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SUMMARY

There has been debate on whether average slip $\overline{D}$ in long ruptures should scale with rupture length L, or with rupture width W. This scaling discussion is equivalent to asking whether average stress drop $\Delta\sigma$, which is sometimes considered an intrinsic frictional property of a fault, is approximately constant over a wide range of earthquake sizes. In this paper, we examine slip-length scaling relations using a simplified 1-D model of spatially heterogeneous slip. The spatially heterogeneous slip is characterized by a stochastic function with a Fourier spectrum that decays as $k^{-\alpha}$, where k is the wavenumber and α is a parameter that describes the spatial smoothness of slip. We adopt the simple rule that an individual earthquake rupture consists of only one spatially continuous segment of slip (i.e. earthquakes are not generally separable into multiple disconnected segments of slip). In this model, the slip-length scaling relation is intimately related to the spatial heterogeneity of the slip; linear scaling of average slip with rupture length only occurs when α is about 1.5, which is a relatively smooth spatial distribution of slip. We investigate suites of simulated ruptures with different smoothness, and we show that faults with large slip heterogeneity tend to have higher $\overline{D}/L$ ratios than those with spatially smooth slip. The model also predicts that rougher faults tend to generate larger numbers of small earthquakes, whereas smooth faults may have a uniform size distribution of earthquakes. This simple 1-D fault model suggests that some aspects of stress drop scaling are a consequence of whatever is responsible for the spatial heterogeneity of slip in earthquakes.

Key words: average stress drop, earthquake slip-length scaling, self-affine fractals, slip heterogeneity, 1-D stochastic model.

INTRODUCTION

Studying the scaling relationships between earthquake rupture parameters provides insight into the physics of the rupture process. One of the most fundamental of these scaling relationships, which is that between average slip and rupture dimensions, has been discussed intensively over the last decade. Does average slip $\overline{D}$ scale with the rupture length L or with the rupture width W? A key element in this debate, however, is the hypothesis that earthquakes are crack-like processes with scale-invariant average stress drops.

The ratios of average slip to rupture dimensions provide direct estimates of the strain change in the vicinity of earthquakes, and hence the average stress drops on rupture surfaces. The average slip $\overline{D}$, from a uniform stress drop $\Delta\sigma$, on a rupture surface of area S, in an elastic whole space of rigidity μ, can be written as (e.g. Kanamori & Anderson 1975)

$$\overline{D} = S^{1/2}\Delta\sigma/C\mu \qquad (1)$$

where C is a dimensionless number whose value depends on the geometry of the rupture and the orientation of the shear stress. Equation (1) can also be shown to apply to a class of half-space problems with sufficient symmetry. It is convenient to characterize rupture surfaces as rectangular, having length L and width W. The relationship between average slip and stress drop for rectangular faults in both whole- and half spaces was first reported by Boore & Dunbar (1977). Parsons *et al.* (1988) found some inconsistencies with these solutions and reported revised results, which can be summarized for a vertical strike-slip fault (Table 1).

Das (1988) demonstrated that these relationships are still approximately true even when the stress drop is spatially heterogeneous, but stress drop is replaced by spatially averaged stress drop. Average stress drop $\Delta\sigma$ was initially suggested to be scale independent from the observation that seismic moment M_0 scales with $S^{3/2}$ over a wide range of earthquake sizes (Aki 1972; Abe 1975; Kanamori & Anderson 1975; Hanks 1977).

A common and seemingly logical interpretation of $S^{3/2}$ scaling of moment is that $\Delta\sigma$ is a material property of faults. For small earthquakes, which are essentially equal dimensional, $S^{1/2}$ could be either L or W in eq. (1), thus constant stress drop means average slip scales linearly with rupture length, as well as with rupture width.

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GJI Seismology

Table 1. Summary of Parsons *et al.*'s (1988) formulae of average stress drop for strike-slip faults.

	Whole space	Half space with surface rupture
Square fault ($L = W$)	$\Delta\sigma = 2.55\,\mu\overline{D}/L$	$\Delta\sigma = 2.04\,\mu\overline{D}/L$
Long, narrow fault ($L \gg W$)	$\Delta\sigma \approx 1.28\,\mu\overline{D}/W$	$\Delta\sigma \approx 0.65\,\mu\overline{D}/W$ (3)

However, if $\Delta\sigma$ is a material property of the fault, then we should expect an upper bound on the average slip for strike-slip earthquakes with very long ruptures. These earthquakes have rupture dimensions that are confined by the thickness of the seismogenic zone in downdip width, but unbounded in rupture length. In this case, average stress drop is determined by the ratio of average slip versus width, and constant stress drop means that average slip will reach a constant value related to the maximum fault width $W_{\max}$. Thus we might expect average slip to depart from linear growth with rupture length, when the rupture length becomes much larger than the fault width. This is sometimes referred to as the *W*-model (*W* represents fault width). The *W*-model assumes that average stress drops are independent of rupture dimensions. Therefore, the *W*-model predicts that $\overline{D} \sim L \sim W$ when $L < W_{\max}$, but $\overline{D} \sim W_{\max}$ when $L \gg W_{\max}$. Equivalently, the *W*-model predicts seismic moment ($M_0 = \mu L W \overline{D}$) scales with L^3 for small events and with L for large events.

