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FAST TRACK PAPER

A new approach to computing accurate gravity time variations for a realistic earth model with lateral heterogeneities

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SUMMARY

We have developed a new elasto-gravitational earth model able to take into account lateral variations, deviatoric pre-stresses and topographies. As a first application, we assume an ellipsoidal earth with hydrostatic pre-stresses, and validate and discuss our numerical model by comparison with previous studies on the M_2 body tide. We then study the response of the ellipsoidal earth to zonal atmospheric loads, and find that global lateral variations within the Earth, such as ellipticity, have a weak impact (about 1 per cent) on the elasto-gravitational deformations induced by atmospheric loading.

Key words: atmospheres, finite-element methods, geodynamics, lateral heterogeneity, tides.

1 INTRODUCTION

At low frequencies, the Earth is deformed mainly by luni-solar tides and by surface loads, including ocean, atmosphere, ice volumes and post-glacial rebound. In this work, we focus our attention on the Earth's body tides and atmospheric loadings.

The most accepted Earth body-tide models presently deal with an ellipsoidal, rotating earth, containing a liquid core and an anelastic mantle with hydrostatic pre-stresses (Wahr 1981; Wahr & Bergen 1986). The Earth, however, is not an exact ellipsoid, but presents lateral variations and deviatoric pre-stresses: there are long-wavelength density anomalies within the mantle, as shown by geoid anomalies and tomography studies (e.g. Romanowicz & Gung 2002). Wang (1994) and Dehant *et al.* (1999) studied the influence of lateral heterogeneities on Earth tides and showed that this effect is small but not necessarily negligible. They did not, however, take into account possible deviatoric pre-stresses: these effects on the Earth's body tides are totally unknown.

In addition to tidal forces, mass changes in the atmosphere also cause deformation and mass redistribution inside the planet, involving both local and global surface motions and variations in the gravity field, which may be observed in geodetic experiments. For several decades, satellite geodesy has provided information on the temporal variation of the Earth's geopotential, and especially on the low-degree zonal harmonics (J_2, J_3 ...) (Gegout & Cazenave 1993), which are essentially controlled by surface loads. These hydrological, atmospheric or oceanic effects on the Earth's gravity field are usually modelled assuming a spherical earth with hydrostatic pre-stress (e.g. Farrell 1972; Wahr *et al.* 1998).

With the advent of the new generation of gravity measurements, one of the challenges of the coming decade will be to provide more realistic earth models that show the variation of gravity with time. In particular, global studies based on gravity data from satellites such as GRACE, GOCE, and future GRACE/GOCE follow-on ones require accurate body-tide deformation models. More realistic gravity variation models are also needed for local and ground measurements, particularly for the very accurate superconducting gravimeters and the associated gravimetric observatory network such as the Global Geodynamic Project (Crossley *et al.* 1999).

The formalism developed to compute this elasto-gravitational model is usually based on spherical harmonic analysis. The addition of lateral variations leads to couplings between spherical harmonics, i.e. to a more complex formalism that requires a large numerical effort (e.g. Wang 1994; Plag *et al.* 1996). We develop here a new approach for a non-radially symmetrical earth model using a finite-element method known as the spectral element method. The efficiency of this method is less dependent on the shape of the lateral heterogeneities than the spherical harmonic method. Our method is therefore well adapted to studying the impact of global and local lateral variations on the Earth deformation.

We solve the elasto-gravitational equations taking into consideration the lateral variations within the Earth by using a first-order perturbation theory (Smith 1974; Dahlen & Tromp 1998). This new model allows us to take into account lateral variations of density and rheological parameters, deviatoric pre-stresses and interface topography.

In order to validate our calculations, we tackle a well-known problem: the impact of the hydrostatic ellipticity on the Earth body tides. An analytical solution for this problem can be derived for a simple model in which the earth is assumed to be homogeneous and incompressible. The gravitational potential and the vertical displacement on the surface of the deformed ellipsoid were first derived by Love (1911) and then corrected by Wang (1994). We have recently extended these analytical results to the tangential surface displacement (Greff-Lefftz *et al.* 2005). We first validate our model with our analytical solutions, and then compare our results with

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previous numerical solutions for a slightly elliptical hydrostatic stratified compressible elastic model (PREM).

