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RECYCLING AND ENDOGENOUS CYCLES

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Abstract
This article investigates the conditions under which deterministic cycles can emerge in a discrete-time model with infinitely lived agents and when the economy is characterized by two sectors producing two perfectly substitutable goods: a virgin good and a recycled one. The occurrence of deterministic fluctuations rests upon the countercyclical behavior of the recycling industry: an increase in present consumption implies a lower future waste activity together with a lower agents' total income, yielding a decline in next period consumption.

Keywords: Endogenous fluctuations, Recycling activity

JEL classification: E32, Q53

1. Introduction

In this paper we consider an infinite horizon economy with a representative agent consuming a homogeneous good and supplying labor elastically. There are two productive sectors. The first one produces a virgin good, according to standard constant returns to scale technology. In addition, a share of previous period consumption is recycled by means of a concave production function including a linear shrinkage cost (Martin, 1982). Profits of the recycling activities are distributed to the agents by means of lump-sum transfers. The hypothesis on the source of waste is similar to that postulated by De Beir *et al.* (2010): the recycling activity involves past-period virgin good production. In such a framework, agents accumulate capital and the dynamics describing intertemporal

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equilibrium is three-dimensional. Stability conditions involve therefore an accurate parameter calibration. Conversely, under our assumption about the origin of waste and by getting rid of capital accumulation, the dynamics boils down to a simple first-order difference equation in terms of past-period and present consumption. We show that under mild assumptions on the recycling cost and provided the waste technology is concave enough, there arise deterministic cycles through a flip bifurcation. This result mimics standard multi-sector models, but the mechanism does not rely upon dramatic reversals in capital intensity across sectors (see, among the others, Boldrin and Rustichini, 1994). Rather, cycles are due to the countercyclical behavior of the recycling-sector profits: when present consumption increases, tomorrow waste activity will be pushed down and agents' total income, together with consumption, will be lower.

Waste recycling activities concern growing industries for a large number of raw materials and these activities involve many kind of waste. For instance, European countries achieve record numbers in the recycling rate of glass (80%), of paper-cardboard (60%) and aluminium (75%). Nowadays, the world production of recycled aluminium is greater than the production of virgin aluminium. Economic literature analyses recycling from three theoretical points of view. First, recycling is integrated into the analysis of industrial organization; following the Alcoa case, studies have analyzed the erosion of monopoly power by competitive firms of recycled products (Grant, 1999). Second, natural resource economics studies point out the usefulness of recycling for economies facing a decrease in the availability of resources; recycling postpones the extraction of mining resources and reduces the intensity of forest exploitation (Dasgupta and Heal, 1979). Third, environmental economics analyses recycling as an instrument to reduce environmental externalities (Lusky, 1976). Yet, these studies do not analyze the consequences in terms of economic stability of the life cycle of the goods allowed by the recovery waste activity. This lacking literature justifies therefore our paper, which focuses on the consequence in terms of economic stability and of the occurrence of deterministic cycles of the recycling activity.

Our contribution improves the existing literature on endogenous fluctuations and it seems to confirm that in order to get such a phenomenon one does not need strong external effects in production, as it the case in Benhabib and Farmer (1994), where a upward sloping labor demand curve is required, or sector specific external effects as in Benhabib and Farmer (1996) where sunspot equilibria are compatible even with a downward sloping labor demand curve. At the same time, the occurrence of cycles, in our paper, is perfectly consistent with the assumption of long-lived agents and does not require the restricted market participation typical of the overlapping generations models (e.g. Azariadis, 1981, Azariadis and Guesnerie, 1984; Grandmont, 1985) where cycles of different periodicity emerge when there are some – although mild – income effects in the consumption-labor arbitrage. In our study periodical cycles are compatible with the individual smoothing behavior and with whatever curvature of the utility functions, provided the economy is located in the region where profits are decreasing. The issue treated in this paper therefore confirms the idea that endogenous fluctuations are a quite pervasive phenomenon since they emerge even in very simple standard framework, once one departs from the basic and very restrictive typical assumptions. In this sense, endogenous cycles can be seen not just as a mere theoretical curiosity, but a phenomenon which is bound to occur even under very slight distortions which imply the failure of the First Welfare Theorem. In our case, all what is required to get cycles is a recycling activity entailing an external

effect on future consumption: today consumption affects, in fact, tomorrow one by means of a direct – although not taken into account by agents – modification of the production possibility frontier.
The remainder of the paper is structured as follows. In Section 2 we develop the theoretical model and derive the difference equation describing intertemporal equilibrium and the stationary solution as well. Section 3 discusses the stability properties of the model and derives conditions that guarantee the emergence of economic fluctuations. Finally, Section 4 contains the main conclusions of the paper and suggests some reliable extension to be carried out in the future.

