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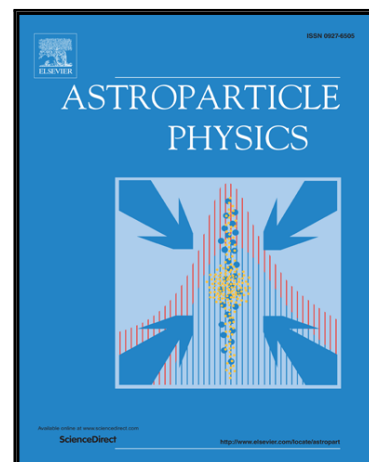
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Highlights

- Reliable solar wind limits on photon mass
- First experimental values in the solar wind through Ampère's law

ACCEPTED MANUSCRIPT

Solar wind test of the de Broglie-Proca massive photon with Cluster multi-spacecraft data

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Abstract

Our understanding of the universe at large and small scales relies largely on electromagnetic observations. As photons are the messengers, fundamental physics has a concern in testing their properties, including the absence of mass. We use Cluster four spacecraft data in the solar wind at 1 AU to estimate the mass upper limit for the photon. We look for deviations from Ampère's law, through the curlometer technique for the computation of the magnetic field, and through the measurements of ion and electron velocities for the computation of the current. We show that the upper bound for m_γ lies between 1.4×10^{-49} and 3.4×10^{-51} kg, and thereby discuss the currently accepted lower limits in the solar wind.

Keywords: Photon mass, Solar wind, Cluster satellites, Ampère's law

PACS: 14.70.Bh, 94.80+g, 95.40.+s, 96.50.Ci

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1. Introduction

We have just witnessed the opening of the gravitational wave window on the universe [1, 2]. Nevertheless, our understanding of the universe at large and small scales will continue to rely also on electromagnetic observations. As photons are the messengers, fundamental physics has a concern in testing their properties, including the absence of mass.

Furthermore, while alternative theories of to general relativity are conceived for solving the questions raised by the dark universe or to couple gravity with the other interactions, less effort is deployed for studying alternative electromagnetism. Nevertheless, electromagnetism may differ from the Maxwellian conception of the nineteenth century and thereby contribute to solve some of the riddles in contemporary physics and cosmology.

Non-Maxwellian theories can be classed into non-linear or massive photon based. The former were initiated by Born and Infeld [3], Heisenberg and Euler [4]. For the latter, the initiator was de Broglie who assessed the photon mass lower than 10^{-53} kg [5, 6], reinterpreted the work of his student Proca [7–11], and achieved a comprehensive formulation of the photon [12]. The de Broglie-Proca, henceforth dBP, equations follow relativity laws for reference frames moving at constant velocities (Lorentz-Poincaré transformations); instead, a change of potential implies a change in the field (not Lorenz gauge invariant). Gauge invariant formulation have been later proposed by several authors; among the early contributions, we have Stueckelberg [13–15], Podolsky [16–18], Chern and Simons [19]. Phenomenologically oriented reviews include [20–23].

Massive photons have been evoked for dark matter, inflation, charge conservation, magnetic monopoles, Higgs boson, redshifts; in applied physics, superconductors and "light shining through walls" experiments. The mass can be considered effective, if depending on given parameters.

How much the foundations of physics are affected by a massive photon is not straightforward to assess, for the variety of the theories, the removal of our ordinary landmarks and the rising of interwoven implications.

The dBP equations [12] differ from the four original Maxwell ones solely in the divergence of the electric field \vec{E} , and in the curl of the magnetic field \vec{B} . They describe a massive spin-1 boson. In SI units, the dBP equations are

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} - \mathcal{M}^2 \phi, \quad (1)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad (2)$$

$$\vec{\nabla} \cdot \vec{B} = 0, \quad (3)$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} - \mathcal{M}^2 \vec{A}, \quad (4)$$

where $\mathcal{M} = m_\gamma c / \hbar = 1/\lambda$, m_γ is the photon mass, $c = 2.99 \times 10^8 \text{ m s}^{-1}$ the speed of light, $\hbar = 1.05 \times 10^{-34} \text{ J s}$ the reduced Planck's constant, λ the reduced Compton wavelength, $\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$ the permittivity, $\mu_0 = 1.26 \times 10^{-6} \text{ N A}^{-2}$ the permeability, ρ the charge density, \vec{j} the current density vector, ϕ and \vec{A} the scalar and vector potential.