We then investigate the effect of the ellipticity of the Earth. This effect is usually neglected in loading deformation studies because it is assumed that, for a surface load, the short-wavelength lateral variations will probably generate much larger perturbations. In this paper, we quantify the real impact of the ellipticity on the loading Love numbers. We then study the perturbations of the temporal zonal coefficients of the geopotential induced by atmospheric loading.

The paper is organized as follows. In the next section, we recall the elasto-gravitational theory for a spheroidal, pre-stressed planet and present some aspects of the spectral element method used in our approach. In the subsequent section, we validate our results with previous analytical and numerical studies. The influence of the ellipticity of the Earth on the deformations induced by M2 tides and by atmospheric loading is investigated in the final section.

2 THEORETICAL AND NUMERICAL APPROACH

2.1 Elasto-gravitational theory

To describe the deformation of a 3-D realistic earth model, we use a first-order perturbation formalism (for more details, see Dahlen & Tromp 1998).

The planet is submitted to volumic forces $\mathbf{f}$, with $\mathbf{f} = -\rho\nabla V$ (luni-solar attraction or gravitational force associated with surface loading), or to surface conditions (pressure or tangential traction), which cause deformations and mass redistributions involving surface motions and gravity variations.

The problem is solved in two steps.

(1) We first solve the elasto-gravitational equations for a spherical hydrostatic pre-stressed planet submitted to $\mathbf{f}$, i.e. we determine the displacement $\mathbf{u}$ and the gravitational potential ϕ within the planet, using a Lagrangian parametrization. As the inertial terms and Coriolis acceleration are extremely small for low-frequency excitation sources, we neglect them. The system is then composed of the static momentum and the mass redistribution equations:

$$\begin{aligned} A\mathbf{u} + \quad \rho\nabla\phi \quad &= \mathbf{f}, \\ \nabla\cdot(\rho\mathbf{u}) + \tfrac{1}{4\pi G}\Delta\phi &= 0, \end{aligned} \tag{1}$$

where A is the elasto-gravitational differential operator, depending on the inner parameters of the planet; ρ is the density, and G is the gravitational constant.

(2) We then introduce lateral variations of density and of rheological parameters, of deviatoric pre-stresses and of interface topography with respect to the initial reference sphere as small perturbations. We solve a first-order perturbed elasto-gravitational system of equations. The unknowns are now the additional perturbed displacement $\delta\mathbf{u}$ and gravitational potential $\delta\phi$:

$$\begin{aligned} A\delta\mathbf{u} + \quad \rho\nabla\delta\phi \quad &= \delta\mathbf{f} - \delta A\mathbf{u} - \delta\rho\nabla\phi, \\ \nabla\cdot(\rho\delta\mathbf{u}) + \tfrac{1}{4\pi G}\Delta\delta\phi &= -\nabla\cdot(\delta\rho\mathbf{u}). \end{aligned} \tag{2}$$

δA is the first-order perturbed elasto-gravitational differential operator, and $\delta\rho$ the perturbed density. Note that the right-hand side of the perturbed elasto-gravitational system depends on the solutions for the spherical planet with hydrostatic pre-stresses.

These equations are solved taking into account boundary conditions on displacements, tractions, gravity and gravity potential, all projected onto the spherical referential earth.

The elasto-gravitational system of equations is solved for the static approximation. There are no inertial modes, nor rotational eigenmodes in our present model. The liquid core is modelled as an elastic solid with a very small rigidity. We found that the error induced by these approximations is negligible in the perturbed solution $\delta\mathbf{u}$ and $\delta\phi$ for an earth made with lateral variations (about 0.5 per cent on the perturbed Love numbers—see the solution for δh_+ and δk_+ for the elliptical earth in Section 3 for example).