2. The model

There exists a continuum of infinitely-lived identical agents whose size is normalized to unity. The objective of the representative agent is to maximize his intertemporal welfare:

$$max\sum_{t=0}^{\infty}\beta^{t}\left[u(c_t)-v(l_t)\right]$$

subject to the dynamic budget constraint:

$$p_t c_t = \omega_t l_t + p_t \pi_t$$

where c stands for consumption, l for labor and $\beta \in (0,1)$ for the discount factor. u is the instantaneous utility function of consumption, v the disutility of labor, p the price of both the virgin and the recycled good, ω the nominal wage and π the real profit obtained in the recycling sector, which is assumed to be given from the point of view of the single agent. The functions u and v satisfy standard assumptions.

Assumption 1. u and v are continuously differentiable as much as needed. In addition, $u^{'}(c)>0, u^{"}(c)\le 0$, $\lim_{c\to 0+} u^{'}(c)>0$, $\lim_{c\to\infty+} u^{'}(c)\ge 0$, $v^{'}(l)>0$, $v^{"}(l)>0$, $\lim_{l\to 0+} v(l)=0$, $\lim_{l\to +\infty} v(l)=+\infty$.

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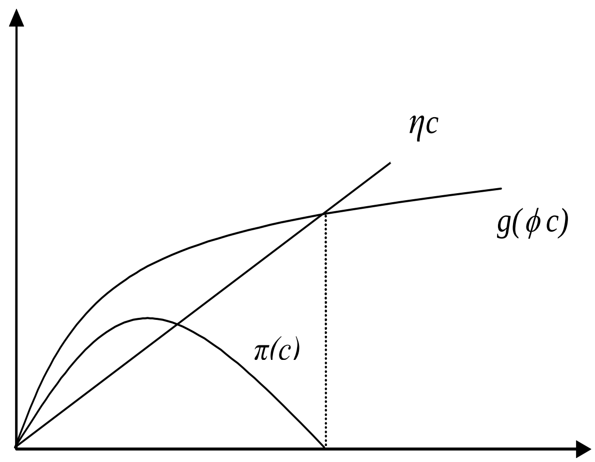

Figure 1: Profit of the recycling sector

The First Order Conditions of the maximization problem yield the arbitrage condition:

$$v'(l_t) = \frac{\omega_t}{p_t} u'(c_t) \tag{1}$$

Condition (1) claims that the effort of supplying one additional unit of labor must be equal to the correspondent increase in utility. The virgin good y^v is produced by means of a linear technology which employs only labor *i.e.* $y^v=l$ where v stands for virgin. The profit in nominal terms is then $p_t y_t^v - \omega_t l_t$. It follows that at equilibrium one has $w_t=p_t$ and therefore profits in this sector are zero for each $t=1,2\ldots$. Recycled goods and virgin goods are perfect substitutes. A recycling sector competes directly with the virgin production sector. The representative firm includes recovery and recycling activities. Following Swan (1980) and Martin (1982), we suppose that the production factor is the recovered waste and that the returns are decreasing (a characteristic of the technology of recovery). Hence, the recycled good is produced according to a concave production function g which recovers a share $\phi \in (0,1)$ of the previous period consumption at the fixed cost[1] $\eta > 0$. As the two goods are *in fine* perfect substitutes, we assume that it is possible to recycle goods an infinity of time.

Assume a mandatory recycling sector implying that there is no profit maximization task: it is indeed a public sector. Moreover, it exists in many European countries, private companies accredited by the public authorities (for example DSD in Germany) which have to sort all the available households waste packaging. Therefore the real net output (i.e. real profit) of the recycling sector will be (see Fig. 1):

$$\pi_t = g(\phi c_{t-1}) - \eta c_{t-1} \tag{2}$$

[1] η is the shrinkage cost and it represents the costs linked to the recycling activity: a share is lost during the waste recovery stage or during the recycling process.