The first estimates are due to Schrödinger [24, 25], who noted that \vec{A} falls off exponentially at large distances

$$\vec{A} = \vec{\nabla} \times \frac{\vec{m}}{r} e^{-r/\lambda}. \quad (5)$$

For a distance comparable to the reduced Compton wavelength, the exponential term lead to the observable deviations from the Maxwell power law scaling, equivalent to determining the photon mass.

A laboratory Coulomb's law test determined the mass upper limit of $2 \times 10^{-50} \text{ kg}$ [26]. de Broglie [12] observed that photon speed would be proportional to the inverse of the square of the frequency, unfortunately like it occurs for dispersion due to the ionised parts of the interstellar medium. For pulsars, delays of lower energy incoming photons are routinely measured, but lacking any independent estimate on the electron density, the differences are solely attributed to plasma. The dispersion-based limit is $3 \times 10^{-49} \text{ kg}$ [27]. In the last years, photon dispersion laws have been extensively analysed in the quest for quantum gravity, e.g., [28–32], where classical relativistic symmetries are broken by Planck-scale effects, with the emergence of a preferred frame, or else [33–36] where relativistic symmetries are only deformed by the Planck scale.

With Jupiter and Earth magnetic fields, a limit around 10^{-51} kg was set [37, 38]. In the solar wind, Ryutov found 10^{-52} kg at 1 AU [39, 40], and $1.5 \times 10^{-54} \text{ kg}$ at 40 AU [40], limit accepted by the Particle Data Group (PDG) [41]. For recent studies of various nature, see [22, 42–46]. Lower limits ($3 \times 10^{-63} \text{ kg}$) have been claimed when modelling the galactic magnetic field [47–49]. The lowest value for any mass is dictated by Heisenberg's principle $m \geq \hbar/\Delta t c^2$, and gives $3.8 \times 10^{-69} \text{ kg}$, where Δt is the supposed age of the Universe.

The large-scale [47–49] and other limits might be legitimate, but remain the outcomes of models and observations rather than strictly experimental limits. Indeed, Goldhaber and Nieto state “Quoted photon-mass limits have at times been overly optimistic in the strengths of their characterizations. This is perhaps due to the temptation to assert too strongly something one knows to be true” [20]. We share this concern.

Estimates from solar wind magnetic fields [39, 40, 50] are partly based on *in situ* measurements, but a close scrutiny reveals that: (i) the magnetic field is assumed exactly always and everywhere a Parker spiral; (ii) the accuracy of particle data measurements (from, *e.g.*, Pioneer or Voyager) has not been discussed; (iii) there is no error analysis, nor data presentation. In [39], Ryutov first refers to the limit of 10^{-51} kg, relative to the Jupiter magnetic field, obtained by others in [37]. Then he discusses that an imbalance of the magnetic forces in the neighbouring areas of the solar wind would have caused violent plasma motions with an average energy exceeding the energy of the ions by three orders of magnitude; since such motions are not observed in the solar wind, the author lowers the estimate in [37] of one order of magnitude, and sets the mass upper limit at 1 AU. In [40], still Ryutov refers to Voyager 1 and Voyager 2 data appeared in previous work [51–53] to justify strict use of Parker’s model, and to adopt the *reductio ad absurdum* approach already used in [39]. Indeed, on the basis that the magnetic field is almost entirely azimuthal, Ryutov considers that the Lorentz force would be increased of a factor $(L_B/\lambda)^2$, where L_B is the magnetic field characteristic length, with respect to the Maxwellian case. Since the deviations from the observed flow structure would become grossly incompatible with the real situation, the mass upper limit is lowered to 1.5×10^{-54} kg at 40 AU. A margin of a factor three constitutes the error budget. Finally, the authors in [50], again without presenting data, argue that the mass upper limit could be lowered of a factor two.

For checking such solar wind estimates, we therefore attempt a more experimental approach. We need either a precise experiment or a large apparatus, since a small m_γ is associated to a large λ . Herein, we focus on the second possibility through the largest-scale magnetic field accessible to *in situ* spacecraft measurements, *i.e.* the interplanetary magnetic field carried by the solar wind.

