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## Effective photon mass by Super and Lorentz symmetry breaking



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### ABSTRACT

In the context of Standard Model Extensions (SMEs), we analyse four general classes of Super Symmetry (SuSy) and Lorentz Symmetry (LoSy) breaking, leading to observable imprints at our energy scales. The photon dispersion relations show a non-Maxwellian behaviour for the CPT (Charge-Parity-Time reversal symmetry) odd and even sectors. The group velocities exhibit also a directional dependence with respect to the breaking background vector (odd CPT) or tensor (even CPT). In the former sector, the group velocity may decay following an inverse squared frequency behaviour. Thus, we extract a massive Carroll–Field–Jackiw photon term in the Lagrangian and show that the effective mass is proportional to the breaking vector and moderately dependent on the direction of observation. The breaking vector absolute value is estimated by ground measurements and leads to a photon mass upper limit of  $10^{-19}$  eV or  $2 \times 10^{-55}$  kg, and thereby to a potentially measurable delay at low radio frequencies.

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We largely base our understanding of particle physics on the Standard Model (SM). Despite having proven to be a very reliable reference, there are still unsolved problems, such as the Higgs Boson mass overestimate, the absence of a candidate particle for the dark universe, as well as the neutrino oscillations and their mass.

Standard Model Extensions (SMEs) tackle these problems. Among them, Super Symmetry (SuSy) [1,2] figures new physics at TeV scales [3]. Since, in SuSy, Bosonic and Fermionic particles each have a counterpart, their mass contributions cancel each other and allow the correct experimental low mass value for the Higgs Boson.

Lorentz Symmetry (LoSy) is assumed in the SM. It emerges [4–7] that in the context of Bosonic strings, the condensation of tensor fields is dynamically possible and determines LoSy violation. There are opportunities to test the low energy manifestations of LoSy violation, through SMEs [8,9]. The effective Lagrangian is given by the usual SM Lagrangian corrected by SM operators of any dimensionality contracted with suitable Lorentz breaking tensorial (or simply vectorial) background coefficients. In this letter,

we show that photons exhibit a non-Maxwellian behaviour, and possibly manifest dispersion at low frequencies pursued by the newly operating ground radio observatories and future space missions.

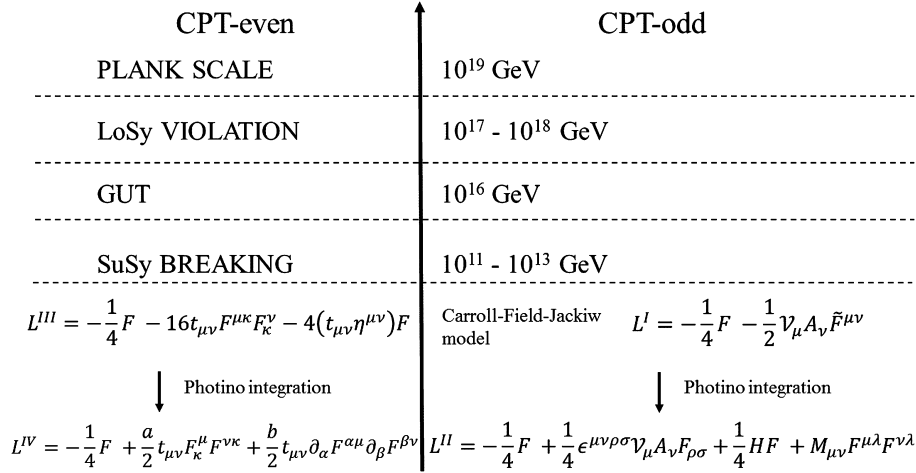
LoSy violation has been analysed phenomenologically. Studies include electrons, photons, muons, mesons, baryons, neutrinos and Higgs sectors. Limits on the parameters associated with the breaking of relativistic covariance are set by numerous analyses [10–12], including with electromagnetic cavities and optical systems [13–19]. Also Fermionic strings have been proposed in the presence of LoSy violation. Indeed, the magnetic properties of spinless and/or neutral particles with a non-minimal coupling to a LoSy violation background have been placed in relation to Fermionic matter or gauge Bosons [20–25].

