



HAL
open science

Intrinsic structure of acoustic emission events during jerky flow in an Al alloy

M.A. Lebyodkin, T.A. Lebedkina, F. Chmelík, T.T. Lamark, Y. Estrin, C. Fressengeas, Jérôme Weiss

► **To cite this version:**

M.A. Lebyodkin, T.A. Lebedkina, F. Chmelík, T.T. Lamark, Y. Estrin, et al.. Intrinsic structure of acoustic emission events during jerky flow in an Al alloy. *Physical Review B: Condensed Matter and Materials Physics (1998-2015)*, 2009, 79 (17), pp.174114. 10.1103/PhysRevB.79.174114. insu-00420864

HAL Id: insu-00420864

<https://insu.hal.science/insu-00420864>

Submitted on 11 Mar 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution 4.0 International License

Intrinsic structure of acoustic emission events during jerky flow in an Al alloy

M. A. Lebyodkin,^{1,*} T. A. Lebedkina,² F. Chmelík,³ T. T. Lamark,⁴ Y. Estrin,^{5,6} C. Fressengeas,¹ and J. Weiss⁷

¹Laboratoire de Physique et Mécanique des Matériaux, Université Paul Verlaine-Metz/CNRS, Ile du Saulcy, 57045 Metz, France

²Institute of Solid State Physics, Russian Academy of Sciences, 142432 Chernogolovka, Russia

³Department of Physics of Materials, Charles University, Ke Karlovu 5, CZ-121 16 Prague 2, Czech Republic

⁴Bergen Engineering AS, N-5862 Bergen, Norway

⁵ARC Centre of Excellence for Design in Light Metals, Clayton, Victoria 3217, Australia

and Department of Materials Engineering, Monash University, Clayton, Victoria 3800, Australia

⁶CSIRO Division of Materials Science and Engineering, Clayton South, Victoria 3169, Australia

⁷Laboratoire de Glaciologie et Géophysique de l'Environnement/CNRS, 54 rue Molière,

Boîte Postale 96, 38402 St. Martin d'Hères, France

(Received 18 February 2009; published 22 May 2009)

Scaling behavior is found in acoustic-emission events associated with stress drops observed in velocity-driven plastic deformation of an Al alloy, which exhibits jerky plastic flow. The occurrence of scaling proves that these acoustic-emission events, which are commonly regarded as “elementary” ones, have a small-scale self-organized structure comprising a group of peaks correlated in time. This structure reveals details of the temporal variation in elementary plastic events at a microsecond scale, which are hardly accessible by other measurement techniques.

DOI: [10.1103/PhysRevB.79.174114](https://doi.org/10.1103/PhysRevB.79.174114)

PACS number(s): 62.20.F-, 05.65.+b, 45.70.Ht, 43.60.+d

I. INTRODUCTION

Jerky flow, or discontinuous yielding, of alloys during plastic deformation has been observed in many dilute alloys. Commonly referred to as the Portevin–Le Chatelier effect (PLC), this phenomenon is governed by the dynamic interaction of dislocations with solute atoms¹ and it is a striking example of the complexity of the spatiotemporal dynamics arising from the collective behavior of defect populations.² In a uniaxial tensile test with a constant imposed overall strain rate (in practice a constant driving velocity), the effect manifests itself as a series of serrations on the stress vs time or stress vs strain curve, associated with bursts of strain rate in the form of localized deformation bands and accompanied with acoustic emission (AE).³ Dynamical⁴ and statistical^{5,6} methods were applied to characterize the intermittency of the stress-time series. Depending on the driving velocity, complex dynamic regimes, such as chaotic dynamics and scale-free critical dynamics,^{6,7} were identified, suggesting self-organization and strong spatiotemporal correlations. Various observations indicate that this behavior has a general character. Specifically, critical-like statistics were observed for jerky flow in pure metals, related to reduction of the specimen size to micron-level scales⁸ or caused by low-temperature catastrophic glide^{9,10} and twinning.⁹ Such statistics were also detected at finer scales by measuring electrical signals accompanying the stress drops.^{9,11} By contrast, the macroscopically smooth plastic deformation of crystalline solids is conventionally viewed as a homogeneous plastic “flow.” However, the fact that plastic deformation can be regarded as a continuous flow only in an approximate sense has been recognized for a long time.¹² In recent years, it has been demonstrated by high-resolution extensometry experiments at large time scales (10^{-1} – 10^2 s) (Ref. 13) and AE at smaller time scales (1 ms–1 s) (Refs. 14–16) that such seemingly “regular” plasticity is also intermittent although the

magnitude of the local strain-rate jumps is much smaller than in jerky flow under the PLC regime. These results suggest that regular plasticity also results from a scale-free correlated avalanche-like collective motion of dislocation ensembles.^{13,17,18}