2. Dealing with Cluster data

Cluster [54] is an ESA mission composed by four spacecraft flying in tetrahedral configuration at 1 AU from the Sun, and separated by distances ranging from 10^5 to 10^7 m. It has allowed the first direct computation of three-dimensional

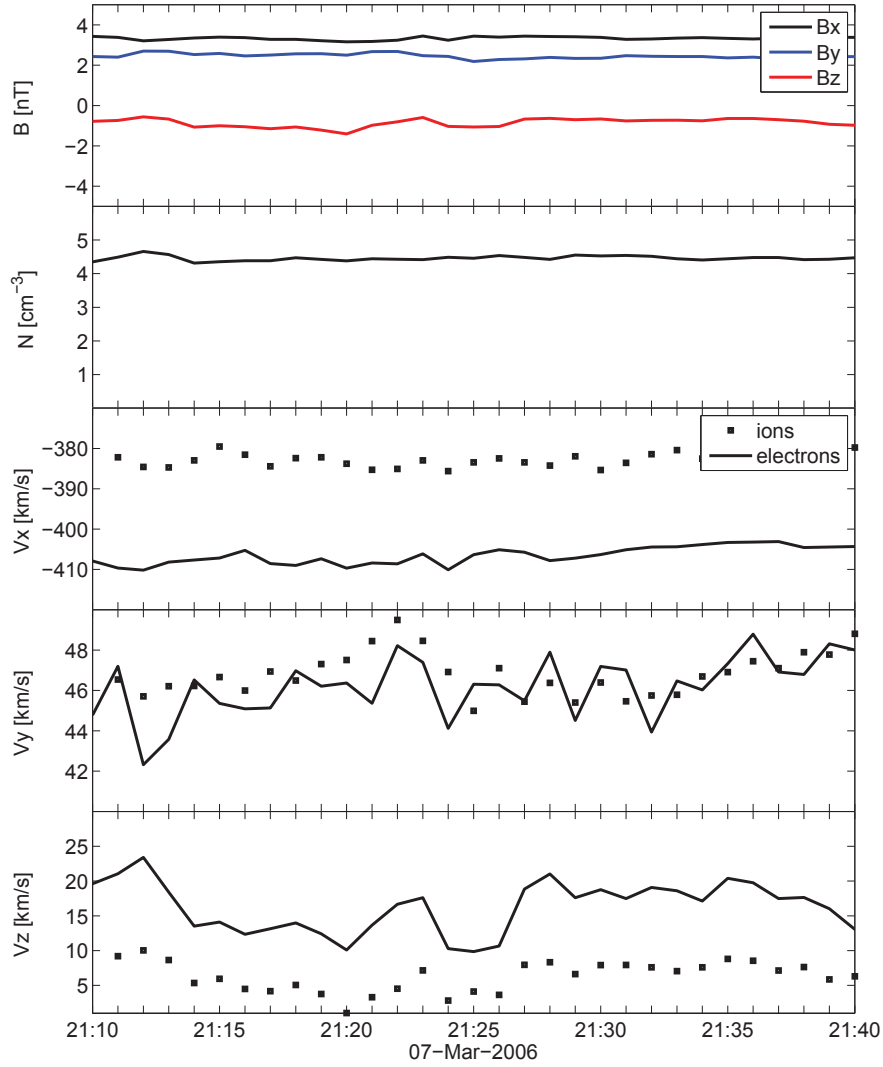


Figure 1: Panel (First from above). The three components of the magnetic field for Cluster 3 in the GSE (Geocentric Solar Ecliptic) coordinate system. The magnetic field (nT) is measured at 22 samples/s and has been re-sampled at the time resolution of the spacecraft position measurements (1 minute). The GSE coordinate system is defined as having the X direction pointing from the Earth to the Sun, the Z direction orthogonal to the ecliptic plane, and the Y direction completing the right-handed system. Panel (Second from above). The average plasma density ($N \text{ cm}^{-3}$). Panels (Third, Fourth, Fifth from above). The v_x , v_x , v_x velocity components (km s^{-1}) of ions (dotted line) and electrons (full line) in GSE, respectively. The ion velocity is measured by one spacecraft (C1). The electron velocity is the average over the tetrahedron of the velocity measured by all four spacecraft (C1,C2,C3,C4).

quantities such as $\vec{\nabla} \times \vec{B}$ from magnetic field four-point measurements in the solar wind.