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The production function g possesses the following properties.

Assumption 2. $g(0) = 0, g'(x) > 0, g''(x) < 0$

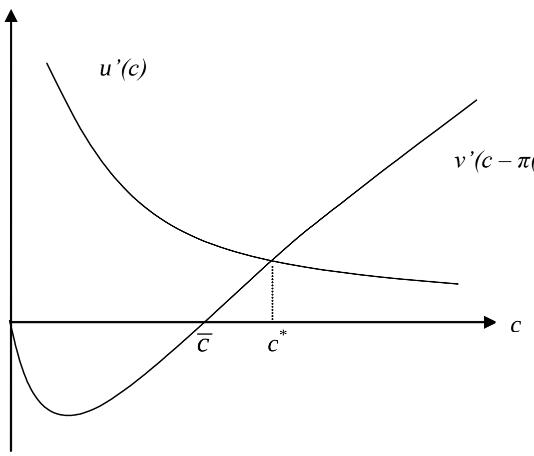

Figure 2: Arbitrage condition and equilibrium

Since $c_t = \pi_t + l_t$ and at equilibrium $p = \omega$, the arbitrage equation (1) boils down to:

$$v'(c_t - \pi(c_{t-1})) = u'(c_t) \quad (3)$$

In view of Assumptions 1 and 2, it is easy to prove the existence of a unique stationary solution. In fact, as depicted in Fig. 2 the right hand side of (3) is non-increasing and continuous, while its left hand side is decreasing first and thereafter negative. Then, for all $c > c_{min}$, where:

$$c_{min} = arg_c min v'(c - \pi(c))$$

it slopes positive and for $c = \bar{c}$ it is $c = \pi(\bar{c})$. Therefore, for all $c > \bar{c}$, $v'(c - \pi(c))$ is positive, increasing and diverges to infinite when $c \to +\infty$. It follows that it will cross $u'(c)$ exactly once, for some $c^* > \bar{c}$. This, in particular, implies that at the steady state $\pi'(c) < 1$.

Proposition *1.* *Equation (3) possesses a unique interior steady state* $c^* > \bar{c} > 0$ *with* $l^* = c^* - \pi(c^*) > 0$.

3. Stability properties

In order to study the stability of equation (3), we evaluate the derivative $\left(dc_t/dc_{t-1}\right)_{c=c*}$ at the steady state. For sake of simplicity in notation we omit the asterisk over the steady state value of a variable. Straightforward computations yield the following expression for $\left(dc_t/dc_{t-1}\right)_{c=c*}$

$$\frac{v''(l)\pi'(c)}{v''(l)-u''(c)} \qquad (4)$$

The sign of (4) is the same of that of $\pi'(c)$, since $v''(l)>0$ and $u''(c)<0$. Suppose first that $\pi''(c)>0$, i.e. that the steady state lies in the increasing locus of the profits function of the recycling sector. As it is shown in the following Proposition, in such a configuration the steady state is monotonically stable.

Proposition 2. *Assume* $\pi''(c)>0$. *Then* $0<\left(dc_t/dc_{t-1}\right)_{c=c*}<1$ *and the steady state is monotonically stable.*

Proof. $\frac{v''(l)\,\pi'(c)}{v''(l)-u''(c)}<1$ requires $v''(l)\,\left[\pi'(c)-1\right]<-u''(c)$. But, by definition of $\overline{c}$, and from the concavity of π, we have $\pi'(\overline{c})<1$. Therefore, again in view of the concavity of π, one has $\pi'(c^*)<\pi'(\overline{c})<1$. It follows that expression (4) is lower than one.