2.2 Spectral element method

The elasto-gravitational operator is identical in the two systems (1) and (2). It is applied to $\mathbf{u}$ and ϕ in system (1) and to $\delta\mathbf{u}$ and $\delta\phi$ in system (2). We thus developed a numerical model to solve these systems independently of the right-hand members. Our approach is based on the spectral element method, developed in theoretical seismology in recent years (Komatitsch & Tromp 1999; Chaljub *et al.* 2003; Chaljub & Valette 2004). The earth is discretized on the 'cubed sphere mesh' based on a Ronchi *et al.* (1996) transformation, extended by Chaljub *et al.* (2003) into the radial dimension. The partial differential equations are solved using variational formulations decomposed on a polynomial basis of high degree. As for finite-element methods in general, the method is easily parallelized. Our program is parallelized on a message passing interface (MPI).

This numerical method taking into account mass redistribution is applied here to static phenomena. The method is fully detailed in Métivier (2004).

3 VALIDATION

In order to check our method, we investigated the effect of the spheroidal shape of the Earth on the semi-diurnal body tides, since (1) for this problem there is an analytical solution for the simple case of an incompressible homogeneous planet, and (2) several authors (Wahr 1981; Wang 1994; Mathews *et al.* 1995; Dehant *et al.* 1999) have already numerically estimated it for the PREM model.

We note ψ_o, the centrifugal potential that induces the spheroidal shape of the Earth:

$$\psi_o(r,\theta,\varphi) = -\frac{\Omega^2 r^2}{3}\left(1 - P_2^0(\cos\theta)\right), \tag{3}$$

where r is the radius, θ is the colatitude and φ the longitude. P_n^m are the non-normalized Legendre polynomials. Ω is the rotational velocity.

The influence of the centrifugal force is partly applied as a perturbation. We solve the problem following Smith (1974): the radial part of the centrifugal potential is taken into account in the spherical reference Earth potential and the elliptical part is applied as a perturbation. This perturbation introduces the elliptical lateral variations of the density and of the topographies (but no deviatoric pre-stress).

Let V_{M_2} be the degree-two tidal potential induced by the semi-diurnal lunar wave M_2:

$$V_{M_2}(r,\theta,\varphi) = -V_o\ P_2^2(\cos\theta)\cos(\sigma t + 2\varphi)\frac{r^2}{a^2}, \tag{4}$$

where V_o is the potential amplitude, σ is the semi-diurnal frequency, and a is the Earth's radius.

The tidal solutions are expressed in terms of Love numbers. We use hereafter the notation of the IERS convention (IERS 2003). We classically note h_2, l_2 and k_2 the classical spherical tidal Love numbers. Because the ellipticity on the M_2 tide generates spheroidal deformation in P_2^2 and P_4^2 and a toroidal deformation in P_3^2, we introduce the perturbed tidal Love numbers δh_0, δh_+ related to the vertical displacement, δl_0, δl_+, $\delta\omega_+$ related to the tangential displacement, and δk_0, δk_+ related to the gravity potential. The

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displacement is then written on the deformed outer surface of the ellipsoid:

$$\mathbf{u} + \delta\mathbf{u} = \frac{V_o}{g_o}\Big\{\Big((h_2 + \delta h_0)P_2^2 + \delta h_+ P_4^2\Big)\cos(\sigma t + 2\varphi)\,\mathbf{n} + \nabla\Big[\Big((l_2 + \delta l_0)P_2^2 + \delta l_+ P_4^2\Big)\cos(\sigma t + 2\varphi)\Big] - \mathbf{e}_r \wedge \nabla\Big[\delta\omega_+ P_3^2 \sin(\sigma t + 2\varphi)\Big]\Big\}, \quad (5)$$

where $\mathbf{n}$ is the outward normal of the ellipsoid and $g_o = GM/a^2$ the referential surface gravity, G is the gravitational constant and M the mass of the Earth. The deformation-induced Eulerian potential in the free space is

$$V_o\left[(k_2 + \delta k_0)\left(\frac{a}{r}\right)^3 P_2^2 + \delta k_+\left(\frac{a}{r}\right)^5 P_4^2\right]\cos(\sigma t + 2\varphi). \quad (6)$$

Our numerical model is built for a compressible earth. We impose the incompressibility condition by setting the compressibility parameter at an arbitrarily high value (typically about 10^{14}). We compute the deformation for a homogeneous incompressible earth with a radius of $a = 6371$ km, a rigidity $\mu = 1.15 \times 10^{11}$ Pa s and a density $\rho = 5520$ kg m^{-3}. The upper part of Table 1 shows that our results are in very good agreement with the analytical solutions: the relative errors are less than 0.007 per cent.