For large-scale steady components of the magnetic field, *i.e.* very low frequencies, the displacement current density in Eq. (4) can be dropped. For v_{sw} the solar wind velocity (for the numerical values, see the text below), we have

$$\epsilon_0 \mu_0 \frac{\partial E}{\partial t} \sim \epsilon_0 \mu_0 \frac{E v_{sw}}{L_B} \sim \epsilon_0 \mu_0 \frac{B v_{sw}^2}{L_B} \sim 2 \times 10^{-22} \text{ A m}^{-2},$$

and Eq. (4) reduces to the dBP Ampère's law

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} - M^2 \vec{A}. \quad (6)$$

For the current densities $\vec{j}_B = \vec{\nabla} \times \vec{B} / \mu_0$ and $\vec{j} = \vec{j}_P = ne(\vec{v}_i - \vec{v}_e)$, n the number density $n = 4.46 \times 10^6 \text{ m}^{-3}$, e the electron charge $1.6 \times 10^{-19} \text{ C}$, \vec{v}_i , \vec{v}_e the velocity of the ions and electrons, respectively, the dBP photon mass is

$$m_\gamma = \frac{k}{|\vec{A}_H|^{\frac{1}{2}}} \left| ne(\vec{v}_i - \vec{v}_e) - \frac{\vec{\nabla} \times \vec{B}}{\mu_0} \right|^{\frac{1}{2}} = \frac{k |\vec{j}_P - \vec{j}_B|^{\frac{1}{2}}}{|\vec{A}_H|^{\frac{1}{2}}}, \quad (7)$$

where $k = \hbar \mu_0^{\frac{1}{2}} c^{-1}$, and \vec{A}_H is the vector potential from the interplanetary magnetic field.

We have selected a few events in Cluster solar wind orbits with the following criteria: (i) an undisturbed solar wind, *i.e.* disconnected from the terrestrial bow shock, and as far as possible from the terrestrial magnetic field; (ii) the closest location of the spacecraft to the equatorial plane; (iii) the widest inter-spacecraft separation, about 10^7 m , assuring the largest differences in the magnetic field among the spacecraft; (iv) the configuration best approaching the tetrahedron; (v) the availability of good quality particle currents. We discuss the event having the most accurate particle data (7 March 2006) since the particle current density is the observable that eventually determines the upper bound. Figure 1 shows the magnetic field measured by Cluster for this event. The magnetic field has $B_x > 0$, $B_y > 0$ and $B_x, B_y \gg B_z$, as expected for a Parker spiral configuration close to the ecliptic plane. It is emphasised, however, that our analysis does not rely on the Parker model, since the magnetic field is measured *in situ*. The choice of an event having Parker's spiral orientation is to have conditions as similar as possible to those presented in [39, 40] which are the only available estimates on the upper bound of m_γ in the solar wind.

We measure j_B using the curlometer technique [55, 56] on the magnetic field data from the fluxgate magnetometer [57]. The average $\vec{\nabla} \times \vec{B}$ is computed over the tetrahedron with no assumptions on the field analytical form (only assuming linear gradients). The error on j_B is

$$\frac{\Delta j_B}{\langle j_B \rangle} = \frac{\Delta B}{\langle \delta B \rangle} + \frac{\Delta R}{\langle R \rangle}, \quad (8)$$

with $\langle j_B \rangle$ being the average current density, ΔB the error on the magnetic field, $\langle \delta B \rangle$ the average value of the magnetic field difference between the satellites, ΔR the error on the spacecraft separation R , $\langle R \rangle$ the average separation between the satellites. An estimate on the deviation from linearity is given by the quantity $|\vec{\nabla} \cdot \vec{B}|/|\vec{\nabla} \times \vec{B}|$. The magnitudes of j_B and of its absolute error Δj_B are shown in Fig. 2. The absolute errors on the magnetic field $\Delta B = 0.1$ nT and on spacecraft separation $\Delta R = 10^3$ m correspond to typical values [56]. The average current density over the duration of the event is $\langle j_B \rangle \approx (3.5 \pm 4.7) \times 10^{-11}$ A m⁻². The relative error and the value of $|\vec{\nabla} \cdot \vec{B}|/|\vec{\nabla} \times \vec{B}| \approx 75\%$ are larger than typical estimates [56]. This is due to the large values of the elongation $E \sim 0.6$ and the planarity $P \sim 0.7$ of the tetrahedron for this event (the ideal estimate occurs for a perfect tetrahedron with $E=P=0$). In absence of an event with both a correct tetrahedron shape and adequate particle measurements, we picked an event where the latter condition is met (conversely, we found a much better accuracy for $\langle j_B \rangle$ in other events - not shown herein). Similar independent results on the error on $\langle j_B \rangle$, through applying random variations on magnetic field measurement and satellite position at each of the satellites, and estimating the standard deviation of the obtained current fluctuations.