Despite this expectation, Scholz (1982) reported that average slip scales linearly with rupture length for both small and large earthquakes. To explain linear slip-length scaling, Scholz (1982) introduced the *L* model in which stress drop and average slip are determined by rupture length L. Unlike the *W*-model, the *L*-model predicts that M_0 scales with $L^2 (\overline{D} \sim L$ and $W = $ constant) for large earthquakes and with L^3 for small events. Shimazaki's (1986) observations on a set of Japanese intraplate earthquakes are also consistent with an *L* model. More recently, Romanowicz (1992) argued that existing data for large earthquakes are compatible with $M_0 \sim L$ scaling, which is evidence for the *W* model. This spurred further debate about whether existing data favoured the *W* model (Romanowicz 1994; Romanowicz & Ruff 2002), or the *L*-model (Scholz 1994a; Pelger & Das 1996; Wang & Ou 1998). Recent modelling and compilations of data (Scholz 1994b; Bodin & Brune 1996; Mai & Beroza 2000; Shaw & Scholz 2001), however, show that neither the *L* model nor the *W* model may be adequate to explain slip-length scaling observations. Current compilations seem to indicate that average slip continues to increase with L when $L \gg W_{\max}$, but at a slower rate than implied by the *L* model. Furthermore, the transition from linear to non-linear length scaling seems to occur at rupture lengths that are relatively large compared to the maximum rupture width. This observation suggests the limitation of static *L* and *W* models, in which fault slip is determined by the average stress drop and the final dimension of fault rupture.

Dynamic rupture models, such as partial stress drop, abrupt locking and self-healing models (Housner 1955; Brune 1970, 1976; Heaton 1990), may help to explain the messy slip-length scaling observations. For example, in the slip-pulse model, Heaton (1990) speculated that the most natural explanation for a correlation between average slip and rupture length (regardless of the rupture width) is that slip pulses with very large slip tend to propagate larger distance. If earthquake rupture grows like a narrow 'slip-pulse' propagating along the fault, then there would be a stochastic element in rupture propagation, and this stochastic element would have consequences in how far the rupture can go, thus affecting the scaling of earthquake rupture parameters.

In this paper, we introduce a simple 1-D stochastic model of fault slip as a function of along-strike distance. This model is purely geo-metric in nature and is not intended to satisfy equations of motion or specific boundary conditions. It does, however, share features that we expect to occur in slip-pulse type models of dynamic rupture. The purpose of this paper is to argue that constant stress drop is not necessarily required to explain slip-length scaling. Specifically, we show that the spatial heterogeneity of fault slip can play a key role in slip-length scaling. We find that our model does not generally result in linear scaling of slip with rupture length, although linear scaling is a special case for a particular degree of rupture smoothness. However, when a suite of faults with different smoothness is considered, the overall pattern of average slip with length is similar to that contained in the data compiled by Wells & Coppersmith (1994).

We restrict our discussion of slip-length scaling for individual earthquake ruptures, not the maximum displacement—length scaling for faults. The latter scaling describes the relationship between the maximum cumulative displacements and the total lengths of faults (e.g. Walsh & Watterson 1988; Cowie & Scholz 1992; Clark & Cox 1996; Manighette *et al.* 2001). These fault parameters represent summed effect of either numerous seismic ruptures or non-seismic fault growth. It is unclear whether the scaling for individual earthquakes also applies to faults.

1-D SLIP VARIATION MODEL

The basic motivation for our 1-D model is the notion that a pulse of slip propagates along a fault. At any given point, x, it has slip of amplitude $D(x)$. As the rupture propagates to an adjacent point, $x + \Delta x$, the amplitude of the slip changes by some random amount, which can be either positive or negative. In this way, the earthquake increases or decreases in intensity in some stochastic way as rupture propagates along a fault. If the slip amplitude decreases to zero, then the earthquake is over.

We demonstrate that the relationship between average slip and rupture length for this class of models is closely related to the assumed spatial heterogeneity of slip. While the physical origin of spatial slip heterogeneity may be a profoundly important issue, we do not address it here. We are simply emphasizing the role that slip heterogeneity plays in determining stress drop.

Our model consists of two simple rules.

Rule 1: Slip as a function of position, $D(x)$, can be approximated by a convolution in the Fourier space between a random function of position and some function with power law dependence. Mathematically,

$$D(x) = D_0|FT^{-1}[\hat{R}(k)k^{-\alpha}]| \qquad (2)$$

where D_o is a constant, FT^{-1} refers to taking inverse Fourier transform, $\hat{R}(k)$ is the Fourier transform of $R(x)$, which is a Gaussian random function of x with zero mean and variance of 1, k is wavenumber, and α is a filtering parameter that determines the smoothness or roughness of the slip function.

Rule 2: Any earthquake consists of a spatially contiguous segment of positive slip. That is, any point at which $D(x) = 0$, defines the end of an individual rupture.