The very occurrence of AE is itself evidence of a collective character of the motion of dislocations,¹³ as opposed to motion of a single dislocation, which hardly produces any measurable acoustic response. However, in classical AE analysis, the recording system automatically individualizes AE “events” whose duration is defined as the time interval over which the signal remains above a user-defined threshold value. Such events are viewed as being elementary and corresponding to a “single” dynamic event, referred to as a dislocation avalanche. The possibility that the avalanche itself might be composed of independent plastic-glide events or a main event with after effects, reflecting a shorter-scale self-organization of dislocation motion, at time scales below 1 ms, remains unexplored. Such large-scale “discretization” of the dynamics into dislocation avalanches is to be questioned, as the underlying assumptions may impact on the scaling laws derived at coarser time scales.

In this paper, we explored intermittency of plastic deformation over the time scales 1 μ s–1 ms by recording AE accompanying weakly correlated PLC bands that occur in alloys driven at slow velocities, close to the lower bound of the PLC range of strain rates (Fig. 1). Such PLC bands are referred to as type C bands.^{1,2} The appeal of such conditions for the present investigation derives from the strong collective effects associated with large stress-drop magnitude and from a low degree of correlation between the stress drops separated by large time intervals.¹⁹ Both the stress-drop magnitude and the width of localized deformation bands (usually, 0.1–1 mm) bear witness to a large number of dislocations involved. As seen in Fig. 1, large stress drops are separated in time by seconds while their duration is of the order of 0.05–0.1 s. At higher strain rates, the bands propagate along

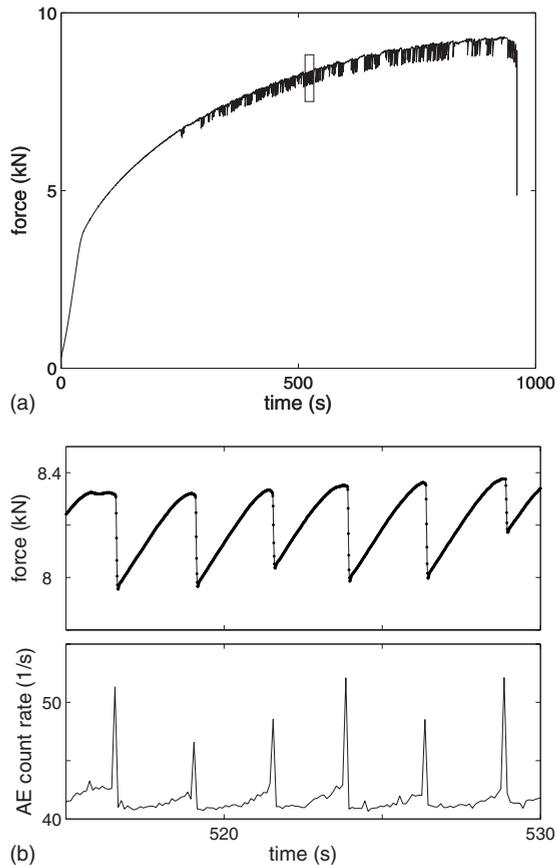


FIG. 1. (a) Example of a deformation curve for a strain rate of $2.9 \times 10^{-4} \text{ s}^{-1}$. The data-acquisition time was 0.01 s. (b) Correlation of a portion of this curve, marked by a rectangle, with AE event rate.

the sample quasicontinuously, suggesting the presence of long-range spatial correlations.^{6,7}