The particle current density is $\langle j_P \rangle \approx (1.86 \pm 3) \times 10^{-8}$ A m⁻², Fig. 2. In all events considered, we have not found significantly lower values of j_P and of its error. An accurate assessment is difficult due to inherent instrument limitations. Ion electrostatic analysers [58] can saturate in the solar wind due to the presence of high fluxes, resulting often in incorrect ion currents. The computation of the currents from electron analyzers is affected by photoelectrons and spacecraft charging issues [59]. The value of the current density j_P for this event (as well as for other events) is much larger than that from the curlometer. This is mostly due to the differences in the velocities, while the estimate on density is reasonable. Figure 2 shows two (for other events up to four) orders of magnitude between j_P and j_B . While we can't exclude that (part of) this difference is due to the dBP massive photon, the large uncertainties of particle currents hint forcefully to instrumental limits.

2.1. The estimations of A_H

Since Eq. (3) h, holds, then $\vec{\nabla} \times \vec{B} = \vec{A}$ still holds in dBP theory in absence of magnetic monopoles. Nevertheless, Eq. (6) depends on the value of \vec{A} . Thus, conversely to Maxwellian electromagnetism, the dBP theory is not Lorenz gauge invariant. The potential is perceived as a measurable quantity, the values of which implies a change in the fields. Turning to A_H due to the interplanetary magnetic field at Cluster location, we arrange three estimations.

2.1.1. Four point estimate

From j_B and $B = 4.23 \times 10^{-9}$ T, we get $L_B \sim B/\mu_0 j_B \approx 9.6 \times 10^7$ m. For our event, the inter-spacecraft separation is 6×10^6 m, thus much smaller than L_B . The characteristic value of the vector potential is $A_H \sim B \times L_B \approx 4.1 \times 10^{-1}$ T m. The variation δ of this estimate is comparable to the observables themselves $B \sim \delta A_H / \delta L \sim A_H / L_B$ [40].

The errors on B and j_B lead to even lower values of A_H (1.1×10^{-1} T m), and thus a higher upper limit on m_γ of a factor two.

The advantages of the curlometer method consist in performing instantaneous measurements, in avoiding assumptions on the local structure of B and on the (or lack of) steadiness of the solar wind. On the other hand, it is based on the first derivatives of B , and thus it may not be suited for small volumes.

2.1.2. Single spacecraft estimate

Alternatively A_H is found through a single spacecraft monitoring B as it advects past the spacecraft. The drawback is that A_H can only be evaluated along the solar wind flow (approximately, the GSE X direction), as opposed to the curlometer estimate being performed in all directions. A different aspects of the same limit, the single spacecraft method works exclusively in presence of temporal variations in the solar wind. Further, without an analytical model of the magnetic field and thus of A_H , from only *in situ* measurements it is not possible to determine the direction where the largest component of A_H lies. We assessed the Z component of A_H due to the Y component of B (being $B^Y > B^Z$) over an interval of about ± 12 hours around the event pointed in Fig. 1. For the event close to the equatorial plane

$$B^Y = \frac{\partial A_H^Z}{\partial x} - \frac{\partial A_H^X}{\partial z} \sim \frac{\partial A_H^Z}{\partial x}, \quad (9)$$

we get

$$\delta A_H^Z \sim A_H^Z \sim \int B^Y(t) dx \sim v_{sw} \int B^Y(t) dt, \quad (10)$$

being $v_{sw} = 4 \times 10^5 \text{ m s}^{-1}$ the typical advection velocity. It appears that A_H is linearly changing in the time interval 14:00-22:00, where B^y is roughly constant, and thus $A_H^z \approx 29 \text{ T m}$. Along the flow direction, it determines $L_B \approx 6.7 \times 10^9 \text{ m}$, about seventy times larger than L_B obtained through the curlometer.

It is not possible to establish which of these two approaches is more reliable.

2.1.3. Parker's model estimate

We wish to investigate in all directions, and we thus recur to the Parker model for a third (non-experimental, *i.e.* without data) assessment. We compute A_H in spherical coordinates from [60]

$$A_r = \frac{2b}{3} \left[1 - \frac{3}{2}x - x \ln(1+x) \right], \quad (11)$$

$$A_\theta = \frac{2b}{3} \sin \theta \left[\frac{x}{1+x} + \ln(1+x) \right] \left(\frac{\cos \theta}{x} \right), \quad (12)$$

$$A_\phi = \frac{a}{r \sin \theta} (1-x), \quad (13)$$

where $x = |\cos \theta|$, $a = 3.54 \times 10^{-9} \text{ T AU}^2$ and $b = 3.54 \times 10^{-9} \text{ T AU}$, and get $A_H = 637 \text{ T m}$ (for $\theta = \pi/2$).

