

449 If the source term is invariant with depth, equation A5 integrates to:

450

$$451 \quad dp/dz = \rho g - q\mu/k + (Q\mu/2k)z \quad (A6)$$

452

453 Here equation A1 has provided the constant of integration.

454 Finally, if all parameters except the pressure are constant, equation A6 integrates to:

455

$$456 \quad p = \rho g z - (q\mu/k)z + (Q\mu/2k)z^2 \quad (A7)$$

457

458 This is the equation of a parabola (Fig. 2C). It is analogous to the equation for production and  
459 flow of heat in the Earth's crust (Carslaw and Jaeger, 1959). In equation (A7), the source and  
460 the permeability both contribute to a quadratic term in  $z$  and their ratio determines the  
461 sharpness of the resulting pressure peak. For example, take the same values as before for the  
462 fluid viscosity, and for the overpressure gradient, intrinsic permeability, and Darcy velocity  
463 in the overburden. Now assume that the fluid comes from an underlying source layer, which  
464 is 2 km thick and 40 times less permeable than the overburden (Fig. 2C). From equation A7,  
465 the rate of fluid generation in the source layer will be  $Q = 2 \times 10^{-14} \text{ s}^{-1}$ . If the upper half of the  
466 source layer accounts for upward flow through the overburden, and the lower half accounts  
467 for downward flow through the substrate, which is taken to be a sink, then the fluid pressure  
468 reaches a lithostatic value about half way up the source layer.

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577 **Figure captions**

578

579 Fig. 1. A. White veins of fibrous calcite (“beef”) within black shale of the “Shales-with-  
580 Beef”, Lyme Regis, Dorset Coast, SW England. B. Nested cones of fibrous calcite (“cone-in-  
581 cone”) from same locality.

582 Fig. 2. Theoretical profiles of fluid overpressure due to Darcy flow. For equations and  
583 parameters used in calculations, see Appendix.

584 A. Linear profile due to uniform upward flow through medium of uniform permeability.

585 Fluid comes from below. Its pressure gradient is greater than hydrostatic, but smaller than

586 lithostatic. B. Pressure ramp due to uniform upward flow through sealing layer. Fluid comes

587 from below. Pressure reaches lithostatic value at base of seal. C. Parabolic hump due to fluid

588 generation within source layer. No fluid comes from below. Pressure reaches lithostatic value

589 about half way up source layer.

590 Fig. 3. Horizontal hydraulic fractures due to upward fluid flow and resulting seepage forces

591 in pack of powdered silica. Container is cylinder of transparent plastic. Horizontal fractures

592 connect to vertical channels near upper surface of powder.

593 Fig. 4. Idealized vertical section through homogeneous column of rock, containing horizontal

594 fracture. Acting on each element of solid framework are force of gravity,  $F_g$ , and seepage

595 force,  $F_s$ . Sides of column are stationary, rigid and frictionless.

596 Fig. 5. Mohr circle for stress and linear Coulomb failure envelope for non-cohesive material.

597 Fig. 6. Mohr circles and envelopes for elastic behaviour or for yield, assuming a minimum

598 stress of 100 MPa.

599 Fig. 7. Reduction in state of effective stress due to overpressure,  $P$ . Mohr circle shifts to left

600 by an amount equal to  $P$ .

601 Fig. 8. Influence of increasing overpressure on position and size of Mohr circle for effective  
602 stress (after Mourgues and Cobbold, 2003).

603 Fig. 9. Influence of fluid overpressure in a material that has true tensile strength. As  
604 overpressure increases, Mohr circles for effective stress become progressively smaller in  
605 compressive field, then larger again in tensile field. Mohr circle may touch failure envelope  
606 at point on horizontal axis. Least principal effective stress is then tensile and vertical, and  
607 failure results in horizontal fracture.