We now compute the M_2 body tides for an ellipsoidal earth model stratified following PREM (Dziewonski & Anderson 1981). We compare our results with the study of Wang (1994). The bottom part of Table 1 shows that we agree with his solutions for δh_+ and δk_+. However our δh_0 and δk_0 differ significantly from Wang's values (see Table 1). Wang did not treat the ellipticity problem in the classical way (i.e. following Smith 1974). He started with a spherical reference model that does not take into account the radial pressure and the radial potential induced by the radial part of the centrifugal potential. He applied all the centrifugal potential as a perturbation. Consequently the sum h_2 and k_2 differ from our h_2 and k_2, the discrepancy being in the ellipticity order; however, $h_2 + \delta h_0$ must be identical, as must $k_2 + \delta k_0$ (Greff-Lefftz *et al.* 2005). In Table 1, we have corrected the Wang values in order to compare the two solutions. To do so, we subtracted the difference between our h_2 (or k_2) and Wang's h_2 (or k_2) solution from Wang's δh_o (or δk_0) solution. Table 1 show that they remain significantly different. We do not know exactly how Wang (1994) calculated the Earth's response to the radial part of the centrifugal potential. Did he consider that the PREM earth responds as an incompressible inviscid fluid on the rotation time-scale, as is assumed in Clairaut's theory for the degree 2 deformation? Or did he assume that the Earth responds as a compressible fluid, which would imply that the mean radial density, the mean radial elastic parameters and the mean radius of the planet would change? As for us, we believe that, as PREM is a mean spherical earth built from seismological observations, the radial part of the fluid deformation induced by the centrifugal potential is already taken into account in the reference model.

4 APPLICATIONS

4.1 The M2 tides and Earth ellipticity

As a first application, we further investigated the predictions of our model for the M_2 tide. Figs 1 and 2 show the ellipticity perturbation of the surface displacement and of the gravity (computed for PREM). The perturbed displacement is about 0.5 mm, and consequently cannot be neglected since, for the sake of space geodesy, it is now necessary to achieve the mm-level in tidal displacements. For gravity, there is a significant degree-four order-two component with an amplitude of about 200 nGal. This perturbation should be detectable with very accurate superconducting gravimeters for which the instrumental precision is about 1 nGal (see the GGP project, Crossley *et al.* 1999), if the oceanic effects are correctly modelled.

4.2 Atmospheric loading and Earth ellipticity

We now investigate the response of an ellipsoidal earth to zonal atmospheric loads. In point of fact, the annual component of the Earth zonal geopotential is notably due to air mass redistribution in the atmosphere (Gegout & Cazenave 1993; Blewitt *et al.* 2001).

The boundary conditions for the loading gravitational potential, denoted V^{L}, are the same as the ones for the tidal potential, but there is now a boundary condition linking the external pressure acting on the ellipsoidal earth to the radial traction induced by elasto-gravitational deformation. The free-space gravitational potential can be written using ΔJ_n zonal coefficients:

$$-g_o a\left[\frac{a}{r} - \sum_{n=2}^{M}\Delta J_n\left(\frac{a}{r}\right)^{n+1} P_n^0(\cos\theta)\right]. \quad (7)$$

The zonal loading potential is expanded in spherical harmonics, such as: $V^{\mathrm{L}} = \sum_{n=1}^{M} V_n^{\mathrm{L}} P_n^0(\cos\theta)$. The ΔJ_n can be expressed with the help of the spherical loading Love numbers k'_n and with perturbed Love numbers $\delta k'_{n-}$, $\delta k'_{n0}$ and $\delta k'_{n+}$:

$$\Delta J_n = \left(1 + k'_n + \delta k'_{n0}\right)\frac{V_n^{\mathrm{L}}}{g_o a} + \delta k'_{n+2-}\frac{V_{n+2}^{\mathrm{L}}}{g_o a} + \delta k'_{n-2+}\frac{V_{n-2}^{\mathrm{L}}}{g_o a}. \quad (8)$$