Although the results reported in the present paper concern a particular case of AE bursts caused by sudden localized dislocation glide, the question of elementary AE events presents a more generic interest because burstlike AE is known to accompany various processes associated with catastrophic microstructure variation, for example, twinning,^{16,20} fracture,²¹ and martensitic transformations.²² Some hints to the possible nonrandom short-time structure of AE events may be found in earlier investigations. In particular, the visual signatures¹⁶ and the Fourier spectra²⁰ of individual AE wave forms were successfully applied to distinguish between the AE caused by twinning and dislocation glide. The observation of electrical signals during the low-temperature jerky flow of pure metals should also be mentioned in this connection because it clearly revealed a complex structure of stress drops at time scales below 1 ms.⁹

The paper is organized as follows. Section II describes experimental details and the object of the analysis. Section III outlines experimental observations of burstlike acoustic signals. Section IV deals with the main focus of the present investigation—the quantitative analysis of the observed signals based on the multifractal analysis technique. It is shown that, alongside random noise and oscillatory components due to sound propagation and reflection from the specimen sur-

faces, the individual wave forms recorded during a PLC instability also contain a correlated component that results in scaling laws. Hypotheses regarding the physical mechanisms associated with such correlations are put forward in the concluding part of the paper, Sec. V.

II. EXPERIMENTAL

In this paper, AE was measured during straining of polycrystalline samples of an Al-Cu alloy (AA2024) exhibiting jerky flow.²³ The samples with gauge length, width, and thickness of 57, 12.5, and 2.1 mm, respectively, were annealed at 493 °C for 40 min, quenched in polyalkylene glycol solution, and were either tested immediately or kept at $T \leq 240$ K before testing to avoid static aging. Uniaxial tensile tests were conducted at room temperature with constant crosshead velocity corresponding to the initial strain-rate range: $\dot{\epsilon}_a = 2.9 \times 10^{-5} - 2.9 \times 10^{-2} \text{ s}^{-1}$. A computer controlled DAKEL-XEDO-3 system²⁴ was used to record AE (see Ref. 3). A miniaturized MST8S transducer (diameter 3 mm) with nearly flat response in a frequency band from 100 to 600 kHz and sensitivity of 55 dB was attached to the sample surface with the help of silicon grease and a spring. The total gain and sampling rate were, respectively, 92 dB and 4 MHz. The equipment allowed for recording wave-form samples, i.e., individual acoustic signals containing 10 000 data points over the total duration of 2.5 ms. In order to avoid losing information on the initial portion of the signal, the saved data contained about 1000 points stored before the continuously measured AE voltage exceeded a preset threshold of 1695 mV. Such wave forms were then evaluated using multifractal analysis. In addition, several standard AE parameters, in particular, the AE count rates were recorded during the whole deformation test in a way similar to that reported in Ref. 3. This made it possible to verify the relationship between the PLC stress drops and the AE activity. In this study, the threshold voltage for the identification of the starting and the end points of an AE event was set at 1695 and 1212 mV, respectively.

III. EXPERIMENTAL RESULTS

AE was recorded in the whole range of strain rates used but jerky flow was found only for $\dot{\epsilon}_a \geq 10^{-4} \text{ s}^{-1}$. It is known that, for type C bands, the bursts in the AE count rate are clearly associated with stress drops (cf. Ref. 3). As seen in Fig. 1(b), a similar correspondence was also found for the alloy investigated although synchronization with wave-form samples could not be checked accurately. In addition, the stress-drop duration (0.05–0.1 s) is close to the estimated response time of the testing machine, which thus limits the measurement accuracy during the drops.

Figure 2 shows some typical examples of the observed acoustic signals. Such wave forms were recorded during the entire duration of the jerky flow, which is consistent with the fact that the deformation process has a qualitatively steady-state character at low-strain rates with stress drops in Fig. 1(a) consistently exhibiting a characteristic C-type pattern.^{1,2} It can be seen from Figs. 2(a) and 2(b) that AE events may

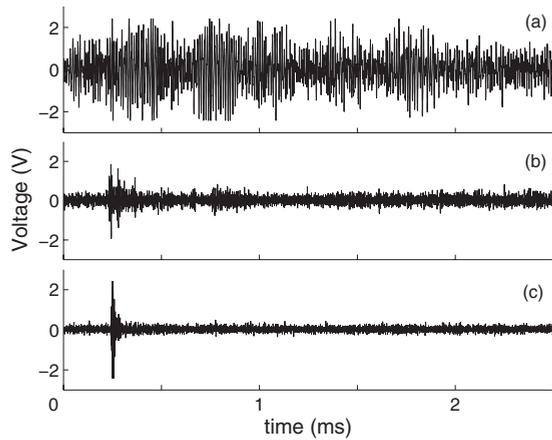


FIG. 2. Examples of acoustic emission events observed during the jerky flow.