It is now immediate to verify that $\left(dc_t/dc_{t-1}\right)_{c=c*}<0$ when $\pi'(c)<0$. We wonder if the steady state may undergo a flip bifurcation, i.e. whether there exists some parameter configuration such that $\left(dc_t/dc_{t-1}\right)_{c=c*}$ = -1, To this end, let us write equality $\frac{v''(l)\,\pi'(c)}{v''(l)-u''(c)}=-1$ in the form $\pi'(c)=\frac{u''(c)}{v''(l)}-1$. Since, at the steady state equilibrium, we have $v'(l)=u'(c)$ we can write $\pi'(c)=-\frac{\varepsilon_u}{\varepsilon_v}\frac{l}{c}-1$ where $\varepsilon_u=-u''(c)c/u'(c)$ is the elasticity of the marginal utility of consumption and $\varepsilon_v=v''(l)l/v'(l)$ is the elasticity of the marginal disutility of working. Since $\pi'(c)=g'(\phi c)\phi-\eta$ one obtains condition $g'(\phi c)\,\frac{\phi c}{g(\phi c)}\frac{g(\phi c)}{c}-\eta=-\frac{\varepsilon_u}{\varepsilon_v}\frac{l}{c}-1$. Setting now $\varepsilon_g\equiv g'(\phi c)\,\phi c/g(\phi c)$ the elasticity of the recycling production function, we can write $\varepsilon_g \frac{g(\phi c)}{c} - \eta = -\frac{\varepsilon_u}{\varepsilon_v}\frac{l}{c} - 1$. Eventually, taking into account that $g(\varphi c) = \pi + \eta c$, we get $\varepsilon_g\left[\frac{\pi}{c}+\eta\right]-\eta=-\frac{\varepsilon_u}{\varepsilon_v}\frac{l}{c}-1$. It follows that the bifurcation value of ε_t is:

$$\varepsilon_g^F = \frac{\eta - \frac{\varepsilon_u}{\varepsilon_v}\left[1-\frac{\pi}{c}\right]-1}{\eta+\frac{\pi}{c}} \qquad (5)$$

We must now verify that $\varepsilon_g^F \in (0,1)$. Since $\pi' < c$ it is immediate to see that $\varepsilon_g^F > 0$ if and only if $\eta > 1 + \frac{\varepsilon_u}{\varepsilon_v}\left[1-\frac{\pi}{c}\right] > 0$. Furthermore, $\varepsilon_g^F < 1$ requires $-\frac{\varepsilon_u}{\varepsilon_v}\left[1-\frac{\pi}{c}\right]-1<\frac{\pi}{c}$, condition always verified in the light of the previous considerations. One can immediately appraise that, as soon as η increases from $1+\frac{\varepsilon_u}{\varepsilon_v}\left[1-\frac{\pi}{c}\right]$ to infinite, the bifurcation value for ε_g increases from zero to one. We have therefore the following Proposition, which is immediately proved once one takes into account that, as shown in Grandmont (2008) and Bosi and Ragot (2011), in nonlinear systems the occurrence of a flip bifurcation entails the existence of 2-period deterministic cycles near the steady state for a whole range of values close to the bifurcation one.

Proposition 3. *Assume* $\pi'(c)<0$ *and* $\eta>1+\frac{\varepsilon_u}{\varepsilon_v}\left[1-\frac{\pi}{c}\right]>0$. *Then:*

(i) For $0<\varepsilon_g<\varepsilon_g^F<1$ *the steady state is unstable and the economy diverges following oscillatory paths.*

(ii) For $0<\varepsilon_g^F<\varepsilon_g<1$ *the steady state is stable and the economy converges following oscillatory paths.*

In addition, when ε_g *goes through* ε_g^F *, the steady state undergoes a flip bifurcation.*

If, conversely, $0<\eta<1+\frac{\varepsilon_u}{\varepsilon_v}\left[1-\frac{\pi}{c}\right]$, *then the steady state is stable and the economy converges following oscillatory paths.*

In order to understand the mechanism on the ground of which cycles may arise, let us suppose that at the steady state profits are decreasing. Let us suppose, in addition, that in period t consumption is larger than its steady state value. It follows that the profits of the waste activity will be lower, in view of the concavity of the function and taking into account that profits are distributed only one period later: it follows that consumption in period $t+1$ will decrease. But following an analogous reasoning, profits in period $t+2$ will be larger and so will be consumption, yielding perpetual oscillations around the steady state. These results open the door for stabilization environmental policies. One of the central parameters, among others, is the η term. Hence, government may use market-based

