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The nonlinear kinetic theory is presented for the ion acoustic perturbations at the foot of the Earth's quasiperpendicular bow shock, that is characterized by weakly magnetized electrons and unmagnetized ions. The streaming ions, due to the reflection of the solar wind ions from the shock, provide the free energy source for the linear instability of the acoustic wave. In the fully nonlinear regime, a coherent localized solution is found in the form of a stationary ion hump, which is traveling with the velocity close to the phase velocity of the linear mode. The structure is supported by the nonlinearities coming from the increased population of the resonant beam ions, trapped in the self-consistent potential. As their size in the direction perpendicular to the local magnetic field is somewhat smaller that the electron Larmor radius and much larger that the Debye length, their spatial properties are determined by the effects of the magnetic field on weakly magnetized electrons. These coherent structures provide a theoretical explanation for the bipolar electric pulses, observed upstream of the shock by Polar and Cluster satellite missions. © *2009 American Institute of Physics*. [doi:10.1063/1.3240342]

I. INTRODUCTION

Low frequency electrostatic perturbations in the frequency range of 10^2–10^3 Hz, which lies both below and above the local electron cyclotron frequency, are commonly observed in the vicinity of a quasiperpendicular shock front. In the classical study,[1] it was shown that these can be attributed to either ion-sound or whistler waves. Usually, waves above the electron cyclotron frequency are thought to be the Doppler shifted ion-sound waves, and those below are whistler waves. At the front of a supercritical, quasiperpendicular shock, various sources of free energy have been identified, that give rise to different plasma instabilities. The generation of the ion acoustic waves is commonly attributed either to instabilities related with the electron temperature gradient, or to a strongly inhomogeneous electric current within the ramp. Other possible candidates for the electrostatic turbulence in this frequency range include the lower-hybrid and electron acoustic waves that can be excited respectively by the Bunemann instability, in the presence of an electron beam, and by the fan instability, in the presence of a hot electron population.

The analysis by Balikhin *et al.*[2] of the data collected by the Cluster 3 satellite on February 26, 2002, revealed the existence of the ion-sound turbulence within the Earth's bow shock, upstream of the ramp. As there are no strong electron-electron temperature gradients in the foot of the shock, it was concluded that these ion acoustic waves are probably generated by electric currents. The appropriate short lived, small scale current layers were not detected by the mission Ref. 2, but they were predicted theoretically, within the nonstationary model of the shock front.[3,4] Similar observations at the Earth's bow shock, under extreme solar wind conditions, were obtained also by the Polar spacecraft.[5] Besides the large amplitude coherent wave packets, with the frequency in the range of 100–4000 Hz and lasting 10–30 cycles, that were the dominant feature of the ion-sound turbulence at the magnetic ramp, a number of coherent solitary structures was also observed. The same kind of electrostatic structures have been identified recently[6] in the Cluster 3 data of February 26, 2002, in the foot region of Earth's quasiperpendicular bow shock (the angle between the shock normal and the magnetic field was ~55°). These structures feature a characteristic bipolar pulse in the electric field, corresponding to a localized negative potential of −4 to −5 V.[6] The amplitude is 10–20 mV/m, which is several times larger than that of the sound waves and packets. They propagate mostly in the solar wind direction toward the shock, at a significant angle to the ambient magnetic field, with the velocity at the acoustic range. The velocity in the solar wind frame was estimated at 620 km/s, which is faster than the ion bulk velocity and the local ion-sound velocity (c_s=40 km/s) but slower than the electron thermal velocity (v_{T_e}=2400 km/s), for T_e=17 eV. Their scale perpendicular to the magnetic field is a few hundred meters, which in the plasma parameters scales as $\sim 20\lambda_D \sim 0.5\rho_{L_e}$. The parallel size is larger, within one order of magnitude, with the typical aspect ratio 0.26, which was interpreted as the propagation at the angle ~75° to the magnetic field.[6]

The actual source of the ion acoustic turbulence at the foot of the shock still remains elusive. There are no strong electron-electron temperature gradients in this region and the existence of the appropriate short lived localized electric currents has not been confirmed. The latter was inferred[2] only from the properties of the observed sound waves. An alternative mechanism has been suggested, based on the observations of an ion beam at the foot region, see, e.g., Refs. 7–9 and references therein. Such beam arises from the reflection

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of the ions of the solar wind from the shock, and the ions propagating roughly along the magnetic field direction are observed at the onset of the event. Later on, the angular distribution often becomes gyrophase-bunched, which is interpreted as the specular[7] or nonspecular[8] reflection. Such ion beams can efficiently produce linearly unstable ion acoustic waves.