involve a large number of individual bursts. In Fig. 2(c) an apparently singular burst, albeit involving about 15 irregular oscillations, is discernible from a noisy background, which possibly includes some lower-amplitude bursts. It is worth recalling that, in all cases, the total interval captured was just 2.5 ms, well below the duration of a single stress drop. That is to say, the microsecond scale bursts captured in Fig. 2 should not be confused with the “bursts” in the AE intensity [Fig. 1(b)], reflecting a sequence of stress drops and corresponding essentially to the larger time scales which are commonly discussed in connection with AE experiments.

It should be noted that the period of the oscillations composing a single burst (about 2–5 μs) is much larger than the estimated travel time of sound waves across the sample thickness. With velocity of sound close to 5200 m/s in polycrystalline Al, the latter amounts to 0.8 μs . This value is of the order of the characteristic duration of the finest fluctuations observed in the signals. Note however that the flat response of the transducer is limited to 1.7 μs , i.e., 600 kHz. Thus, the singular AE event in Fig. 2(c) must involve a complex combination of bursts due to dislocation glide and subsequent elastic wave reflections. Hence, the data in Fig. 2 raise the following questions, which are addressed in the rest of the paper: (i) Are there hidden correlations in the burst sequences in Figs. 2(a) and 2(b)?; (ii) Is the background signal in Fig. 2(c) random?; (iii) Are apparently singular bursts like the one shown in Fig. 2(c) elementary events?

IV. MULTIFRACTAL ANALYSIS OF ACOUSTIC SIGNALS

The presence of bursts in AE signals results in broad Fourier spectra with some harmonics being possibly associated with elastic reflections. Therefore, spectral analysis is not relevant for their quantitative characterization. Faced with similar problems with stress drops, previous investigations of jerky flow^{7,19,25,26} have used multifractal analysis,²⁷ whose robustness with regard to random or oscillatory noise was proven.^{19,26} In the present work, multifractal analysis is also applied to the AE signal, $V(t)$. It should be emphasized again that, while in Refs. 7 and 25 the time scales of the stress drop

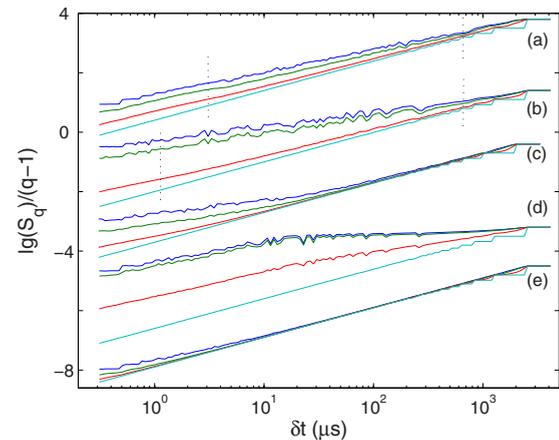


FIG. 3. (Color online) Partition functions for the AE signals in: (a) Fig. 2(a), (b) Fig. 2(b), (c) Fig. 2(c) as the latest portion between 0.5 and 2.5 ms, (d) Fig. 2(c) as the entire record, and (e) random noise. The moment q takes on the values 40, 5, 1, and 0 for each set from the upper curve downward. The sets are deliberately shifted along the ordinate axis to avoid superposition. The dotted vertical lines show the δt intervals used to compute the multifractal spectra.