We use the procedure described by Turcotte (1997, p. 149–155) to generate a fractal series $D(x)$. The series is then rescaled to obtain the

desired mean and standard deviation, and we then call it the parent series. The parent series consists of alternative segments of positive and negative values. We then take the absolute value of the parent series, so that both positive and negative segments are retained as analogues of earthquake slip functions, which are assumed to be non-negative functions. Accordingly, the length of a rupture is thus taken to be the distance between zeroes of the parent series (rule 2). That is, an earthquake rupture consists of only one spatially continuous segment of slip. In this way, each parent series is split into a set of earthquake slip functions of varying length with approximately the same α. Segments that are represented by fewer than 10 data points are not used in our statistical analysis, because they may be too short to be statistically stable. Indeed, statistics show more scatter in average displacement for short segments. However, this choice of truncation is arbitrary, as we do not see a sharp change in statistics for segments of various sizes. In this article, we define α, the slope of its Fourier spectral amplitude with wave number, as the smoothness of the slip function. Fig. 1 shows $D(x)$ and $\hat{D}(k)$ for typical simulated events with smoothness α of 1.0, 1.25 and 1.5. The combination of rules 1 and 2 implies that a model with a rougher slip distribution (lower α) will produce more short events because those functions are more likely to pass through zero.

The parent series described by the first rule is known as 1-D fractional Brownian motion (fBm) (Mandelbrot 1985). A fBm function has the property that $D(x)$ is statistically similar to $\frac{D(rx)}{r^H}$, where H is the Hausdorff measure and r is any scale length. The theoretical relationship between H and α is (Turcotte 1997)

$$H = \alpha - 0.5 \qquad 0.5 < \alpha < 1.5. \qquad (3)$$

While there is no conclusive evidence that $D(x)$ is fractal for real earthquakes, the use of fractal slip functions is motivated by numerous observations suggesting fractal fault traces, fractural surfaces, internal structure of fault zones and fault networks (e.g. Brown & Scholz 1985; Scholz & Aviles 1986; Power *et al.* 1987; Chester *et al.* 1993; Schmittbuhl *et al.* 1995). Furthermore, fractal slip distributions have been adopted in earthquake models (e.g. Andrews 1980; Herrero & Bernard 1994; Mai & Beroza 2002; Lavallée & Archuleta 2003). However, previous studies have focused mostly on self-similar fractals. In contrast, we have considered the more general self-affine fractals and exploited the implication of the assumption of anisotropic fractal slip functions in the relationship between average slip and length.

Our spatially heterogeneous slip functions imply that stress and strain changes are even more spatially heterogeneous. That is, strain
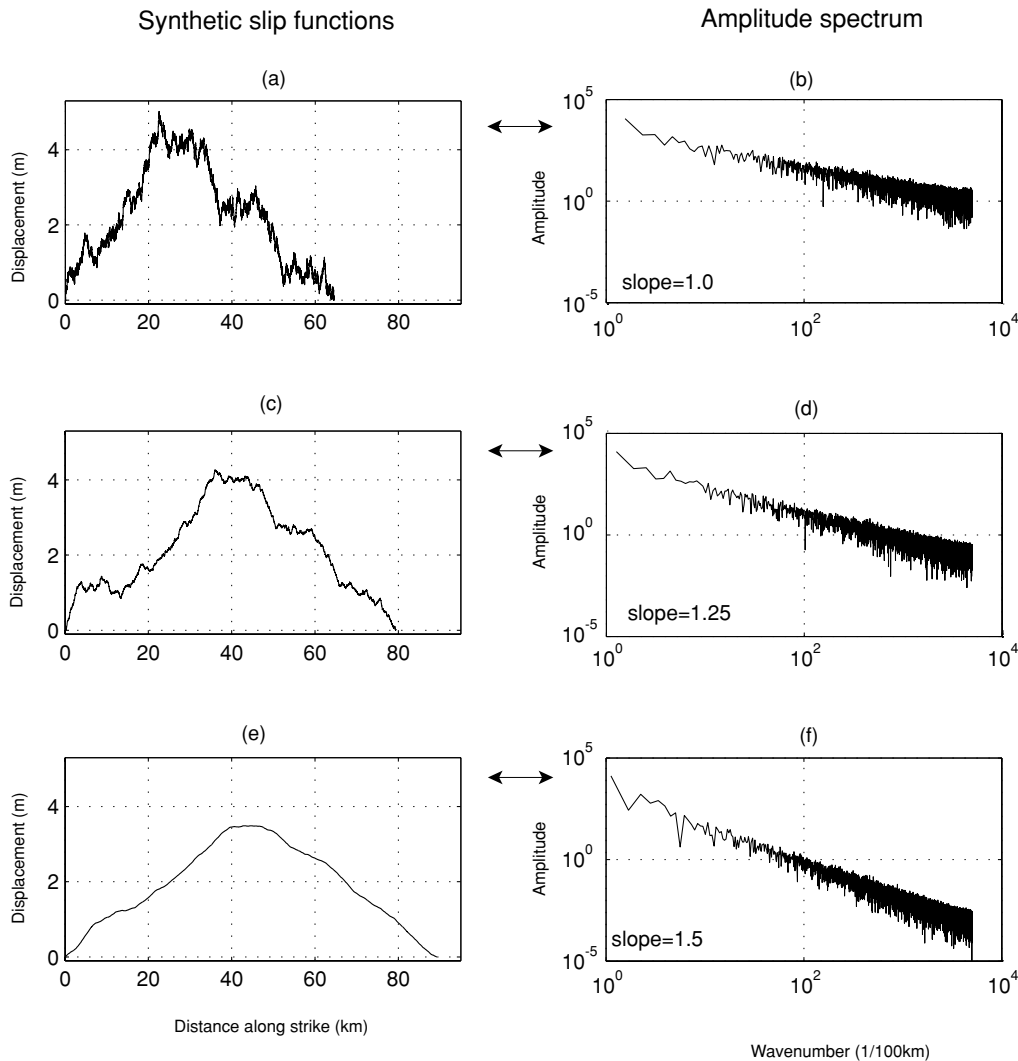
Figure 1. Examples of synthetic self-affine slip functions of different variability (a,c,e) and their Fourier amplitude spectra (b,d,f). As the slip function gets smoother from (a) to (c) to (e), the slope of the amplitude spectrum grows from 1.0 (b) to 1.25 (d) to 1.5 (f).
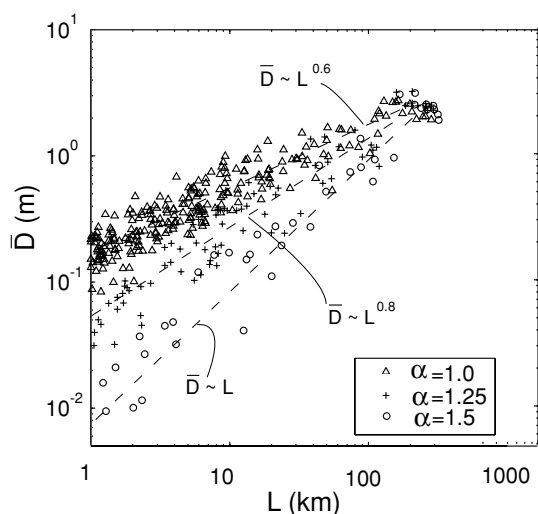
changes on the fault are a function of first derivatives of slip with respect to distance. If slip can be described by spatially filtered random functions with a spectral decay of $k^{-\alpha}$, then stress and strain change functions will have a spectral decay of $k^{-\alpha+1}$. Thus $\alpha = 1$ is consistent with Gaussian random stress changes and $\alpha < 1$ corresponds to stress change functions that are deficient in long-wavelength variations compared to random.
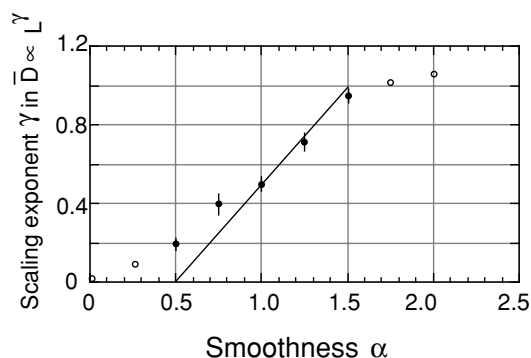
MODELLED RESULTS