In the present paper, we present the nonlinear kinetic theory of electrostatic perturbations associated with acoustic perturbation in an ion beam-plasma system in a weakly magnetized plasma. Using the standard Schamel's model for the ion distribution in the stationary state,[10] we find a coherent localized structure in the form of an ion hump, that is supported by the increased population of the beam ions that are trapped in the self-consistent potential. In contrast to the standard ion holes and humps in unmagnetized plasmas, whose spatial dispersion is governed by the ion thermal effects and the charge separation, in weakly magnetized plasma in front of the shock, that is characterized by small Debye length and large electron Larmor radius, $\lambda_D \ll \rho_{L_e}$, the spatial properties of the ion humps are determined by the (large) electron Larmor radius effects. In Secs. I and II we derive the appropriate nonlinear descriptions of the ions and electrons, respectively. In Sec. III we summarize the linear properties of the unstable ion acoustic waves and in Sec. IV we present a numerical solution of the full nonlinear system, in the form of a coherent ion hump. The conclusions are given in Sec. V.

II. THE DESCRIPTION OF IONS

We study coherent nonlinear kinetic plasma structures at the foot of the Earth's bow shock, whose typical size is somewhat larger than the electron Larmor radius and the ions are essentially unmagnetized. The ion velocity distribution is typically non-Maxwellian. As a simple model, we take that the ion population consists of stationary and streaming (i.e., beam) ions. The ion beam originates from the reflection of the solar wind from the bow shock. The unperturbed distribution is adopted as

$$f_i^{(0)} = \frac{(1-\beta)n_0}{\sqrt{2\pi v_{T_{iS}}^2}} \exp\left(-\frac{v^2}{2v_{T_{iS}}^2}\right) + \frac{\beta n_0}{\sqrt{2\pi v_{T_{iB}}^2}} \exp\left[-\frac{(\vec{v}-\vec{v}_B)^2}{2v_{T_{iB}}^2}\right], \quad (1)$$

where the subscripts S and B denote the stationary and beam ions, and $\vec{v}_B$ is the beam velocity. The unperturbed ion densities and temperatures are taken to be homogeneous, since we study the phenomena whose typical size is much shorter than the characteristic length of inhomogeneity. In the presence of the electrostatic potential ϕ that is stationary in the reference frame propagating with the velocity $\vec{u}$, one easily finds an integral of motion of the ion Vlasov equation, which is the ion energy W, $W=m_i(\vec{v}-\vec{u})^2/2+e\phi$. In the simple case when the potential is strictly one-dimensional (1D), the energy is the only conserved quantity. Taking that the potential vanishes in the infinity, that it was adiabatically switched on at some distant past, and that the entire evolution was adiabatic, we can express the initial ion velocity $\vec{v}^{(0)}$ as

$$\vec{v}^{(0)} = \vec{u} + \frac{\vec{v}-\vec{u}}{|\vec{v}-\vec{u}|}\sqrt{(\vec{v}-\vec{u})^2 + \frac{2e}{m_i}\phi}. \quad (2)$$

Obviously, negative potentials may trap ions, i.e., the "initial velocity" is a complex quantity for the particles whose velocities satisfy $(\vec{v}-\vec{u})^2+2e\phi/m_i<0$. Then, for the ions we adopt the standard Schamel's 1D model distribution function[10]

$$f_i = \frac{(1-\beta)n_0}{\sqrt{2\pi v_{T_{iS}}^2}} \exp\left[-\frac{(\mathrm{Re}\,\vec{v}^{(0)})^2 - \alpha_S(\mathrm{Im}\,\vec{v}^{(0)})^2}{2v_{T_{iS}}^2}\right] + \frac{\beta n_0}{\sqrt{2\pi v_{T_{iB}}^2}} \exp\left[-\frac{(\mathrm{Re}\,\vec{v}^{(0)}-\vec{v}_B)^2 - \alpha_B(\mathrm{Im}\,\vec{v}^{(0)})^2}{2v_{T_{iB}}^2}\right]. \quad (3)$$