series extend from seconds to hours, the AE events investigated in the present study occur at a millisecond time scale. Data processing was based on a procedure similar to that used in the analysis of deformation curves in Refs. 19 and 26. For this purpose, the time axis is subdivided into intervals δt . A local probability measure $\mu_i(\delta t)$ of the i th interval is defined as the area under the curve $y(t)=V^2(t)$ within the interval, normalized by the total area calculated over the entire time. Hence, this measure reflects the time distribution of AE energy released by dislocation avalanches. The overall sum of μ_i values equals unity, consistent with the definition of probability. In practice, a proxy for the area is the sum of all $y(t)$ magnitudes within the interval. Alternative choices, e.g., $y(t)=\text{abs}[V(t)]$, were also used to check the robustness of the observed scaling behavior. This resulted in redistribution of scaling indices but did not affect the qualitative trends. The generalized dimension spectrum, $D(q)$, $q \in \mathbb{Z}$, is computed by constructing the family of partition functions $S_q(\delta t) = \sum_i \mu_i^q$ for different values of q , where the sum is taken over all time intervals. By definition, the generalized fractal dimension $D(q)$ characterizes scale-invariant behavior when $S_q(\delta t) \sim \delta t^{(q-1)D(q)}$ as $\delta t \rightarrow 0$ (see Fig. 3). Large values of the measure dominate in $S_q(\delta t)$ for large positive q values, whereas small measures correspond to large negative q values. A varying $D(q)$ function reflects multiscale time correlations in the underlying physical process, a property referred to as “multifractality.” A complementary, and equivalent, representation is also possible in terms of the singularity spectrum, $f(\alpha)$, related to $D(q)$ via Legendre transformation. Here, α reflects local singularity of the measure and $f(\alpha)$ represents the fractal dimensions of the subsets of singularities with strength α .²⁷ In this paper, the singularity spectrum was calculated using a direct method²⁸ thus avoiding the inaccuracies resulting from the derivatives in the Legendre transform.

The partition functions computed from the signals in Figs. 2(a) and 2(b) are shown in Figs. 3(a) and 3(b), respectively.

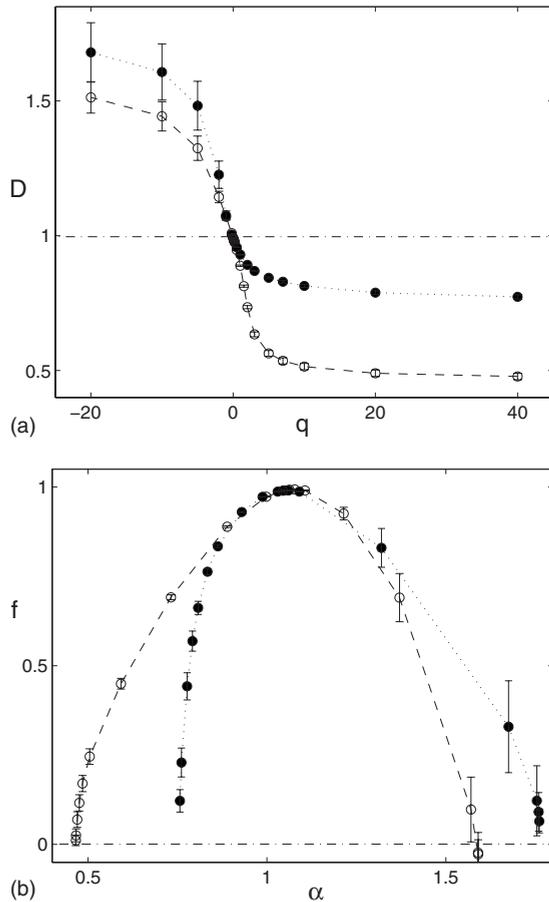


FIG. 4. (a) Generalized dimensions spectra $D(q)$ and (b) singularity spectra $f(\alpha)$ for the acoustic events exhibiting a complex temporal structure. Solid symbols represent spectra for the signal shown in Fig. 2(a); open symbols correspond to Fig. 2(b). The dashed-and-dotted line in Fig. 4(a) designates the slope equal to one in a scaling relation, $D=1$.

They clearly exhibit straight lines over large time intervals on the logarithmic scale with q -dependent slopes. This proves both the scaling behavior and the multifractal character of the signal. Deviations from the linear trend at large time scales are an artifact stemming from the finite size of the signal and those at small time scales are due to its discreteness. Nevertheless, the scaling regime ranges from several microseconds to about 0.5–0.8 ms. The multiscale behavior results in large multifractal spectra, as shown in Fig. 4. This is contrasted by the partition functions for a computer-generated random signal shown in Fig. 3(e) for the sake of comparison. In this case, the curves are bent at small time scales and quickly converge to the unique slope, $D(q)=1$. The same trend is observed in computer-generated periodic oscillations—another kind of signal showing uniformity at large scales [curve 1 of Fig. 5(b)]. Some qualitative arguments derived from the spectra in Fig. 4 confirm the soundness of the calculations. First, the smaller values of D_{\min} and α_{\min} , and the larger span of D and α in the range $q > 0$ (left part of the singularity spectrum) indicate stronger singularity and heterogeneity of the signal in Fig. 2(b), as compared to the more regular oscillatory signal in Fig. 2(a). Second, a finite positive value of $f(\alpha_{\min})$ is found for the latter signal,