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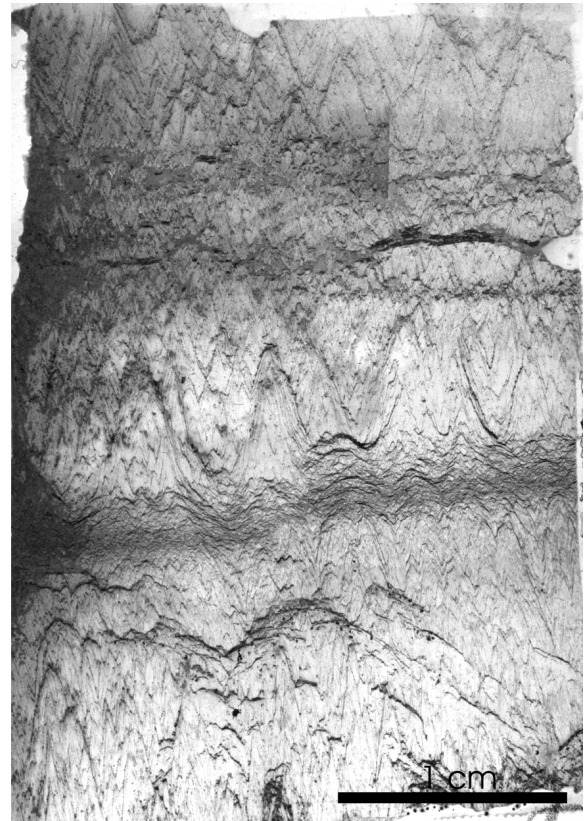
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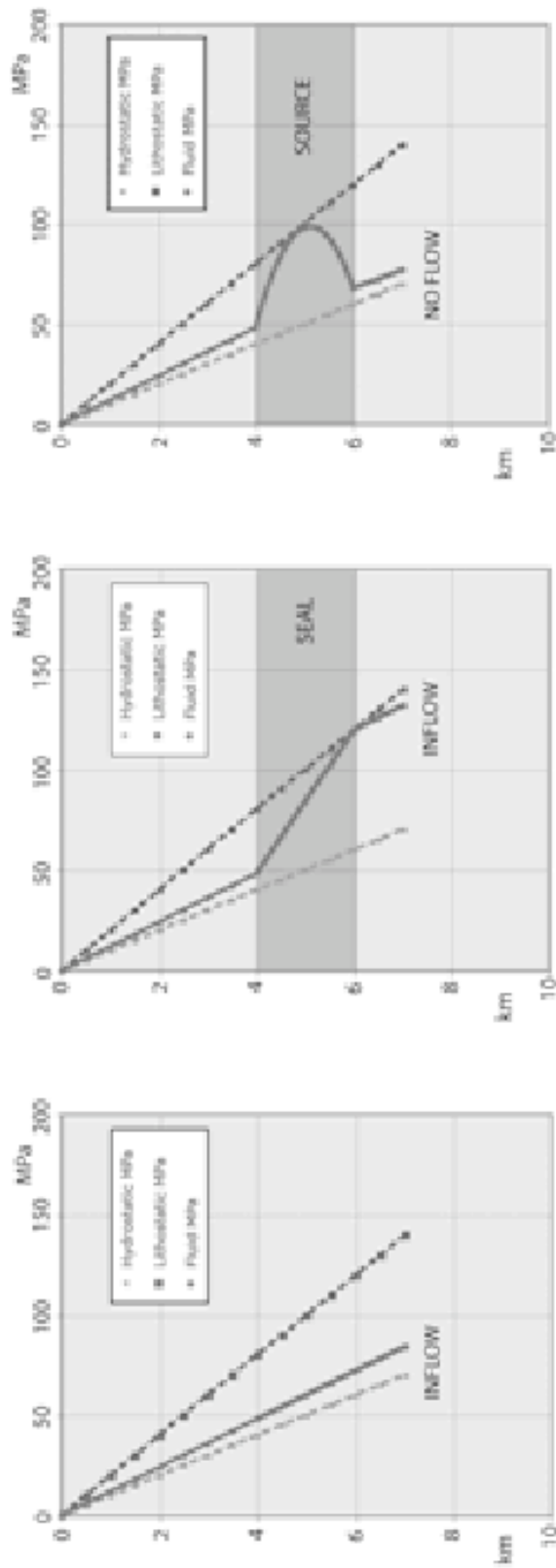