The vector potential, Eqs. (11-13), is computed in Coulomb's gauge $\nabla \cdot \vec{A} = 0$. The dBP equations are not Lorenz gauge invariant but automatically satisfy the Lorenz gauge, that is $\nabla \cdot \vec{A} + 1/c^2 \partial \phi / \partial t = 0$. Thus, in our Coulomb gauge case, the scalar potential ϕ must be constant in time. This latter condition inserted in the time derivative of $\vec{E} = \nabla \phi - \partial \vec{A} / \partial t$, and recalling that we deal with a static case for which $\partial \vec{E} / \partial t = 0$, implies that Eqs. (11-13) are valid only if A_H varies at most linearly in time. Indeed, the event under scrutiny fulfills this feature.

2.2. The mass upper limits

We retain from the previous sections that the value of A_H is result of estimates and not of a direct measurement. At best, it depends indirectly on measurements (four point and single spacecraft estimates); else, it is the outcome of a model (Parker estimate). Further, the values of A_H itself vary greatly in the three estimates. Thus, considering its errors would appear misplaced. Indeed, it seems appropriate that our experimental approach measures an upper limit for $A_H^{1/2} m_\gamma$, Eq. (7), and that we compute three estimates of m_γ , one for each value of A_H . Finally, we consider k an error-free constant (in dBP theory c depends on the frequency, but herein we don't deal with dispersion issues).

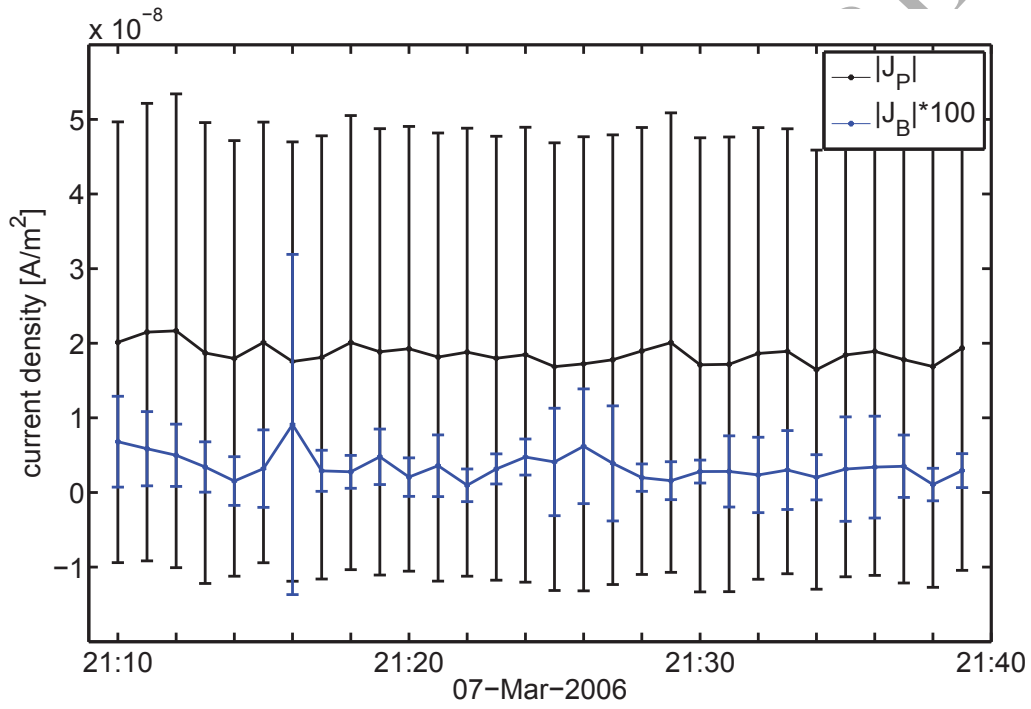


Figure 2: The current densities j_B (computed with the curlometer) and j_P (inferred from the particle detectors) and the associated experimental errors. Δj_B is estimated by using Eq. (8), while Δj_P is obtained through the propagation of maximal errors on density n , ion velocity v_i and electron velocity v_e taken as $\Delta n = 10\%$, $\Delta v_i = \Delta v_e = 5\%$ [61].