We first obtain the V_n^{L} components from a running average, with an average length of about one year, of the surface pressure coefficients, with an inverted barometer correction, from the NCEP/NCAR

Table 1. Perturbed tidal Love numbers for two earth models. Top: incompressible homogeneous earth model. Comparison of our numerical results with the analytical values. The parameters used are: earth radius $a = 6371$ km, rigidity $\mu = 1.15 \times 10^{11}$ Pa s and density $\rho = 5520$ kg m^{-3}. Bottom: PREM. Comparison of our results with the ones obtained by Wang (1994). The Wang (1994) solutions have been corrected in order to enable comparison of the solutions (see text).

$\times 10^{-3}$	δh_0	δh_+	δk_0	δk_+	δl_0	δl_+	δw_+
			Homogeneous incompressible earth[a]				
Analytical solution	1.587130	−0.332207	2.136239	−0.406726	1.077985	−0.063861	0.224459
Numerical solution	1.587247	−0.332230	2.136248	−0.406729	1.077947	−0.063846	0.224460
Relative error	−0.0074 per cent	−0.0069 per cent	−0.0004 per cent	−0.0007 per cent	0.0035 per cent	0.0235 per cent	0.0005 per cent
			PREM				
Wang solution	1.23	−0.10	1.25	−0.19	—	—	—
Our solution	0.742	−0.107	1.090	−0.195	0.655	−0.534	0.230

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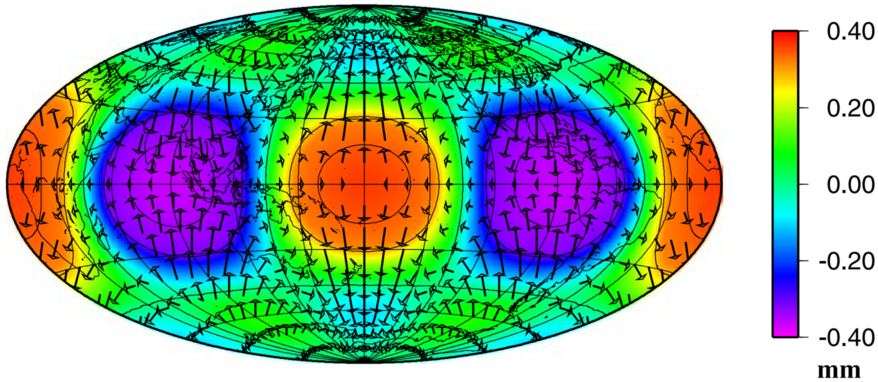

Figure 1. The perturbed M2 body-tide displacement due to ellipticity. The colour scale represents the vertical displacement in millimetres, and the arrows represent the perturbed tangential displacement (maximum of 0.3 mm), on the ellipsoid. The reference earth model is PREM.

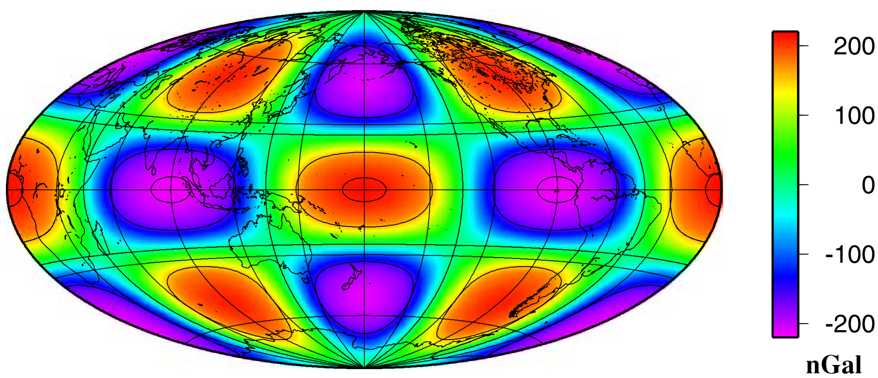

Figure 2. The M2 body-tide gravity perturbation (in nGal) due to the ellipticity calculated on the deformed surface, for PREM.