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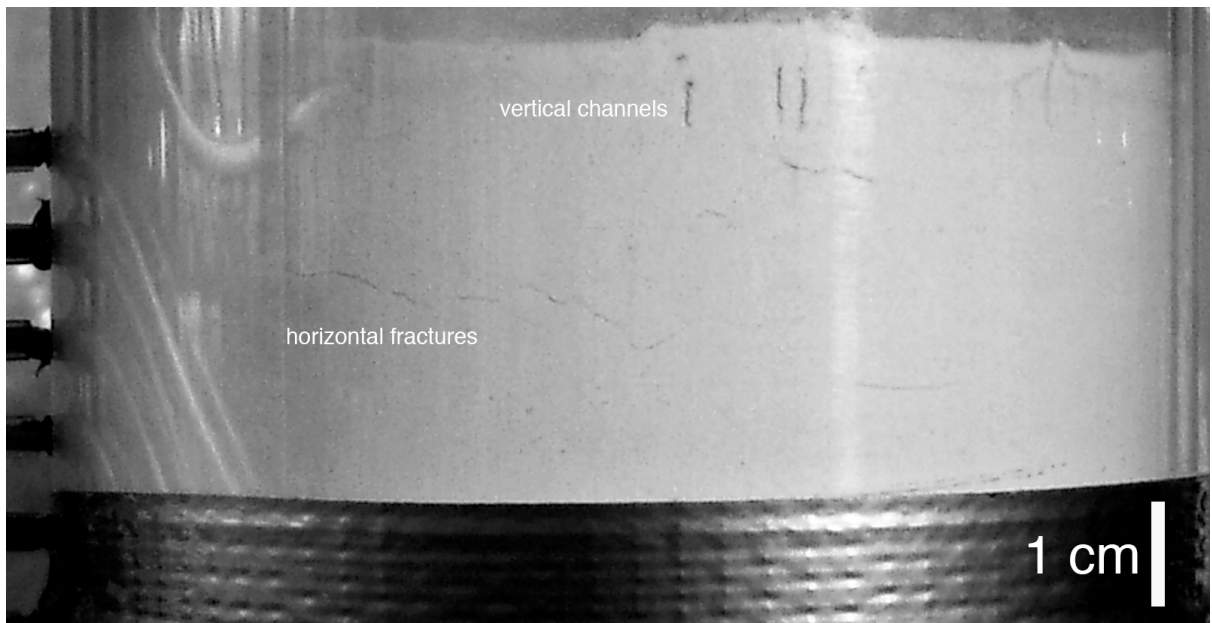
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630 Figure 2