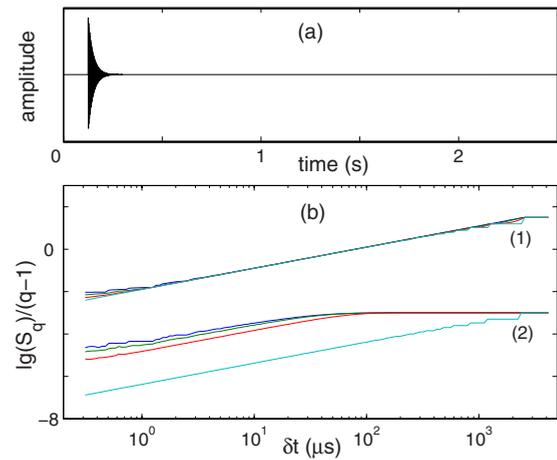


FIG. 5. (Color online) Analysis similar to that in Fig. 3, applied to oscillatory signals. (a) Wave form for a computer generated signal containing 10 000 data points and representing exponentially attenuated periodic oscillations with a period of $2.5 \mu\text{s}$, triggered at the 500th step. (b) Partition functions for the computer-generated signals: (1) periodic oscillations without attenuation; (2) decaying signal shown above.

which indicates a high density of large-scale fluctuations. This feature might also be related to a higher degree of homogeneity of this signal. These remarks suggest that the deformation processes, which give rise to the AE events shown in Figs. 2(a) and 2(b), are characterized by short-time correlations resulting in scale invariance. Such multifractal character of time evolution is often seen for processes developing in a cascade manner.²⁹

The signal in Fig. 2(c) is obviously not scale invariant because it involves two distinct scales corresponding to the short singular acoustic event and the background signal. Hence, the analysis is divided into two parts. Figure 3(c) displays the partition functions for the latest part of this signal between 0.5 and 2.5 ms, i.e., the time after the large initial burst has decayed. The curves resemble those for uncorrelated random noise albeit with slower convergence to the unit slope. This behavior may reflect correlated plastic activity at very small time scale although the scaling range is quite short. The partition functions obtained from the entire signal in Fig. 2(c), including the initial burst, are shown in Fig. 3(d). Since the amplitude of the elastic waves associated with this burst is larger than the background level, they dominate the large positive q values. In these curves, scaling behavior is observed at small time scales only. Possible occurrence of correlations at larger time scales is screened by the noisy elastic wave-related component. By contrast, curves obtained for small positive q values display scaling behavior at all time scales with slopes identical to their counterparts in Figs. 3(a) and 3(b). Note that the latter are distinctly different from the slope obtained from the random signal. Additional tests showed that trivial time correlation coming from the attenuated elastic waves following a single burst cannot explain this scaling behavior, i.e., it has no bearing on the validity of our analysis. The results of such calculation for a computer-generated noise-free attenuated periodic signal [see Fig. 5(a)] are presented in curves labeled “2”

in Fig. 5(b). In the absence of noise, the curves for all positive q values are related to the “burst” and are close to each other. It can be seen that, in contrast to the data of Fig. 3(d), all curves 2 in Fig. 5(b) are parallel at small scales except for the range close to the resolution limit where deviations are seen.

Hence, the analysis suggests that the scaling laws in Fig. 3(a) probably extend to the shortest time scales although some distortion is present in that range due to limited time resolution. Furthermore, indications of persistent correlation are found in very short acoustic events [Fig. 2(c)], despite the noise and the trivial self-correlation at ultrashort time scales associated with attenuation of the elastic waves. A systematic investigation of a large number of AE signals using an AE technique with smaller time resolution would be required to ascertain these two conjectures.