instruments (tax and subsidies) to influence this cost or command and control instruments (like ϕ), such as economic fluctuations are avoided. Actually, European governments mix these two instruments to achieve environmental objectives. For example, in the case of the packaging waste policy, since 1994, a state-registered eco-organisation subsidizes the recovery sector in each EU country; moreover, the 1994 EU Packaging Directive determines minimum recovery and recycling targets for packaging waste.
One may wonder at this point how our results would be different if, instead of assuming that the recycled good is produced according to a concave function, it were produced according to other types of technologies. The answer is quite complex, but there is one point that must be emphasised. Indeed, as we have shown, the occurrence of endogenous cycles is dramatically linked to the emergence of a decreasing profit function. Within an alternative technology, this could be no more the case and, as a consequence, the phenomenon would completely disappear. For instance, if the production function were convex, profit would always be increasing in the amount of the recycled good and therefore the countercyclical behavior of the profit function, which is responsible for the occurrence of cycles, would no more be at work. The same is true if we assume a linear technology which, along with the assumption of a linear shrinkage cost, would entail profits either always defined positive (but not countercyclical in the amount of the input) or always negative (and therefore there would be no recycling activity at all).

Another interesting question is concerned with the behavior of the economy under the assumption that the shrinkage cost has a variable component instead of considering it only with a fixed cost component. Here the answer is straightforward: in the case that the shrinkage cost is convex, the countercyclical behavior of the profit function would be exalted and therefore endogenous fluctuations would be even more likely to occur. In the opposite case where the shrinkage cost is concave, the profit would conversely be always increasing in the waste activity and, as a consequence, no countercyclical behavior would emerge. As a result, there would be no room for endogenous cycles.

In our paper we have assumed that the virgin good and the recycled good are perfect substitutes. This means that, from the point of view of the representative household, they share the same market price and therefore he is completely indifferent in respect to which of the two goods to consume. If, on the contrary, we assume that the two goods are not perfect substitutes, there are two possible configurations that may emerge. If, on the one hand, the utility function of the representative agent is defined exclusively on one good, the market arbitrages would drive the price of the two goods to the same value. Indeed, if it were not the case, agents would buy exclusively the cheaper good, driving down the price of the other. This process would be at work until the two prices would become equal. If, on the other hand, the utility function of the representative household is defined over two different commodities, their relative price could be different from unity. In such a case, agents would make opportune arbitrages in terms of consumption streams, by equalizing the marginal rate of substitution with the relative price of the two goods, as in a standard model with several commodities.

4. Conclusion

The European Countries have adopted the European Directive of 2002 concerning the disposal of electrical and electronic waste. Producers of these goods contribute to one of the four State-registered eco-organisations (Eco-Systemes, ERP, Ecologic, and Recyclum). This Directive imposes also a minimum level for the recycling rate, depending on the kind of waste. More generally, the European Commission encourages the development of the recovery and the recycling of waste because it could be a tool for sustainable development policies. However, these mechanisms can be a source of economic fluctuations and therefore may be handled with some care. As a matter of fact, in this paper, within a very simple model, we have shown that for some values of the elasticity of the recycling technology, the development of the recycling activities may lead the economy towards cyclical as well as unstable trajectories. Stabilization polices involve an accurate control of the required rate of the industrial goods recycling activity and, perhaps, the introduction of some upper bound on the recovering waste activity in order to avoid unpleasant effects in terms of economic stability. As we have shown, indeed, cycles appear when economic waste activity lies in the region where profits in this sector are declining, i.e. when the recovery rate is high enough.

The paper can be improved and generalised by means of several extensions. As we have seen, one could assume the two goods, the virgin one and the recycled one, to be not perfect substitutes. In such a case there would emerge, at equilibrium, a relative price not equal to unity. We would indeed obtain a genuine two-sector model, although the inputs used in the two sectors would be not the same. Another improvement is concerned with the assumption of a separable utility function in consumption and labor. Indeed, in the case of a non-separable utility function the requirements needed to get endogenous fluctuations would be probably relaxed and the latter would emerge for a wider parameter configuration. One could also be interested in analyzed the possibility of endogenous fluctuations under the hypothesis of more general production technologies, as we have discussed at the end of Section 3. Of course, some empirical exercise could be done to test our theoretical results. Finally, it would be interesting to study the effects of the presence of a recycled good in an overlapping generation model: in such a configuration, beside agents with a finite life cycle there would be a good with its own life cycle and the combination of the two time structures could give interesting and new insights.

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