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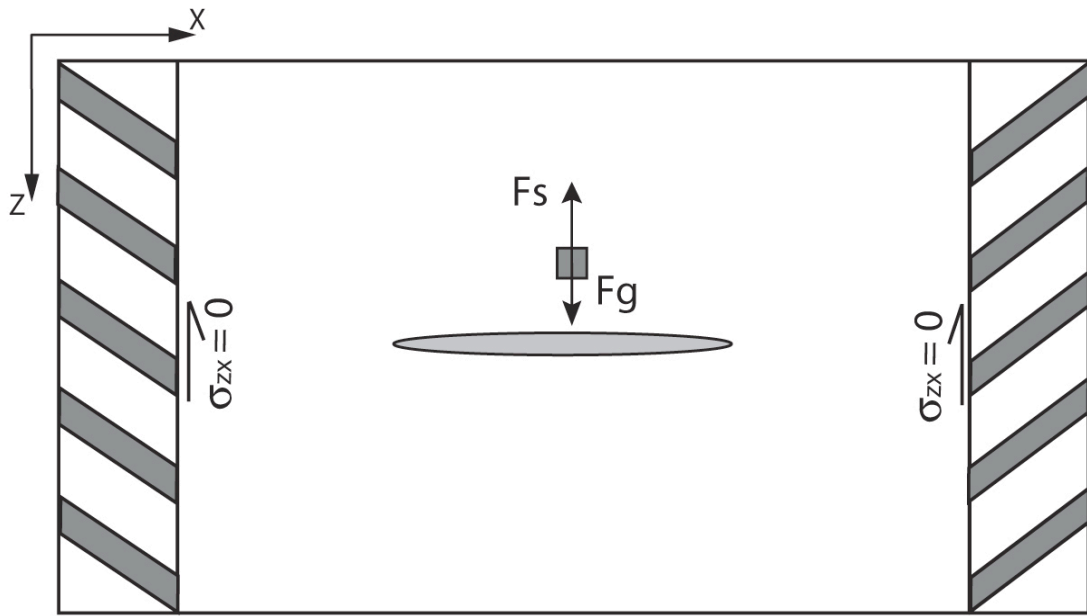
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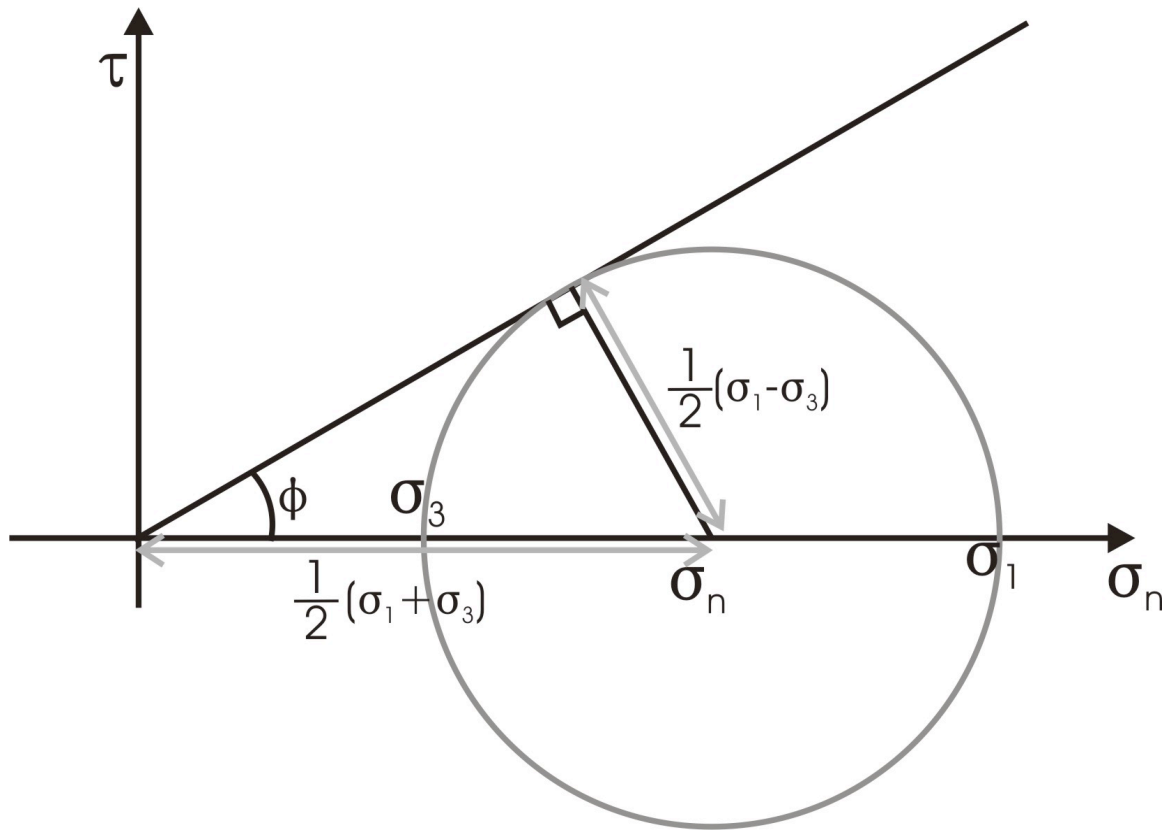