In order to judge the plausibility of the conjectures made, possible experimental artifacts should be ruled out. Indeed, the observed wave forms are inevitably influenced by different factors including the properties of the material in which sound propagates from the AE source to the receiving transducer, the characteristics of the transducer itself, and the geometry of the experimental setup, which may lead to multiple sound reflections. It is however obvious that the former factors, leading to distortion of the waveforms, may only mask existing correlations between bursts but can neither result in a spurious scale-invariant structure nor generate virtual bursts. Furthermore, structures correlated in time due to multiple sound reflections also seem very unlikely for several reasons. First, this is suggested by the comparison (see Sec. III) of the characteristic time of sound wave propagation through the specimen (microseconds) and the large time interval (up to 0.8 ms) in which the scaling behavior is observed. Second, the best multifractal scaling is found for Fig. 2(b) in which AE bursts are well separated in time. The scaling is less pronounced for the signal in Fig. 2(a), whose oscillatory character testifies to a larger contribution of sound reflections. The latter observation is consistent with the fact that no multifractal scaling would be observed for a periodic signal or for a periodic signal with exponential attenuation (Fig. 5).

V. SUMMARY AND CONCLUDING REMARKS

In conclusion, it was shown that part of acoustic events accompanying jerky flow in the Al-Cu alloy investigated has a complex multifractal character. This observation suggests the occurrence of cascadelike correlated deformation processes at a very short time scale, below 1 ms, unexplored so

far. Two major effects may lead to AE generation in this material: avalanchelike dislocation glide (e.g., nucleation of PLC bands), or breaking/decohesion of coarsened precipitates. As pointed out in Sec. II, the specimens were deformed immediately after solution treatment in order to rule out the latter factor. Several experimental observations confirm that the AE signals are related to dislocation processes: (i) a clear relationship between the bursts in the AE count rate and the stress drops obviously governed by the PLC mechanism (type C behavior^{1,2}), (ii) persistence of the observed wave forms during the jerky flow, which exhibits nearly steady state character, and (iii) similarity between these wave forms and those reported in literature for other alloys exhibiting PLC instability.³ Although quantitative analysis of short-time scale correlations was not performed in earlier works, the bulk of the literature data on the PLC effect bears evidence that the observed behavior is of general nature, at least in the case of PLC jerky flow. Moreover, our analysis raises a general question of the existence of a structure in seemingly elementary acoustic events characterizing catastrophic deformation processes of various nature, including fracture, solid-state phase transformations, thermomechanical instabilities, etc.

The time scales for the scaling behavior observed range from microseconds to milliseconds. It is not clear yet whether this short-range scaling behavior is akin to self-organization associated with the long-range internal stresses, which was discovered earlier by the analysis of entire stress-drop series over much larger time scales (exceeding those in the present study by several orders of magnitude).^{4-7,19,25,26} It may as well reflect concurrent short-range interactions occurring during dislocation motion. Recent developments of real-time AE technique,³⁰ allowing for continuous data storage during a whole tensile test, seem promising in a search for a possible link between these two distinct time-scale levels. Further progress may also come from similar investigations of the intermittency of macroscopically homogeneous plastic deformation.

ACKNOWLEDGMENTS

One of the authors (T.A.L.) would like to acknowledge support from Université Paul Verlaine-Metz and the ARCUS program (Project Lorraine/Russia) for her stays in Metz during 2007. F.C. acknowledges financial support from the Research Plan MSM under Grant No. 0021620834 financed by the Ministry of Education, Youth and Sports of the Czech Republic and from the Czech Science Foundation under Grant No. 106/07/0218.

*Corresponding author. lebedkin@univ-metz.fr

¹Y. Estrin and L. P. Kubin, *J. Mech. Behav. Mater.* **2**, 255 (1989).

²L. P. Kubin, C. Fressengeas, and G. Ananthakrishna, in *Dislocations in Solids*, edited by F. R. N. Nabarro and M. S. Duesbery (Elsevier, Amsterdam, 2002), Vol. 11, Chap. 57, pp. 101–192.

³F. Chmelík, A. Ziegenbein, H. Neuhäuser, and P. Lukáč, *Mater. Sci. Eng., A* **324**, 200 (2002).