Integrating the Schamel's distribution, Eq. (3) and taking, for simplicity, that the potential propagates in the direction of the beam, we obtain the ion density perturbation that, in the small amplitude regime $|e\phi/T_i| \ll 1$, with the accuracy to $\sim\mathcal{O}(|e\phi/T_i|^{3/2})$ can be written as[10]

$$\frac{\delta n_i}{n_0} = -a_S\frac{e\phi}{T_{i_S}} - b_S\frac{4}{3}\left|\frac{e\phi}{T_{i_S}}\right|^{3/2} - a_B\frac{e\phi}{T_{i_B}} - b_B\frac{4}{3}\left|\frac{e\phi}{T_{i_B}}\right|^{3/2}, \quad (4)$$

where the coefficients a_S, b_S, a_B, and b_B are given by

$$a_S = \frac{1-\beta}{\sqrt{2\pi v_{T_{i,S}}^2}} \int_{-\infty}^{\infty} \frac{v\,dv}{v-u} \exp\left(-\frac{v^2}{2v_{T_{iS}}^2}\right), \quad (5)$$

$$b_S = \frac{1-\beta}{\sqrt{\pi}}\left(1-\alpha_S-\frac{u^2}{v_{T_{iS}}^2}\right)\exp\left(-\frac{u^2}{2v_{T_{iS}}^2}\right), \quad (6)$$

$$a_B = \frac{\beta}{\sqrt{2\pi v_{T_{iB}}^2}} \int_{-\infty}^{\infty} \frac{v\,dv}{v-u+v_B} \exp\left(-\frac{v^2}{2v_{T_{iB}}^2}\right), \quad (7)$$

$$b_B = \frac{\beta}{\sqrt{\pi}}\left[1-\alpha_B-\frac{(v_B-u)^2}{v_{T_{iB}}^2}\right]\exp\left[-\frac{(2v_B-u)^2}{2v_{T_{iB}}^2}\right], \quad (8)$$

and α_S and α_B are the appropriate Schamel's trapping coefficients for the stationary and moving ion populations. The ion particle trapping, which produces the nonlinear terms $\propto|e\phi/T_i|^{3/2}$ exist only if $\phi<0$.

The velocity of the solar wind, relative to the Earth, is supersonic. Consequently, the velocity of the reflected ions in the solar wind frame, denoted by v_B in Eqs. (7) and (8), is of the same order, i.e., it is (much) bigger than the ion thermal speed. Thus, for the structures that propagate with the velocity close to that of the ion beam, and thus may be in resonance with the beam, viz., $|u-v_B| \lesssim v_{T_{iB}}$, the nonmoving (stationary) fraction of the unperturbed ion population can be regarded as cold, $u \gg v_{T_{iS}}$. Thus, we take that the particle trapping occurs only in the ion beam, and consequently

$$a_S = -(1-\beta)\frac{v_{T_{iS}}^2}{u^2}, \quad b_S = 0, \tag{9}$$

so that we may write

$$\frac{\delta n_i}{n_0} = -\left(a_B + a_S \frac{T_{i_B}}{T_{i_S}}\right)\frac{e\phi}{T_{i_B}} - b_B \frac{4}{3}\left|\frac{e\phi}{T_{i_B}}\right|^{3/2}. \tag{10}$$

III. THE DESCRIPTION OF ELECTRONS

The typical size of the coherent nonlinear structures observed at the foot of the Earth's bow shock is somewhat bigger than the electron Larmor radius.[5,6] Thus, the simple model of Boltzmann distributed electrons, that is used in the classical theory of ion holes in an unmagnetized plasma, may not be appropriate. Noting that for relatively small negative potentials $0 > e\phi/T_i \gg -1$ and in the regime when the electron temperature is comparable with the ion temperature, within the accuracy of Eq. (4) (i.e., when the electron trapping does not take place and the quadratic nonlinearities are negligible), we can use the linear description for electrons. Using the cylindrical coordinates in the velocity space and solving the Vlasov equation, assuming that the magnetic field is constant, one readily obtains the textbook expression for the Fourier component $\delta f_e(\omega, \vec{k}, \vec{v})$ of the perturbation of the electron distribution function, as the series expansion in terms of the modified Bessel functions, see, e.g., Ref. 11,

$$\delta f_e = f_e^{(0)} \frac{e\phi}{T_e} \sum_{m=-\infty}^{\infty} i^m J_m\left(\frac{k_\perp v_\perp}{\Omega_e}\right) \exp\left(i \frac{k_\perp v_\perp}{\Omega_e} \cos\theta\right) \times \sum_{l=-\infty}^{\infty} \frac{l\Omega_e - k_\parallel v_\parallel}{\omega + l\Omega_e - k_\parallel v_\parallel} i^{-l} J_l\left(\frac{k_\perp v_\perp}{\Omega_e}\right), \tag{11}$$