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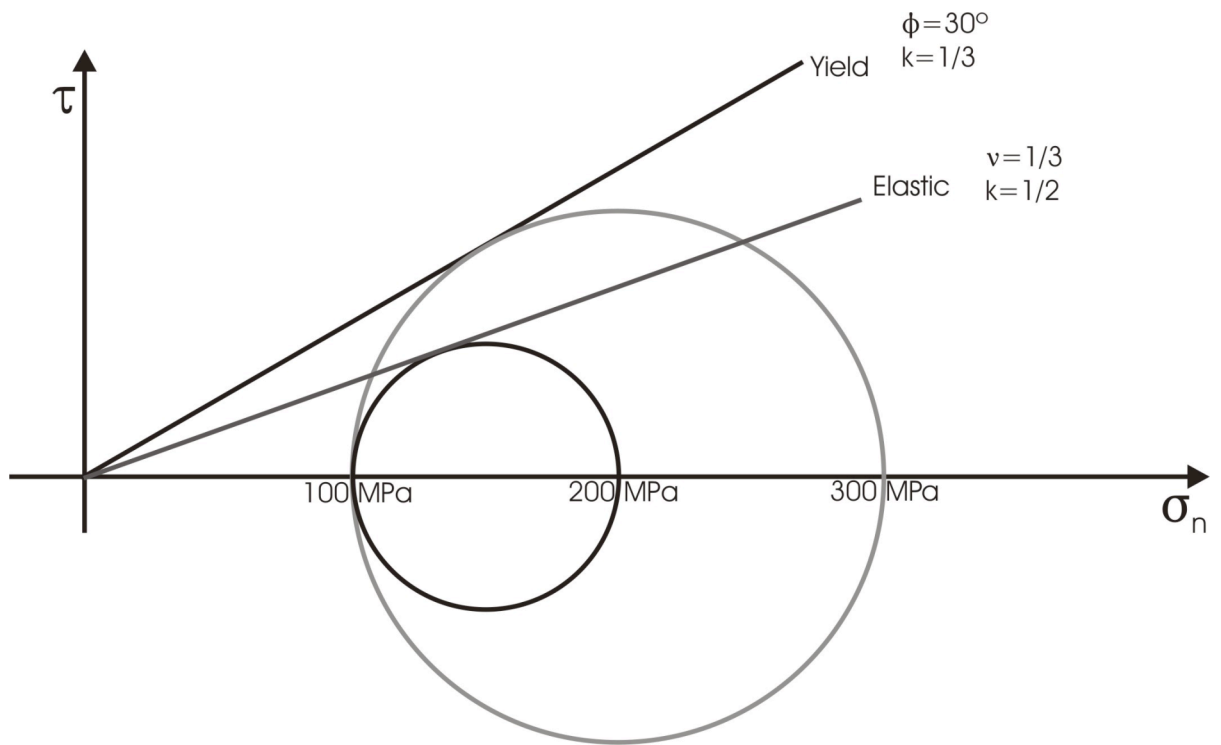
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663 Figure 6