Table 1: The values of m_γ (according to the estimate on A_H).

A_H [T m]	0.4	29 (Z)	637
m_γ [kg]	1.4×10^{-49}	1.6×10^{-50}	3.4×10^{-51}

Error propagation analysis indicates for the worst case [62]

$$A_H^{\frac{1}{2}} (m_\gamma + \Delta m_\gamma) = A_H^{\frac{1}{2}} \left(m_\gamma + \left| \frac{\partial m_\gamma}{\partial j_P} \right| \Delta j_P + \left| \frac{\partial m_\gamma}{\partial j_B} \right| \Delta j_B \right) = k \left[(j_P - j_B)^{\frac{1}{2}} + \frac{\Delta j_P + \Delta j_B}{2(j_P - j_B)^{\frac{1}{2}}} \right]. \quad (14)$$

Considering j_P and Δj_P of the same order, $j_P = 0.62 \Delta j_P$, and both much larger than j_B and Δj_B , Eq. (14), after squaring, obviously leads to

$$A_H^{\frac{1}{2}} (m_\gamma + \Delta m_\gamma) \sim k (j_P + \Delta j_P)^{1/2}. \quad (15)$$

The outcome, Eq. (15), can be drawn directly by Eq. (7) using the same previous assumptions. Table (1) displays the values of m_γ , for three different estimated values of A_H . The upper limits are all well above those appeared in [39, 40].

3. Discussion

The most stringent limitation comes from the particle detectors. The difference between ion and electron velocities is $v_{i-e} \sim (j_P + \Delta j_P)/ne = 6.8 \times 10^4 \text{ ms}^{-1}$. Recasting Eq. (15), we derive v_{i-e} that particle detectors should measure to resolve the m_γ upper bound for a given A_H

$$v_{i-e} \sim \frac{A_H m_\gamma^2}{k^2 ne}. \quad (16)$$

This is plotted in Fig. 3 where the vertical lines refer to earlier literature and to our Cluster spacecraft test. The upper limit 10^{-52} kg in [39, 40] requires resolving a difference v_{i-e} of approximately 3.6×10^{-2} , 2.6 , $5.7 \times 10^1 \text{ m s}^{-1}$ (for the three values of A_H) which is not possible with currently available particle detectors, onboard Cluster or other spacecraft. Otherwise stated, one would need $A_H = 7.5 \times 10^5 \text{ T m}$, to obtain an upper limit of 10^{-52} kg with the measured $v_{i-e} \approx 6.8 \times 10^4 \text{ m s}^{-1}$. Such high values of A_H are not stated in [39, 40]; conversely a value of 10^3 T m is stated in [63].

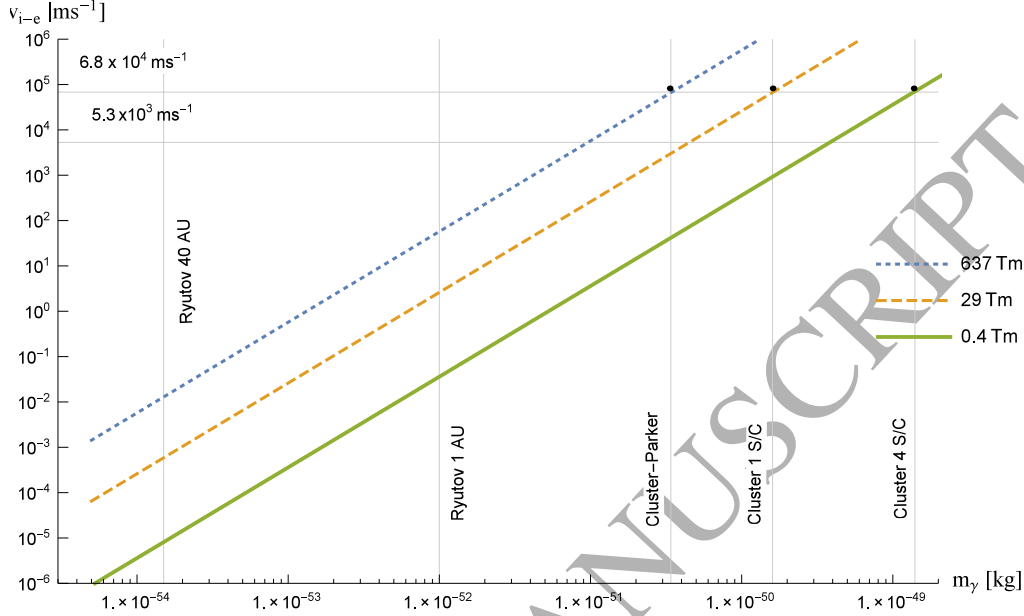


Figure 3: The ion-electron velocity difference v_{i-e} versus m_{γ} , for three A_H values (0.4, 29, 637 Tm). The vertical dotted lines represent solar wind [39, 40] and Cluster limits, Tab. (1). The horizontal dotted lines represent $v_{i-e} = 6.8 \times 10^4$ and $v_{i-e}^Y = 5.3 \times 10^3$ m s⁻¹ for Cluster.