We have considered seven cases with slip smoothness α corresponding to 0.5, 0.75, 1.0, 1.25, 1.5, 1.75 and 2.0. For each case, 10 parent series consisting of $N = 65\,536$ points were constructed, and the distance between adjacent data points is assumed to be $\Delta L = 10$ m. This value was adopted such that the longest synthetic rupture lengths are comparable to those in the real world.

Fig. 2 shows the modelled relationship (log–log) between average displacement $\overline{D}$, and length L for earthquakes with intermediate values of slip smoothness, $\alpha = 1.0, 1.25$ and 1.5. The lengths of the earthquakes range from 1 to 300 km. Several features can be seen in this plot. (1) For a given average displacement, earthquakes with smoother slip functions tend to have longer ruptures. (2) Ruptures with a slip smoothness of $\alpha = 1.5$ produce average slips that are linearly proportional to L. However, linearity breaks down for rougher slip functions ($\alpha < 1.5$). That is, $\overline{D} \sim L^{0.8}$ for ruptures of $\alpha = 1.25$, and $\overline{D} \sim L^{0.6}$ for $\alpha = 1.0$.

The scaling exponent γ in the relationship of $\overline{D} \sim L^\gamma$ depends nonlinearly on the slip smoothness, α, as shown in Fig. 3. Theoretically, the exponent γ in the slip-length scaling is equivalent to the Hausdorff measure (H), and linear scaling of D and L implies $H = 1$, corresponding to smooth or nearly differentiable slip functions. Note that the value of γ deviates systematically from the theoretical line of $H = \alpha - 0.5$. Similar discrepancy also exists between power-law spectral exponent and fractal dimension (Fox 1989; Pickering *et al.* 1999). The reason for the deviation is two-fold. One, the simple relation of theoretical prediction can only be an approximation. Hausdorff measure must lie between 0 and 1 by definition. This corresponds to $0.5 \leq \alpha \leq 1.5$. However, it is possible to construct
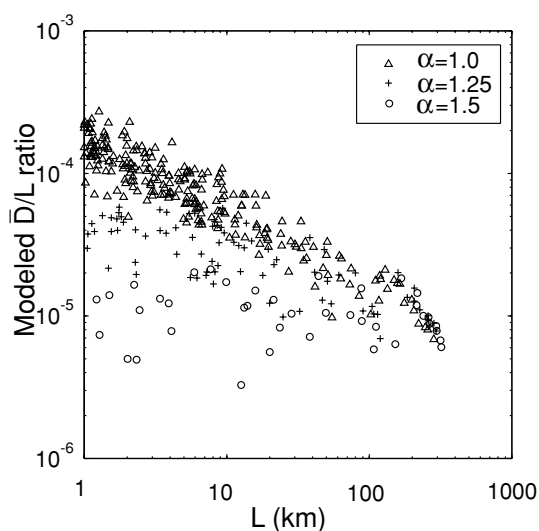