⁴S. J. Noronha, G. Ananthakrishna, L. Quouire, C. Fressengeas, and L. P. Kubin, *Int. J. Bifurcation Chaos Appl. Sci. Eng.* **7**, 2577 (1997).

- ⁵M. A. Lebyodkin, Y. Bréchet, Y. Estrin, and L. P. Kubin, *Phys. Rev. Lett.* **74**, 4758 (1995).
- ⁶G. Ananthakrishna, S. J. Noronha, C. Fressengeas, and L. P. Kubin, *Phys. Rev. E* **60**, 5455 (1999).
- ⁷M. S. Bharathi, M. Lebyodkin, G. Ananthakrishna, C. Fressengeas, and L. P. Kubin, *Phys. Rev. Lett.* **87**, 165508 (2001).
- ⁸D. M. Dimiduk, C. Woodward, R. LeSar, and M. D. Uchic, *Science* **312**, 1188 (2006).
- ⁹M. A. Lebyodkin, V. Ya. Kravchenko, and V. S. Bobrov, *Physica B* **165-166**, 267 (1990); *Mater. Sci. Eng., A* **164**, 252 (1993).
- ¹⁰M. A. Lebyodkin, L. R. Dunin-Barkowski, and T. A. Lebedkina, *Pis'ma Zh. Eksp. Teor. Fiz.* **76**, 714 (2002) [*JETP Lett.* **76**, 612 (2002)].
- ¹¹E. D. Schmitter, *Phys. Lett. A* **368**, 320 (2007).
- ¹²E. Orowan, *Z. Phys.* **89**, 634 (1934).
- ¹³J. Weiss, T. Richeton, F. Louchet, F. Chmelik, P. Dobron, D. Entemeyer, M. Lebyodkin, T. Lebedkina, C. Fressengeas, and R. J. McDonald, *Phys. Rev. B* **76**, 224110 (2007).
- ¹⁴J. Weiss and J.-R. Grasso, *J. Phys. Chem. B* **101**, 6113 (1997).
- ¹⁵J. Weiss and M.-C. Miguel, *Mater. Sci. Eng., A* **387-389**, 292 (2004).
- ¹⁶T. Richeton, P. Dobron, F. Chmelik, J. Weiss, and F. Louchet, *Mater. Sci. Eng., A* **424**, 190 (2006).
- ¹⁷M. C. Miguel, A. Vespignani, S. Zapperi, J. Weiss, and J. R. Grasso, *Nature (London)* **410**, 667 (2001).
- ¹⁸M. Zaiser, *Adv. Phys.* **55**, 185 (2006).
- ¹⁹M. A. Lebyodkin and T. A. Lebedkina, *Phys. Rev. E* **77**, 026111 (2008).
- ²⁰A. Vinogradov, D. L. Merson, V. Patlan, and S. Hashimoto, *Mater. Sci. Eng., A* **341**, 57 (2003).
- ²¹L. I. Salminen, A. I. Tolvanen, and M. J. Alava, *Phys. Rev. Lett.* **89**, 185503 (2002).
- ²²E. Vives, J. Ortín, L. Mañosa, I. Ràfols, R. Pérez-Magrané, and A. Planes, *Phys. Rev. Lett.* **72**, 1694 (1994).
- ²³T. Böhlke, G. Bondár, Y. Estrin, and M. A. Lebyodkin, *Comput. Mater. Sci.* **44**, 1076 (2009).
- ²⁴Full information on the XEDO system may be found on URL: <http://www.dakel.cz/>
- ²⁵M. Lebyodkin and Y. Estrin, *Acta Mater.* **53**, 3403 (2005).
- ²⁶M. A. Lebyodkin and T. A. Lebedkina, *Phys. Rev. E* **73**, 036114 (2006).
- ²⁷T. C. Halsey, M. H. Jensen, L. P. Kadanoff, I. Procaccia, and B. I. Shraiman, *Phys. Rev. A* **33**, 1141 (1986).
- ²⁸A. Chhabra and R. V. Jensen, *Phys. Rev. Lett.* **62**, 1327 (1989).
- ²⁹J. Feder, *Fractals* (Plenum, New York, 1988).
- ³⁰R. Král, P. Dobroň, F. Chmelík, V. Koula, M. Rydlo, and M. Janeček, *Kovove Mater.* **45**, 159 (2007).