LoSy violation occurs at larger energy scales than those obtainable in particle accelerators [26–32]. At those energies, SuSy is still an exact symmetry, even if we assume that it might break at scales close to the primordial ones. However, LoSy violation naturally induces SuSy breaking because the background vector (or tensor) – that implies the LoSy violation – is in fact part of a SuSy multiplet [33], Fig. 1.

The sequence is assured by the supersymmetrisation, in the CPT (Charge-Parity-Time reversal symmetry) odd sector, of the Carroll–Field–Jackiw (CFJ) model [34] that emulates a Chern–Simons [35]

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**Fig. 1.** Breaking energy values and the Lagrangians. A different hierarchy of LoSy, SuSy breaking and Grand Unification Theories (GUT) does not interfere with the dispersion laws of the photonic sector at low energies.

term and includes a background field that breaks LoSy, under the point of view of the so-called (active) particle transformations. The latter consists of transforming the potential  $A^{\mu}$  and the field  $F^{\mu\nu}$ , while keeping the background vector  $\mathcal{V}^{\mu}$  unchanged. For the photon sector, when unaffected by the photino contribution, the CFJ Lagrangian reads (Class I)

$$L^I = -\frac{1}{4}F - \frac{1}{2}\mathcal{V}_{\mu}A_{\nu}\tilde{F}^{\mu\nu}, \quad (1)$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}, \quad (2)$$

where  $F = F^{\mu\nu}F_{\mu\nu}$ . The term in Eq. (2) couples the photon to an external constant four vector and it violates parity even if gauge symmetry is respected [34]. If the CFJ model is supersymmetrised [36], the vector  $\mathcal{V}^{\mu}$  is space-like constant and is given by the gradient of the SuSy breaking scalar background field, present in the matter supermultiplet. The dispersion relation yields, denoting  $k^{\mu} = (\omega, \vec{k})$ ,  $k^2 = (\omega^2 - |\vec{k}|^2)$ , and  $(\mathcal{V}^{\mu}k_{\mu})^2 = (\mathcal{V}^0\omega - \vec{\mathcal{V}} \cdot \vec{k})^2$ ,

$$k^4 + \mathcal{V}^2k^2 - (\mathcal{V}^{\mu}k_{\mu})^2 = 0. \quad (3)$$

If SuSy holds and the photino degrees of freedom are integrated out, we are led to the effective photonic action, *i.e.* the effect of the photino on the photon propagation. The Lagrangian (1) is recast as (Class II) [33]

$$L^{II} = -\frac{1}{4}F + \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\mathcal{V}_{\mu}A_{\nu}F_{\rho\sigma} + \frac{1}{4}HF + M_{\mu\nu}F^{\mu\lambda}F^{\nu\lambda}, \quad (4)$$

where  $H$ , the tensor  $M_{\mu\nu} = \tilde{M}_{\mu\nu} + 1/4\eta_{\mu\nu}M$ , and  $\tilde{M}_{\mu\nu}$  depend on the background Fermionic condensate, originated by SuSy;  $M_{\mu\nu}$  is traceless,  $M$  is the trace of  $M_{\mu\nu}$  and  $\eta_{\mu\nu}$  the metric. Thus, the Lagrangian, Eq. (4), in terms of the irreducible terms displays as

$$L^{II} = -\frac{1}{4}(1 - H - M)F + \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\mathcal{V}_{\mu}A_{\nu}F_{\rho\sigma} + \tilde{M}_{\mu\nu}F^{\mu\lambda}F_{\lambda}^{\nu}. \quad (5)$$

The corresponding dispersion relation reads

$$k^4 + \frac{\mathcal{V}^2}{(1 - H - M)^2}k^2 - \frac{1}{(1 - H - M)^2}\mathcal{V}^{\mu}k_{\mu} = 0. \quad (6)$$

The dispersion law given by Eq. (6) is just a rescaling of Eq. (3) as we integrated out the photino sector. The background parameters are very small, being suppressed exponentially at the Planck scale; they render the denominator in Eq. (6) close to unity, imply-

ing similar numerical outcomes for the two dispersions of Classes I and II.