reanalyses (Kalnay *et al.* 1996). We calculate the associated ΔJ_2 and ΔJ_3. The classical spherical solution [i.e. $(1 + k'_n)\, V^{\mathrm{L}}_n / g_\mathrm{o}\, a$] is plotted at the top of Fig. 3 and its perturbation due to the ellipticity is shown at the bottom. This figure shows that the ellipticity of the Earth has two effects in the ΔJ_n coefficients: first, it weakly perturbs its amplitude (by about 0.01×10^{-10} for the ΔJ_3 component), and second, it creates a phase shift (of less than one hour for the ΔJ_3 annual component) between the degree *n* component of the source and the degree *n* gravitational potential. These perturbations are smaller than the accuracy of the observations and consequently we have to conclude that, for surface loads, the long-wavelengths lateral variations such as ellipticity can be neglected in deformation theory. The next step of our work will be to test whether the short-wavelength lateral variations will generate much more important perturbations for surface loads.

5 CONCLUSION

We have developed a new Earth elasto-gravitational model able to take into account lateral variations, deviatoric pre-stresses and topographies. This numerical model has been validated by comparison with the analytical solution of the ellipticity perturbation on the M_2 body tide for a homogeneous incompressible earth, and with numerical PREM solutions. We have found some discrepancies with previous studies, probably due to different hypotheses about the initial reference sphere. We confirm that the impact of ellipticity on body tides is very large, considering present-day instrumental accuracy.

We have determined the response of the ellipsoidal earth to atmospheric loading, and have found that the ellipticity has a very small impact on the time-variable zonal gravity potential. We conclude that global lateral variations within the Earth will have a weak impact on the elasto-gravitational deformation induced by atmospheric loading. Local lateral variations would probably develop a more important perturbation in the Earth's response to atmospheric loading; this problem will be addressed in the future.

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REFERENCES

Blewitt, G., Lavallée, D., Clarke, P. & Nurutdinov, K., 2001. A new global mode of Earth deformation: seasonal cycle detected, *Science*, **294,** 2342–2345.

Chaljub, E. & Valette, B., 2004. Spectral element modeling of three dimensional wave propagation in a self-gravitating Earth with an arbitrarily stratified outer core, *Geophys. J. Int.*, **158,** 131–141.

Chaljub, E., Capdeville, Y. & Vilotte, J.-P., 2003. Solving elastodynamics in a fluid-solid heterogeneous sphere: a parallel spectral element approximation on non-conforming grids, *J. comput. Phys.*, **187,** 457–491.

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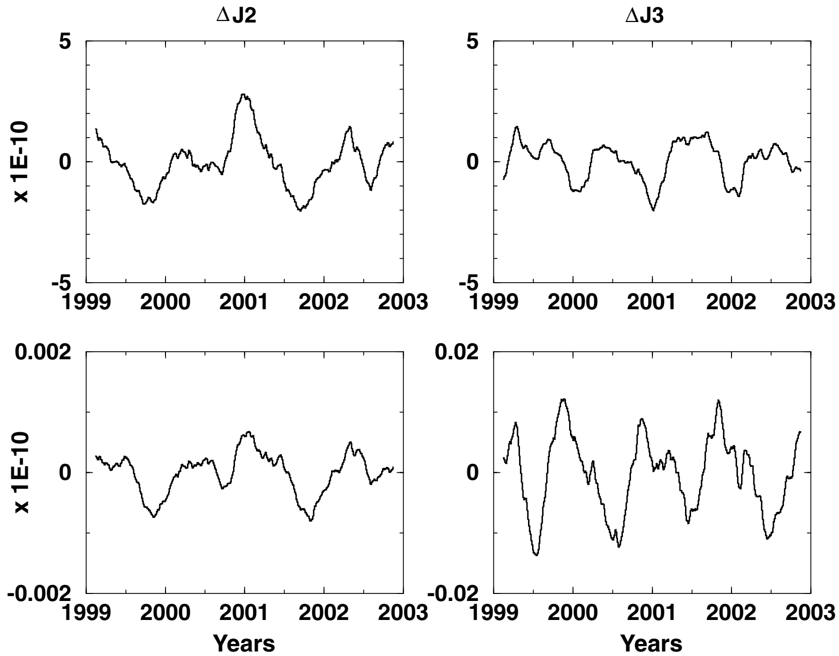

Figure 3. The computed temporal variation of ΔJ_2 and ΔJ_3 due to the atmospheric loading calculated between 1999 and 2003. The classical spherical solutions are represented at the top of the figure and the perturbations due to ellipticity, at the bottom.