Figure 3. The dependence of slip-length scaling on slip heterogeneity α. It is not well constrained for $\alpha < 0.5$ or $\alpha > 1.5$. Large values of α tend to make series in which D diverges with L. The line indicates the theoretical prediction of $\gamma = \alpha - 0.5$, if γ is the Hausdorff exponent.

a power-law function with any value of α. For example, a random noise of $\alpha = 0$ implies $H = -0.5$ if $H = \alpha - 0.5$. This is beyond the possible range of Hausdorff measure. It is equally possible to construct a series $\alpha = 2$, which implies a Hausdorff exponent of 1.5, larger than the possible upper bound of 1. Another factor for the discrepancy is that spectral analysis has its limitations, even though it is the most common technique applied in literature to describe the scaling properties of series. For example, Fourier analysis assumes an infinite length, stationary function. In practice, the data series is finite and may be non-stationary.

We also ran the model with α larger than 1.5. As α becomes greater than 1.5, the slip becomes so smooth that distances between zero crossings become a significant fraction of the total length of the parent series and the statistical analysis becomes unreliable. When $\alpha > 2$, the parent series becomes so smooth that it tends to diverge with distance. That is, zero crossings disappear and this modelling procedure is inappropriate.

In Fig. 4, we plot $\overline{D}/L$ ratios as a function of length L. There are two noteworthy features seen in Fig. 4. (1) rougher slip distributions

Figure 2. A log–log plot of average slip $\overline{D}$ versus rupture length L for modelled ruptures with slip smoothness $\alpha = 1.0, 1.25$ and 1.5. The data can be fitted by power scaling relations whose exponents depend on α. $\overline{D}$ scales linearly with L only for earthquakes with smooth slip ($\alpha = 1.5$).

Figure 4. Modelled slip-length ratio $\overline{D}/L$ as a function of rupture length L. Spatially rougher slip (smaller α) produce higher $\overline{D}/L$ ratios. $\overline{D}/L$ ratios generally decrease with increasing rupture length, except when $\alpha = 1.5$, in which case synthetic earthquakes have constant $\overline{D}/L$ ratios.

(smaller α) produce higher $\overline{D}/L$ ratios, and (2) $\overline{D}/L$ ratio decreases with increasing rupture length, except when $\alpha = 1.5$.

The first feature, higher $\overline{D}/L$ ratios for rougher slip distributions, is a simple result of the fact that, for a given slip, a rupture is more likely to terminate in a short distance if the distribution is rough. The second feature, $\overline{D}/L$ ratios decrease with rupture length if $\alpha < 1.5$, is a result of the fact that there is always the chance that some percentage of the slip distributions will have a long rupture length. These features of our modelled $\overline{D}/L$ ratios are the results of a kind of statistical game of chance, which is determined by the variability of our stochastic slip function.

COMPARISON TO OBSERVATIONS

Can our simple rupture simulation help us to interpret observations of slip-length scaling for real earthquakes? To answer this question, we examine Wells and Coppersmith's (1994) data set, the most complete compilation of earthquake rupture parameters to date. We only use their strike-slip events for which the seismic moment, subsurface rupture length and rupture width are simultaneously given. We have supplemented the Wells & Coppersmith (1994) compilation with additional events, which have occurred since their compilation (see Table 2).

We take the rupture length to be the estimated subsurface rupture length, since some events have little or no surface rupture. However, we do assume that the lengths of surface rupture of several very long pre-instrumental earthquakes (e.g. 1857 Fort Tejon, or 1872 Owens Valley) are close to their subsurface rupture lengths. For shorter ruptures, Wells & Coppersmith (1994) use the distribution of early aftershocks to estimate subsurface rupture length. Although this is not a direct measurement of rupture length, Mogi (1968), Wyss (1979) and Kanamori & Given (1981) demonstrated that it seems to provide an adequate estimate. We determine average slip using $\overline{D} = M_0/(\mu L W)$, where μ is assumed to be 3.0×10^{10} Pa, L is the subsurface rupture length and W is the down-dip width of the rupture plane. W is also estimated from the aftershock distributions. When the aftershock distribution is unknown (e.g. pre-instrumental large ruptures), W is assumed to be the estimated thickness of the seismogenic zone (Wells & Coppersmith 1994).

Since the importance of tectonic environment in scaling relations is still a matter of debate (Scholz *et al.* 1986; Kanamori & Allen 1986; Kanamori & Anderson 1975; Wells & Coppersmith 1994; Romanowicz 1992), we further separate the data into interplate and non-interplate events. Earthquakes that occurred on the San Andreas fault, California, the Fairweather fault, Alaska, the North Anatolian
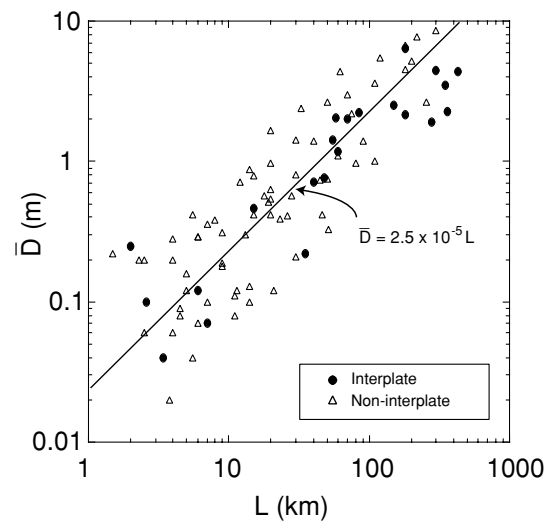
Figure 5. Slip-length relation of real earthquakes in a log–log plot. The solid line indicates a linear relationship. Data are modified from Wells & Coppersmith (1994), plus several earthquakes occurred since their compilation. Open symbols represent interplate earthquakes, and solid ones are non-interplate events.