The even sector [33] assumes that the Bosonic background, responsible of LoSy violation, is a background tensor  $t_{\mu\nu}$ . For the photon sector, if unaffected by the photino contribution, the Lagrangian reads (Class III)

$$L^{III} = -\frac{1}{4}F - 16t_{\mu\nu}F^{\mu\kappa}F_{\kappa}^{\nu} - 4(t_{\mu\nu}\eta^{\mu\nu})F. \quad (7)$$

The dispersion relation for Class III [37] is

$$\omega^2 - (1 + \rho + \sigma)^2|\vec{k}|^2 = 0, \quad (8)$$

where  $\rho = 1/2\tilde{K}^{\alpha}_{\alpha}$ ,  $\sigma = 1/2\tilde{K}^{\alpha\beta}\tilde{K}_{\alpha\beta} - \rho^2$ , and  $\tilde{K}^{\alpha\beta} = t^{\alpha\beta}t^{\mu\nu}p_{\mu}p_{\nu}/|\vec{k}|^2$  are associated to Fermionic condensates.

Integrating out the photino [33], we turn to the Lagrangian of Class IV

$$L^{IV} = -\frac{1}{4}F + \frac{a}{2}t_{\mu\nu}F_{\kappa}^{\mu}F^{\nu\kappa} + \frac{b}{2}t_{\mu\nu}\partial_{\alpha}F^{\alpha\mu}\partial_{\beta}F^{\beta\nu}, \quad (9)$$

where  $a$  is a dimensionless coefficient and  $b$  a parameter of dimension of mass<sup>-2</sup> (herein,  $c = 1$ , unless otherwise stated). For the dispersion relation, we write the Euler–Lagrange equations, pass to Fourier space and set to zero the determinant of the matrix that multiplies the Fourier transformed potential. However, given the complexity of the matrix in this case and the smallness of the tensor  $t_{\mu\nu}$ , we develop the determinant in a series truncated at first order and get [37]

$$btk^4 - k^2 + (3a + bk^2)t^{\alpha\beta}k_{\alpha}k_{\beta} = 0, \quad (10)$$

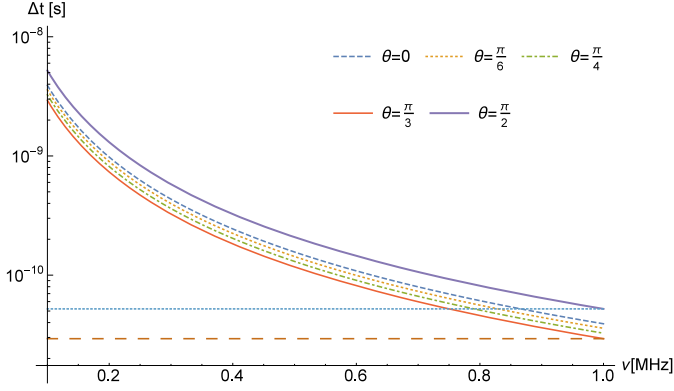
where  $t = t_{\mu}^{\mu}$ .

For determining the group velocity, we first consider  $\mathcal{V}_0 = 0$  for Class I [38,39] and obtain

$$\omega^4 - (2|\vec{k}|^2 + |\vec{\mathcal{V}}|^2)\omega^2 + |\vec{k}|^4 + |\vec{k}|^2|\vec{\mathcal{V}}|^2 - (\vec{\mathcal{V}} \cdot \vec{k})^2 = 0. \quad (11)$$