We further note that the upper limit in [39, 40] of 10^{-52} kg requires a very low value of $j_P + \Delta j_P$, even for the highest value of the potential, that is $A_H = 637$ Tm. Thus, assuming the Parker model and $j_P + \Delta j_P \simeq j_B + \Delta_B \sim 10^{-11}$ A m⁻², we get the currently accepted limits at 1 AU. Nevertheless, we don't find such low values of particle current in our data.

Few improvements could still be achieved with Cluster data. One could select only those components of j_P having the best accuracy. Figure 1 shows the differences between ion and electron velocities in the GSE coordinate system. The smallest difference $\approx 10^3$ m s⁻¹ is in the Y direction, *i.e.*, in the equatorial plane. This is the most accurate component for this event, also considering that both ion and electron velocities are close to the solar wind velocity along Y due to solar wind aberration (on average $\approx 3 \times 10^4$ m s⁻¹). When using $j_P^Y = (4.21 \pm 33.97) \times 10^{-10}$ A m⁻², we lower the m_{γ} upper limit of a factor four, and v_{i-e}^Y is around 5.3×10^3 m s⁻¹. However, this implies that A_H has its largest component in the Y direction, as indicated by the Parker model.

Our analysis does not assume that the solar wind is in steady-state (Parker's

model), except for the third estimate on A_H . Therefore, our approach is indeed different from [39, 40]. Therein, the average large scale properties of the solar wind are taken from statistical datasets; and m_γ is obtained through *reductio ad absurdum* arguments on the integral effect of magnetic stresses ($\vec{j} \times \vec{B}$) with respect to the dynamic solar wind pressure, see Eq. (11) in [40], assuming a steady-state Parker's model of the magnetic field. In our approach, we estimate the local vector potential A_H , and we measure the local currents j_B , j_P with few assumptions on the structure of the magnetic field and on the steadiness of the solar wind. This implies that our estimates on A_H , notably the former two, are not only related to the large scale average field described by Parker's model, but also to the always present fluctuations in the solar wind.

4. Conclusions

We have reported three new estimates for the upper limit of the dBP photon mass by using Cluster multi-spacecraft measurements in the solar wind. We have found larger values than previous estimates, our test being based on fewer assumptions. First, we directly assess $\vec{\nabla} \times \vec{B}/\mu_0$ from four-point magnetic field measurements; this was before impossible with a single spacecraft. Second, for two estimates we do not assume that the interplanetary magnetic field is a Parker spiral, though we have chosen events compatible to the Parker spiral for comparison with earlier findings [39, 40]. For the third estimate, we assume the Parker model, and yet we get a larger upper limit than in [39, 40]. Third, it is the only solar wind test that takes into account in detail the experimental errors. The domain between our findings ($m_\gamma < 10^{-49}$ kg) and the results from *ad hoc* model in the solar wind ($m_\gamma < 10^{-52}, 10^{-54}$ kg) is still subjected to assumptions and conjectures, though fewer now, and not to clear-cutting outcomes from experiments. Our experiment is limited by the resolution of the velocity difference between ions and electrons.

5. Perspectives

For achieving more reliable results, the improvement of particle detector performances and planning multi-spacecraft mission appear both mandatory. Already with Cluster, a resolution of 10^3 m s⁻¹ is attainable when considering the best measured Y component of the flow that nicely reproduces the expected aberration of the solar wind. Further improvements could come from better estimates on the

moments in the solar wind, that is mainly electrons which are more affected by the spacecraft potential.

The MMS satellites might represent another opportunity to apply our method, given the features of the mission and of the measured currents [64]. The impact of the limited separation distance can be assessed, and a study of the impact of the time and velocity resolutions of the particle detectors on the upper limit of the photon mass would be desirable.

We hope that payload designers will be attentive to the requirement that fundamental physics measurements pose. If so, new avenues of research could be explored.

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