fault in Turkey and the Motagua fault in Guatemala are classified under the interplate group. Earthquakes that clearly occurred within a plate, and those that occurred in a diffuse zone surrounding plate boundaries, are classified as non-interplate earthquakes. This qualitative classification is often ambiguous, and varies somewhat with different authors. Furthermore, Scholz *et al.* (1986) and Kanamori & Allen (1986) argued that slip rates and repeat times provide a more meaningful categorization. Unfortunately, we cannot use this approach since slip rates and repeat time data are unknown for most of the earthquakes in Wells and Coppersmith's (1994) data set. Nevertheless, faults that we categorize as interplate (open symbols) all have high slip rates and large total geologic offsets.

Fig. 5 shows the relationship between $\overline{D}$ and L obtained from data on a log–log scale. At first impression, a linear relationship $\overline{D} = 2.5 \times 10^{-5}L$, seems to fit this data (Fig. 5). This seems compatible with constant stress drop scaling. But when we plot $\overline{D}/L$ as a function of L (Fig. 6a), we see that the data seem to fall into a wedge-shaped region with a flat bottom at 6×10^{-6} and a top that decreases from 2×10^{-4} at $L = 1$ km to 2×10^{-5} at $L = 1000$ km. Fig. 6(b) shows the suite of $\overline{D}/L$ values obtained for our 1-D simulation and it includes only the points where $\alpha = 1.25$ and

Table 2. Major strike-slip earthquakes that occurred since Wells & Coppersmith's (1994) compilation.

Event	Date	M_0 (Nm)	L (km)	Downdip width W (km)	Average slip $\overline{D}$(m)*	D/L ratio ($\times 10^{-5}$)	Event type	Ref.
Kobe, Japan	01/16/1995	2×10^{19}	45	20	0.74	1.65	a	1
Manyi, China	11/08/1997	2.23×10^{20}	170	20	2.19	1.29	a	2
Izmit, Turkey	08/17/1999	1.47×10^{20}	100	20	2.45	2.45	b	3
Duzce,Turkey	11/12/1999	4.7×10^{19}	55	20	1.42	2.59	b	3
Hector Mine, California	10/16/1999	6.2×10^{19}	84	16	1.56	1.85	a	4
Kokoxili, China	11/14/2001	5.27×10^{20}	450	20	1.95	0.43	a	5,6
Denali, Alaska	11/03/2002	7.5×10^{20}	340	15	4.9	1.44	b	7

Notes: a. Non-interplate type; b. Interplate type.
*Average slip is determine using $\overline{D} = M_0 \mu$LW), where μ is assumed to be 3.0×10^{10} Pa, L is the rupture length and W is the down-dip width of the rupture plane, W is generally assumed to be the estimated thickness of the seismogenic zone.
References: 1. Ide *et al.* (1996); 2. Peltzer *et al.* (1999); 3. Tibi *et al.* (2001); 4. Ji *et al.* (2002); 5. Antolik *et al.* (2004); 6. Klinger *et al.* (2005); 7. Eberhart-Phillips *et al.* (2003).
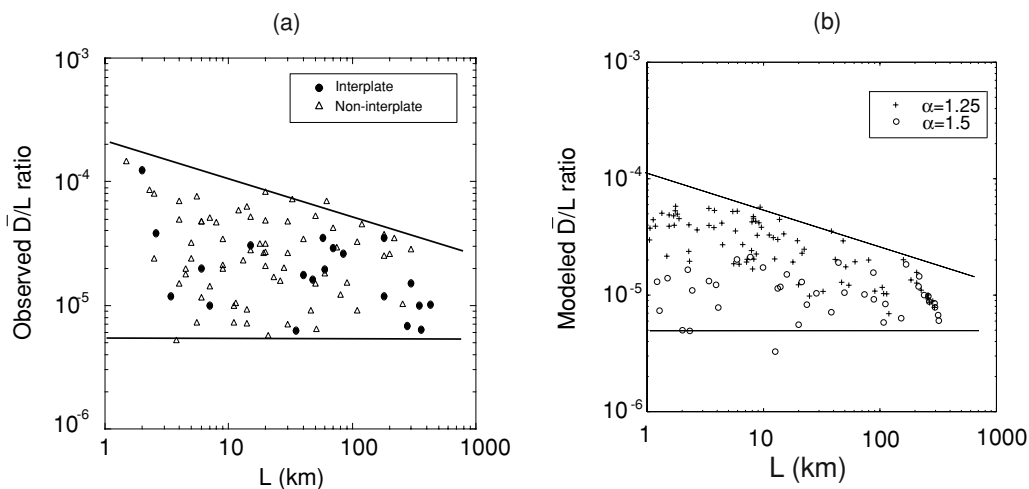
Figure 6. (a) Slip-length ratios ($\overline{D}/L$) versus lengths (L) of real earthquakes. Data seem to fall into a wedge-shaped region, as outlined by two solid lines. There is a larger variation in the slip-length ratio among small earthquakes than among large events. (b) A suite of modelled $\overline{D}/L$ values with $\alpha = 1.25$ and 1.5 show a similar pattern as in (a).