In [39], the authors do not exploit the consequences of the dispersion relations and do not consider a SuSy scenario. Dealing with Eq. (11), we have neglected the negative roots; it turns out that the two positive roots determine identical group velocities  $dw/dk$  up to second order in  $\vec{\mathcal{V}}$ . For  $\theta$ , the angle between the background vector  $\vec{\mathcal{V}}$  and  $\vec{k}$ , we get

$$v_g^I|_{\mathcal{V}_0=0}^{\theta \neq \pi/2} = 1 - \frac{|\vec{\mathcal{V}}|^2}{8\omega^2}(2 + \cos^2\theta), \quad (12)$$



**Fig. 2.** For Class I, we plot the delays [s], Eq. (16), for different angles, Eqs. (12), (13), using  $|\vec{\mathcal{V}}| = 10^{-19}$  eV [40], versus frequency. We have supposed the source to be at a distance of 4 kpc. The frequency range 0.1–1 MHz has been chosen since it is targeted by recently proposed low radio frequency space detectors, composed by a swarm of nano-satellites; see [41] and references therein. There is a feeble dependence of the delays on  $\theta$ . The delay is of about 50 ps at 1 MHz for  $\theta = \pi/2$ , Eq. (13), and around half of this value for  $\theta$  approaching  $\pi/2$ , Eq. (12).

for  $\theta \neq \pi/2$ . Instead for  $\theta = \pi/2$ , one of the two solutions coincides with the Maxwellian value, while the other is dispersive

$$v_g^{I1}|_{\mathcal{V}_0=0} = 1, \quad v_g^{I2}|_{\mathcal{V}_0=0} = 1 - \frac{1}{2} \frac{|\vec{\mathcal{V}}|^2}{\omega^2}. \quad (13)$$

For  $\mathcal{V}_0 \neq 0$ , we suppose that the light propagates along the  $z$  axis ( $k_1 = k_2 = 0$ ) which for convenience is along the line of sight of the source. We then obtain

$$\omega^4 - [2k_3^2 + \mathcal{V}_1^2 + \mathcal{V}_2^2 + \mathcal{V}_3^2]\omega^2 + 2\mathcal{V}_0\mathcal{V}_3k_3\omega + k_3^4 + (\mathcal{V}_1^2 + \mathcal{V}_2^2 - \mathcal{V}_0^2)k_3^2 = 0. \quad (14)$$

We now set  $\mathcal{V}_3 = 0$ , that is, the light propagates orthogonally to the background vector. Further, for  $\mathcal{V}$  spacelike and  $4\mathcal{V}_0^2k_3^2/|\vec{\mathcal{V}}|^4 \ll 1$ , we get two group velocities, one of which is dispersive

$$v_g^{I1}|_{\mathcal{V}_3=0} = 1 - \frac{\mathcal{V}_0^2}{|\vec{\mathcal{V}}|^2}, \quad v_g^{I2}|_{\mathcal{V}_3=0} \simeq \alpha \left(1 - \frac{1}{2} \frac{|\vec{\mathcal{V}}|^2}{\omega^2}\right). \quad (15)$$

The solution  $v_g^{I1}|_{\mathcal{V}_3=0}$  is always subluminal for  $\mathcal{V}$  spacelike. The solution  $v_g^{I2}|_{\mathcal{V}_3=0}$  assumes  $\omega \gg |\vec{\mathcal{V}}|$ . Since  $\alpha = 1 + \mathcal{V}_0^2/|\vec{\mathcal{V}}|^2$ ,  $v_g^{I2}|_{\mathcal{V}_3=0}$  is superluminal for  $\sqrt{2}\omega > |\vec{\mathcal{V}}|(1 + |\vec{\mathcal{V}}|^2/\mathcal{V}_0^2)^{1/2}$ . Further, the value of  $\alpha$  is not Lorentz–Poincaré invariant. Superluminal behaviour is avoided assuming for both solutions  $\mathcal{V}_0 = 0$ .