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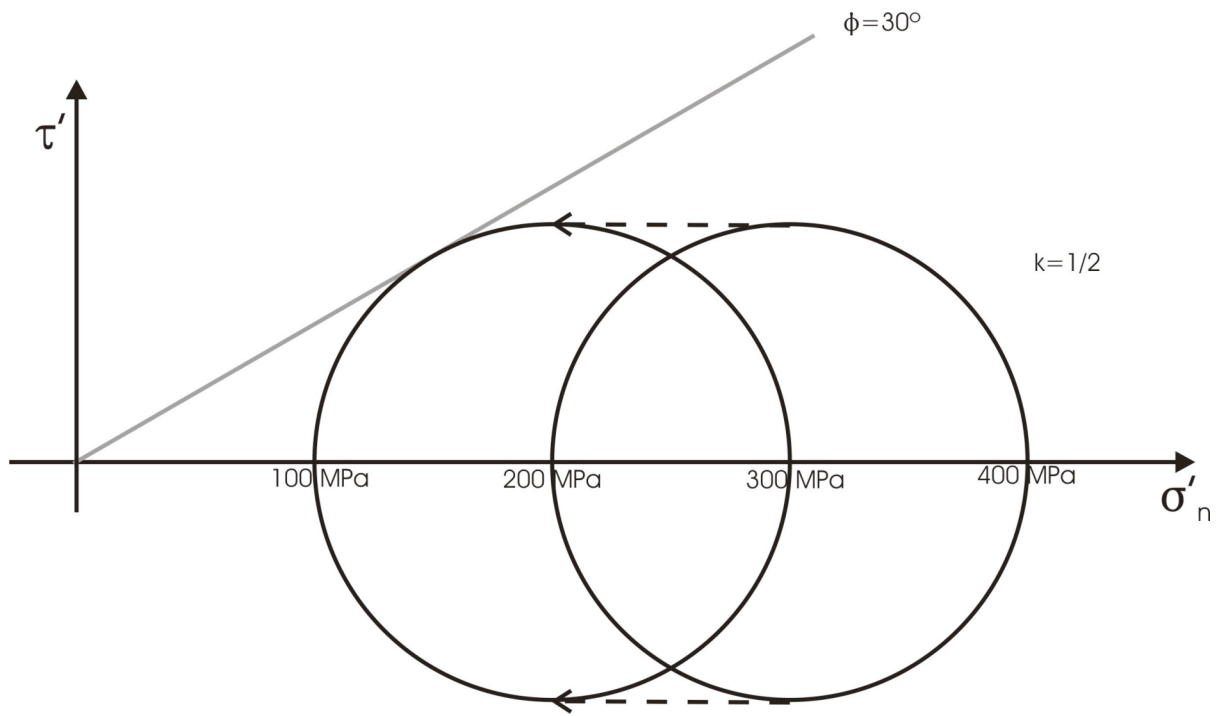
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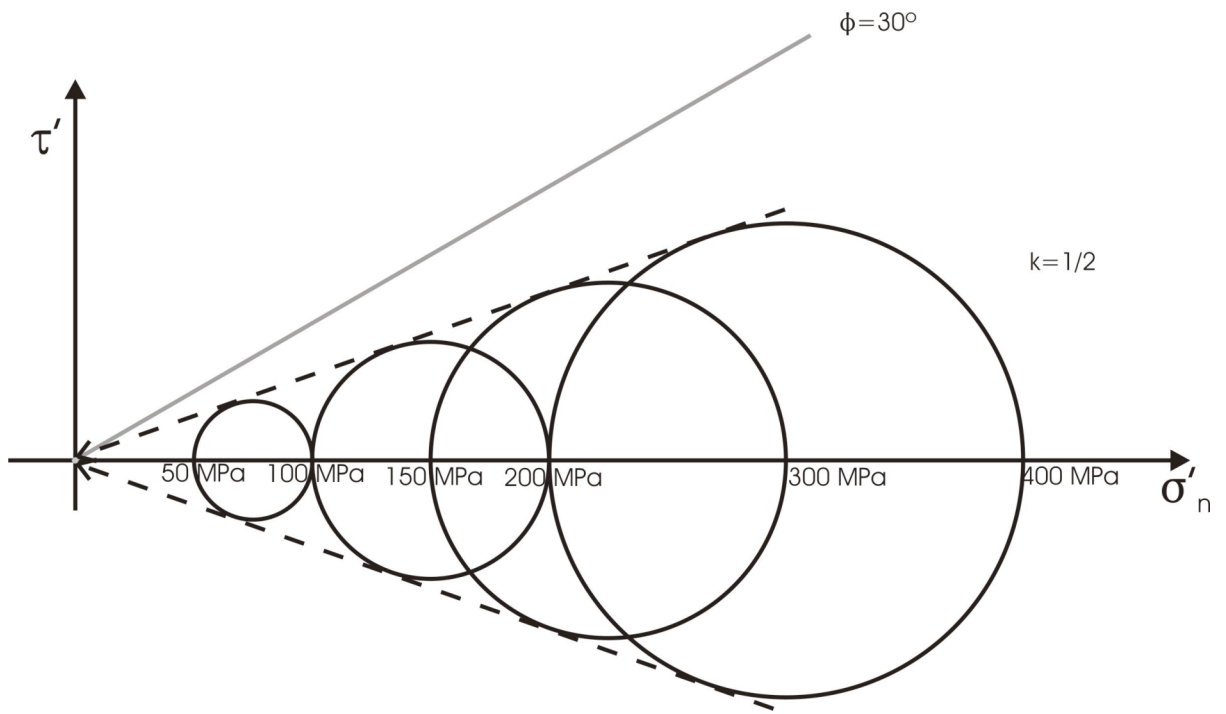
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677 Figure 7