$\alpha = 1.5$. Viewed in this way, it appears that the actual data (Fig. 6b) is similar to our 1-D model assuming that there is a combination of faults having slip smoothness between $\alpha = 1.25$ and $\alpha = 1.5$.

DISCUSSION

There has been a lot of discussion about whether scaling of slip with rupture dimensions should be interpreted with an 'L model' or a 'W model'. A key issue is whether the average stress drop increases with rupture length for very large earthquakes. The decay of geodetically determined ground displacement with distance from the rupture surface of large California strike-slip earthquakes clearly indicates that the bottom of the rupture surface is in the vicinity of 15 km (Thatcher 1975). In this sense, it seems clear that the stress drops of long strike-slip ruptures are determined by the average slip and the width of the seismogenic zone. Observations indicate that average slip increases even for very long ruptures whose widths are saturated. Therefore, average stress drop increases perhaps by a factor of 2 or 3 as rupture gets longer.

Our simple 1-D model of slip heterogeneity offers a different interpretation of this scaling data. Determining the slip and the length of an event becomes a simple game of chance. The stress drop is only determined by how the 'dice roll'. In this model, L and W are statistically linked by a game of chance to D. For earthquakes in which L is less than the width of the seismogenic zone, W and L are more or less comparable; both L and W are statistically linked to D. For long ruptures, L is still linked to D by a game of chance even though W becomes fixed at the width of the seismogenic zone. Thus, although W is of fundamental importance in determining the stress drop, it may not be important for determining the average fault slip when $L \gg W$.

There are several key consequences of the type of model that we introduced. We expect that faults with more heterogeneous slip will have higher $\overline{D}/L$ ratios. This implies that, for a given earthquake moment, we expect to see higher $\overline{D}/L$ ratios on faults with more heterogeneous slip. This is a simple robust result of rule 2; ruptures are contiguous. If there is variability in the roughness of slip from one fault to another, then it seems clear that the faults with rougher slip distributions will tend to have higher $\overline{D}/L$ ratios.

The model also predicts that $\overline{D}/L$ for a particular fault will, in general, decrease with magnitude unless it has $\alpha \sim 1.5$ $\overline{D}/L$ ratio is sometimes taken as a rough measure of average stress drop. In this notion, our model suggests decreasing stress drop with increasing rupture size unless $\alpha \sim 1.5$. When $\alpha \sim 1.5$. $\overline{D}/L$ ratio is invariant over all sizes of length, thus average stress drop is constant. $\alpha \sim 1.5$ can be translated to Hausdorff measure $H \sim 1$ and fractal dimension $D \sim 1$ for 1-D slip functions. Thus the constant stress drop model requires that slip functions are essentially Euclidean. This is incompatible with evidence of many highly irregular slip functions (e.g. McGill & Rubin 1999; Mai & Beroza 2002; Rockwell *et al.* 2002). Our results are in general agreement with Mai & Beroza (2002). They characterized earthquake slip complexity of published finite-source rupture models and found that stress drop is inversely proportional to source size. The constant stress drop model may not apply to the published slip maps.

Unlike Mai & Beroza (2002), we emphasize that in real world there may be a combination of faults having different slip smoothness. If the Earth is considered to have a suite of faults with different α, then we expect to observe larger variability of $\overline{D}/L$ for small earthquakes than we do for large ones. In fact, there is compelling evidence that the stress drops of some small earthquakes are in the range of 100 MPa (e.g. Abercrombie & Leary 1993). However, we sincerely hope that we do not encounter such a stress drop for a large earthquake, since the average slip would have to be in the vicinity of 100 m.

We base our modelling mainly in 1-D. It should be possible to extend our simple model into 2-D, but implementing the model in 2-D has not been straight forward, as one has to make ad hoc assumptions as to how slip tapers to zero at the bottom for a long and narrow rupture. In literature, people have used composite 2-D models consisting of several 1-D horizontal layers (Lavallée & Archuleta 2003), or finite 2-D fault planes of small widths or square lattices, which cannot be scaled realistically to represent shallow and long ruptures (e.g. Andrews 1980; Herrero & Bernard 1994; Mai & Beroza 2002). The 1-D model, as over-simplified as it is, captures the basic features of slip-pulse type models of dynamic rupture. In the slip-pulse models, the width of slip pulse is smaller than that of the seismogenic zone, thus the propagation of the pulse in the horizontal direction may be independent from that in the vertical

direction. Pulses with large slip tend to propagate larger distance, regardless of the final width of the rupture.

Nonetheless, we have tested a 2-D model of a square element matrix, and the results from this model are similar to those in one dimension. That is, a wedge-shaped $\overline{D}/L$ ratios as a function of L, for a suite of smoothness. For our 2-D simulations,

$$D(x, y) = D_0 F T_{2d}^{-1} \frac{R(kx, ky)}{\alpha'\sqrt{kx^2 + ky^2}} \tag{4}$$

where α' is the measure of smoothness of slip in two dimensions. α' is different from, but is mathematically linked to that in 1-D, $\alpha' = \sqrt{2}\alpha$.