If dealing only with a null  $\mathcal{V}_0$  and with dispersive group velocities, for a source at distance  $\ell$ , the time delay of two photons at different frequencies, A and B, is given by (in SI units)

$$\Delta t_{\text{CFJ}} = \frac{\ell|\vec{\mathcal{V}}|^2}{2c\hbar^2} \left( \frac{1}{\omega_A^2} - \frac{1}{\omega_B^2} \right) x, \quad (16)$$

where  $x$  takes the values  $(2 + \cos^2\theta)/4$ , for Eq. (12), and 1 for Eqs. (13), (15). The delays, Eq. (16), are plotted in Fig. 2. Comparing with the de Broglie–Proca (dBP) delay

$$\Delta t_{\text{dBP}} = \frac{\ell m_\gamma^2 c^3}{2\hbar^2} \left( \frac{1}{\omega_A^2} - \frac{1}{\omega_B^2} \right), \quad (17)$$

we conclude that the background vector induces an effective mass to the photon,  $m_\gamma$ , of value

$$m_\gamma = \frac{|\vec{\mathcal{V}}|}{c^2} x. \quad (18)$$

Equation (18) is gauge-invariant, but not Lorentz–Poincaré invariant. Nevertheless, there is a subset of Lorentz–Poincaré transformation that leave the value of Eq. (18) unchanged. Under the assumption of  $\mathcal{V}_0 = 0$  and thus  $|\vec{\mathcal{V}}|$  constant, the value of  $m_\gamma$  is constant when the origin of the reference is translated along the line of sight of the observer to the source and/or under the rotation group  $SO(3)$ . The mass appears as the pole of the transverse component of the photon propagator [39].

Class II, just a rescaling of Class I, implies identical solutions, differing by a numerical factor only.

The group velocities of Classes III and IV show no sign of dispersion; they are slightly smaller than  $c$  – as light travelling through matter, but suffer from anisotropy to a larger degree than in Classes I and II. Indeed, the isotropy is lost due to the tensorial nature of the LoSY and SuSy breaking perturbation. The feebleness of the corrections is due to the coefficient  $\mathcal{T}$  being proportional to the powers of the tensor  $t_{\mu\nu}$  components, of  $10^{-19}$  eV magnitude [37]

$$v_g^{III,IV} = 1 - \mathcal{T} \left( t_1 \sin^2\theta \cos^2\varphi + t_2 \sin^2\theta \sin^2\varphi + t_3 \cos^2\theta \right), \quad (19)$$

where  $\theta$  and  $\varphi$  are the azimuthal and planar angles of  $\vec{k}$  with respect to the axes respectively.

Having seen a massive-like photon behaviour in the group velocities of the odd sector, we rewrite Eq. (1) in terms of the potentials to let a massive-like term emerge

$$L = \frac{1}{2} (\vec{\nabla}\phi + \dot{\vec{A}})^2 - \frac{1}{2} (\vec{\nabla} \times \vec{A})^2 + \mathcal{V}_0 \vec{A} \cdot (\vec{\nabla} \times \vec{A}) - 2\vec{\nabla}\phi \cdot (\vec{\nabla} \times \vec{A}) - \vec{\nabla} \cdot (\vec{A} \times \dot{\vec{A}}).$$

Since the  $\phi$  field appears only through its gradient, in the absence of  $\phi$  time derivatives and thereby of dynamics,  $\vec{\nabla}\phi$  acts as an auxiliary field and can be integrated out from the Lagrangian. Defining  $\chi = \vec{\nabla}\phi + \dot{\vec{A}} - 2\vec{\nabla} \times \vec{A}$ , we get

$$L = \frac{1}{2} \chi^2 - 2(\vec{\nabla} \times \vec{A})^2 + \vec{\nabla} \cdot (\vec{A} \times \dot{\vec{A}}) - \frac{1}{2} (\vec{\nabla} \times \vec{A})^2 + \mathcal{V}_0 \vec{A} \cdot (\vec{\nabla} \times \vec{A}). \quad (20)$$