Crossley, D. *et al.*, 1999. Network of superconducting gravimeters benefits a number of disciplines, *EOS, Trans. Am. geophys. Un.,* **80,** no11, 121–126.

Dahlen, F.A. & Tromp, J., 1998. *Theoretical Global Seismology*, Princeton University Press, Princeton.

Dehant, V., Defraigne, P. & Wahr, J.M., 1999. Tides for a convective Earth, *J. geophys. Res.*, **104,** 1035–1058.

Dziewonski, A.M. & Anderson, D.L., 1981. Preliminary Referential Earth Model, *Phys. Earth planet. Inter.*, **25,** 297–356.

Farrell, W.E., 1972. Deformation of the Earth by surface loads, *Rev. Geophys. Space Phys.*, **10,** 761–797.

Gegout, P. & Cazenave, A., 1993. Temporal variations of the Earth gravity field for 1985–1989 derived from LAGEOS, *Geophys. J. Int.*, **114,** 347–359.

Greff-Lefftz, M., Métivier, L. & Legros, H., 2005. Analytical solutions of Love numbers for an hydrostatic ellipsoidal incompressible homogeneous Earth Celest. Mech. Dyn. Astr., in press, doi: 10.1007/s10569-005-6424-3.

IERS Conventions, 2003.McCarthy, D.D. & Petit, G., IERS Technical Note 32, Verlag des Bundesamts für Kartographie und Geodäsie, Frankfurt am Main 2004.

Kalnay, E. *et al.*, 1996. The NCEP/NCAR 40-year reanalysis project, *Bull. Am. met. Soc.*, **77,** 437–471.

Komatitsch, D. & Tromp, J., 1999. Introduction to the spectral element method for three-dimensional seismic wave propagation, *Geophys. J. Int.*, **139,** 806–822.

Love, A.E.H., 1911. *Some problems of Geodynamics*, Dover Publications, New York.

Mathews, P.M., Buffet, B.A. & Shapiro, I.I., 1995. Love numbers for a rotating spheroidal Earth: New definitions and numerical values, *Geophys. Res. Lett.*, **22,** 579–582.

Métivier, L., 2004. Influence des variations latérales de densité et de paramètres rhéologiques sur la déformation de la Terre, *PhD thesis*, Institut de Physique du Globe de Paris.

Plag, H.-P., Jüttner, H.-U. & Rautenberg, V., 1996. On the possibility of global and regional inversion of exogenic deformations for mechanical properties of the Earth's interior, *J. Geodyn.*, **21,** 287–308.

Romanowicz, B. & Gung, Y., 2002. Superplumes from the core-mantle boundary to the lithosphere: implications for heat flux, *Science*, **296,** 513–516.

Ronchi, C., Iacono, R. & Paolucci, P.S., 1996. The 'Cubed Sphere': A new method for the solution of partial differential equations in spherical geometry, *J. comput. Phys.*, **124,** 93–114.

Smith, M.L., 1974. The scalar equations of infinitesimal elastic-gravitational motion for a rotating, slightly elliptical Earth, *Geophys. J. R. astr. Soc.*, **37,** 491–526.

Wahr, J.M., 1981. Body tides on an elliptical, rotating, elastic and oceanless Earth, *Geophys. J. R. astr. Soc.*, **64,** 677–703.

Wahr, J.M. & Bergen, Z., 1986. The effects of mantle anelasticity on nutations, Earth tides, and tidal variations in rotation rate, *Geophys. J. R. astr. Soc.*, **64,** 633–668.

Wahr, J., Molenaar, M. & Bryan, F., 1998. Time variability of Earth's gravity field: Hydrological and oceanic effects and their possible detection using GRACE, *J. geophys. Res.*, **103,** 30 205–30 229.

Wang, R., 1994. Effect of rotation and ellipticity on Earth tides, *Geophys. J. Int.*, **117,** 562–565.

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