While we have emphasized the relationship between $\overline{D}/L$ and L for this simple model, we can also investigate the statistical behaviour of the frequency of occurrence of different size events. Fig. 2 shows that when $\alpha < 1.5$, there are more small events than there are large ones. This is slightly suggestive of real earthquake phenomena. However, we cannot directly compare the statistics of our 1-D model with actual earthquake statistics since they involve fault systems that are at least 2-D. We ran our preliminary 2-D model multiple times with different values of α, and magnitudes of events were calculated using

$$M_0 = \mu \iint_S D \, dx \, dy \tag{5}$$

and

$$M \equiv \frac{\log M_0}{1.5} - 6.033, \text{ where } M_0 \text{ in N m} \tag{6}$$

The number frequency statistics is shown in Fig. 7. The number frequency statistics with $\alpha = 1.5$ and 1.25 are closest to the Gutenberg–Richter relationship. But none of the statistic with a single value of α fit the Gutenberg–Richter relationship over the entire range of earthquake size. While $\alpha = 1.5$ seem to fit at high moment range, but does not fit at low moment range. On the other hand, $\alpha = 1.25$ seems to produce a fit of the G-R relationship in the low moment range, but has misfit at high moment range. Perhaps a combination of earthquakes with α between 1.5 and 1.25 would produce close to
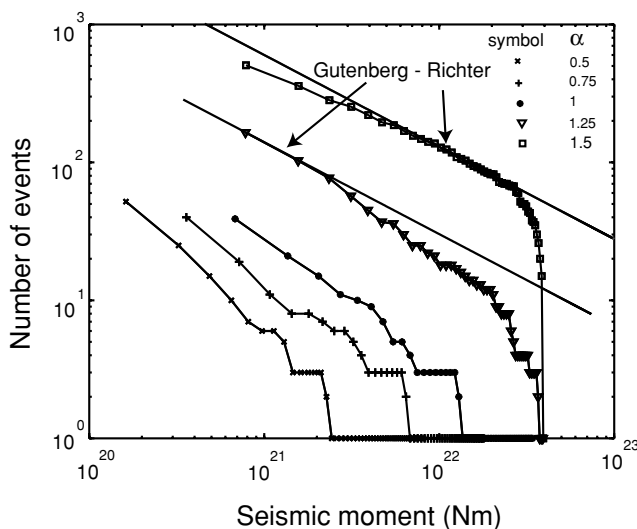
Figure 7. Frequency-magnitude statistics for a 2-D stochastic model with a set of smoothness α. Solid lines represent Gutenberg-Richter law. A combination of earthquakes with $\alpha = 1.5$ and 1.25 would produce close to Gutenberg-Richter statistics.

Gutenberg–Richter statistics. If it is true, then the combination of α between 1.25 and 1.5 indicated by the frequency-size distribution is coincident with that by the pattern of $\overline{D}/L$ ratio with L that seems appropriate for slip-length scaling.

Although we have hypothesized that slip is spatially heterogeneous at all length scales, we have not provided any physical explanation for such behaviour. One hypothesis is that the slip heterogeneity is related to complexities in the geometry of the fault zone or in the frictional properties of the fault zone (Rice 1993; Andrews 1994; Aki 1995); that is, spatially heterogeneous slip is the result of spatially heterogeneous media. However, others have demonstrated that heterogeneous slip can be spontaneously generated by nonlinear processes that arise from positive feedback between slip and friction in some classes of dynamic rupture simulations (Shaw 1995; Cochard & Madariaga 1996; Aagaard *et al.* 2001).

We chose a simple fractal form for our slip heterogeneity, but we are unable to provide compelling evidence that this particular form is a close approximation of real earthquake slip heterogeneity. For instance, our slip distributions are constructed from random number sequences; changing this assumption is likely to change the specifics of our relationship between $\overline{D}$ and L. Nevertheless, it does seem clear that slip is an irregular function of position on a fault (McGill & Rubin 1999; Rockwell *et al.* 2002). Furthermore, it has been shown that finite-fault source inversions of earthquake slip can be well described with stochastic models with power law decay (e.g. Mai & Beroza 2002; Lavallée & Archuleta 2003). We speculate that the existence of heterogeneity, together with the notion that ruptures are spatially contiguous, are sufficient conditions for average stress drop to increase with increasing heterogeneity; in geography, islands with rough topography have higher average elevations than similar sized islands with smooth topography.

CONCLUSIONS

We introduced a simple 1-D model of stochastic slip variation along fault to explore the effect of slip heterogeneity on slip-length scaling and average stress drop. The key assumptions are that (1) slip heterogeneity has a power law decay α with respect to wave number, and (2) individual events are spatially connected regions.

Our model shows that for a given average displacement, earthquakes with smoother slip tend to have longer ruptures. Using only these simple assumptions, we find that the relationship of average slip, $\overline{D}$, and rupture length L depends on the degree of slip heterogeneity. Slip-length scaling is approximately linear only for relatively smooth slip heterogeneity (e.g. smoothness $\alpha = 1.5$). This type of model is motivated by slip pulse models, in which rupture length is determined by the average slip and conditions on a fault (e.g. fault roughness), and static stress drop may not be a controlling factor in rupture dynamics.

The modelled average slip to length ratio $\overline{D}/L$ and therefore average stress drop decreases with increasing rupture length, except when $\alpha \sim 1.5$, in which case $\overline{D}/L$ ratio is constant. This general pattern is consistent with the real data. Our model also implies that slip heterogeneity affects average stress drop; average stress drops are generally higher on faults with rougher slips than on those with smoother slips.

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