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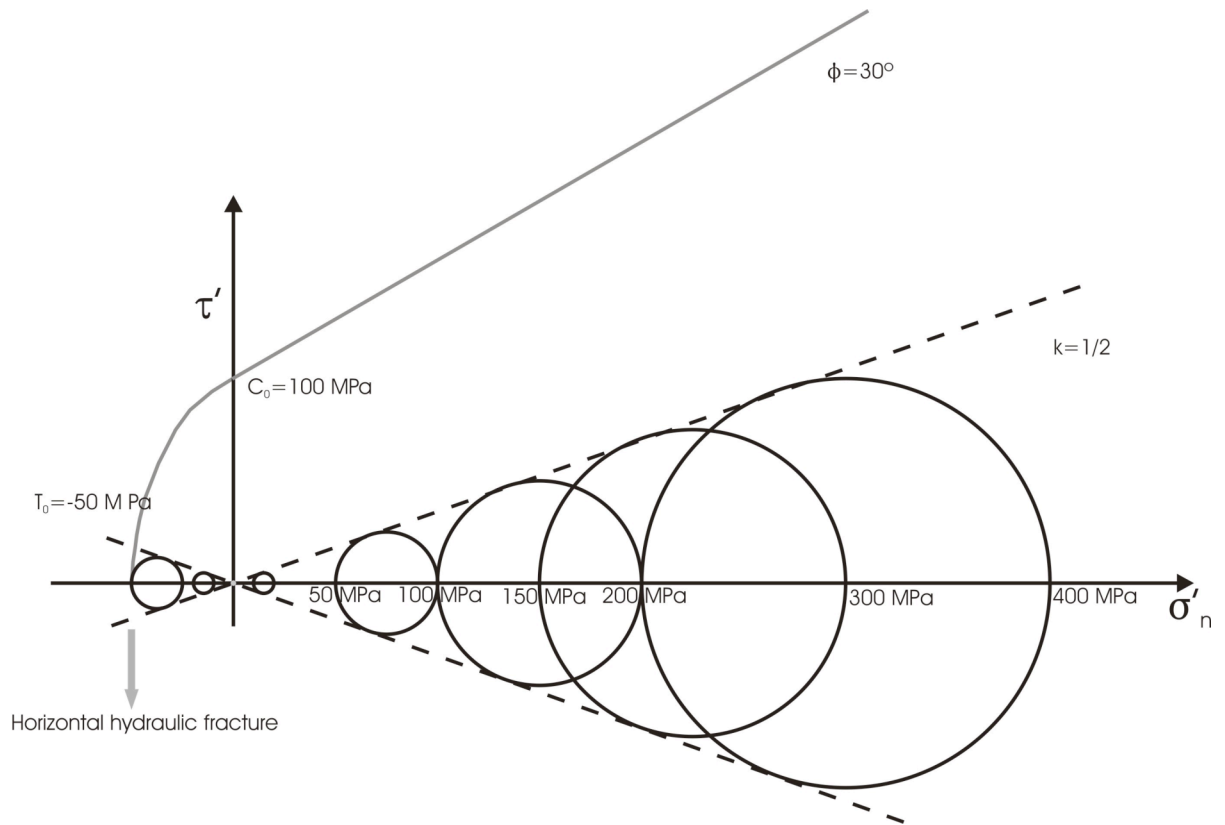
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Figure 8



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698 Figure 9