The Euler–Lagrangian equation for  $\chi$  is disregarded since  $\chi = 0$ . The term  $(\vec{\nabla} \times \vec{A})^2$  is expanded as  $(|\vec{\mathcal{V}}|^2 \delta_{kn} - \mathcal{V}_k \mathcal{V}_n) \times A_k A_n := M_{kn}(\vec{\mathcal{V}}) A_k A_n$ , where  $M_{kn}$  is a symmetric diagonalisable matrix, thanks to a suitable matrix of the  $SO(3)$  rotation group. Performing such a change in Eq. (20), the term under discussion changes into

$$\tilde{A}_i \tilde{M}_{ij} A_j = |\vec{\mathcal{V}}|^2 \tilde{A}_2^2 + |\vec{\mathcal{V}}|^2 \tilde{A}_3^2, \quad (21)$$

thereby showing a massive-like photon term as in the de Broglie–Proca Lagrangian.

The quest for a photon with non-vanishing mass is definitely not new. The first attempts can be traced back to de Broglie who conceived an upper limit of  $10^{-53}$  kg, and achieved a comprehensive formulation of the photon [42], also thanks to the reinterpretation of the work of his doctorate student Proca. To the Lagrangian of Maxwell’s electromagnetism, they added a gauge breaking term proportional to the square of the photon mass. A laboratory Coulomb’s law test determined the mass upper limit of  $2 \times 10^{-50}$  kg [43]. In the solar wind, Ryutov found  $10^{-52}$  kg at 1 AU [44,45], and  $1.5 \times 10^{-54}$  kg at 40 AU [45]. These limits were accepted by the Particle Data Group (PDG) [46], but recently

put into question [47].<sup>1</sup> The lowest value for any mass is dictated by Heisenberg's principle  $m \geq \hbar/\Delta t c^2$ , and gives  $1.3 \times 10^{-69}$  kg, where  $\Delta t$  is the supposed age of the Universe.

In this letter, we have focused on SuSy and LoSy breaking and derived the ensuing dispersion relations and group velocities for four types of Lagrangians. All group velocities show a non-Maxwellian behaviour, in the angular dependence and through sub or superluminal speeds. Superluminal behaviour is exclusive to the odd CPT sector, and may occur only if the time component of the perturbing vector is non-null. Further, in the odd CPT sector, the effective mass shows a dispersion, proportional to  $1/\omega^2$ , as in dBP formalism. Conversely, to the dBP photon, where mass is imposed *ab initio*, the CFJ photon acquires a mass through a mechanism, namely from LoSy violation through the background vector. The other differences lie in the lack of Lorentz–Poincaré invariance and in the angular dependence of the CFJ photon mass.

The delays are more important at lower frequencies and the opening of the 0.1–100 MHz window would be of importance [41]. Elsewhere, we have analysed the polarisation and evinced the transversal and longitudinal (massive) modes [37].

From the rotation of the plane of polarisation from distant galaxies, or from the Cosmic Microwave Background (CMB), it has been assessed that  $|\mathcal{V}_\mu| < 10^{-34}$  eV [12,34,48]. This result is comparable to the Heisenberg mass limit value at the age of the universe. A less stringent, but interesting, limit of  $10^{-19}$  eV [40] has been set through laboratory based experiments involving electric dipole moments of charged leptons or the inter-particle potential between Fermions and the associated corrections to the spectrum of the Hydrogen atom. These latter estimates imply, Eq. (18), a mass upper limit of  $10^{-55}$  kg.

The detection of the CFJ massive photon can be pursued by other means, e.g., through analysis of Ampère's law in the solar wind [47]. Incidentally, the odd and even CPT sectors can be experimentally separable [12].

What is the role of a massive photon for SMEs? String theory has hinted to massive gravitons and photons [5,6], while Proca electrodynamics was investigated in the context of LoSy violation, but outside a SuSy scenario [20]. However, if LoSy takes place in a supersymmetric scenario, the photon mass may be naturally generated from SuSy breaking condensates [33,36]. We point out that the emergence of a massive photon is pertinent also to other SME formulations.

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<sup>1</sup> In [47], there are references to even more optimistic, and less safe, limits ( $3 \times 10^{-63}$  kg), claimed when modelling the galactic magnetic field. For recent studies on propagation limits see [49,50].