where J_l is the Bessel function of the order l, $k_\perp$ and $k_\parallel$ are the components of the wave vector that are perpendicular and parallel to the magnetic field $\vec{B}$, Ω_e is the electron gyrofrequency, $\Omega_e = -eB/m_e$, and the cylindrical coordinates of the electron velocity are given by $v_\parallel = \vec{v}\cdot\vec{B}/B$, $v_\perp = |\vec{v}\times\vec{B}|/B$, and $\theta = \arccos[\vec{e}_y \cdot (\vec{v}\times\vec{B})/|\vec{v}\times\vec{B}|]$. Taking that the unperturbed electron distribution function is Maxwellian, viz.,

$$f_e^{(0)} = \frac{n_0}{(2\pi v_{T_e}^2)^{3/2}} \exp\left(-\frac{v^2}{2v_{T_e}^2}\right), \tag{12}$$

and making use of the recursive properties and orthogonality of the Bessel function, we can integrate the electron distribution function, Eq. (11), with respect to the perpendicular electron velocity components, yielding

$$\delta f_{\parallel e} = \frac{e\phi}{T_e} \frac{n_0}{\sqrt{2\pi v_{T_e}^2}} \exp\left(-\frac{v_\parallel^2}{2v_{T_e}^2} - k_\perp^2 \rho_{L_e}^2\right) \times \sum_{l=-\infty}^{\infty} \frac{l\Omega_e - k_\parallel v_\parallel}{\omega + l\Omega_e - k_\parallel v_\parallel} I_l(k_\perp^2 \rho_{L_e}^2), \tag{13}$$

or, equivalently,

$$\delta f_{\parallel e} = \frac{e\phi}{T_e} \frac{n_0}{\sqrt{2\pi v_{T_e}^2}} \exp\left(-\frac{v_\parallel^2}{2v_{T_e}^2}\right) \times \left[1 - \exp(-k_\perp^2 \rho_{L_e}^2) \sum_{l=-\infty}^{\infty} \frac{\omega}{\omega + l\Omega_e - k_\parallel v_\parallel} I_l(k_\perp^2 \rho_{L_e}^2)\right], \tag{14}$$

where $\delta f_{\parallel e} = \int_0^{2\pi} d\theta \int_0^\infty v_\perp dv_\perp \delta f_e$ and ρ_{L_e} is the electron Larmor radius, $\rho_{L_e} = v_{T_e}/|\Omega_e|$.

A. Small Larmor radius or perpendicular propagation

Using the symmetry property of the Bessel functions $I_{-l} = I_l$, we conveniently rewrite Eq. (14) as

$$\delta f_{\parallel e} = \frac{e\phi}{T_e} \frac{n_0}{\sqrt{2\pi v_{T_e}^2}} \exp\left(-\frac{v_\parallel^2}{2v_{T_e}^2}\right) \times \left[1 - \frac{\omega}{\omega - k_\parallel v_\parallel} I_0(k_\perp^2 \rho_{L_e}^2) \exp(-k_\perp^2 \rho_{L_e}^2) - \exp(-k_\perp^2 \rho_{L_e}^2) \sum_{l=1}^{\infty} \frac{2\omega(\omega - k_\parallel v_\parallel)}{(\omega - k_\parallel v_\parallel)^2 - l^2 \Omega_e^2} I_l(k_\perp^2 \rho_{L_e}^2)\right]. \tag{15}$$

We note that the last term in Eq. (15), proportional to the infinite sum, is small and that it can be neglected with the accuracy to the leading order, if either $k_\perp \rho_{L_e} \ll 1$ (i.e., the electron Larmor radius is small compared with the characteristic perpendicular wavelength) or $|\omega - k_\parallel v_\parallel| \ll |\Omega_e|$ (i.e., in the drift regime). Taking $\omega \lesssim k_\parallel v_\parallel$, the latter condition can be rewritten as $k_\perp \rho_{L_e} < (k_\perp/k_\parallel)(v_{T_e}/v_\parallel)$, which allows also large values of $k_\perp \rho_{L_e}$ if the propagation is quasiperpendicular to the magnetic field, $k_\perp \gg k_\parallel$.

Dropping the infinite series in Eq. (15) and integrating for $v_\parallel$, we readily obtain the expression for the electron density, including the Larmor radius corrections, viz.,

$$\delta n_e \approx \frac{e\phi}{T_e} \frac{n_0}{\sqrt{2\pi v_{T_e}^2}} \int_{-\infty}^{\infty} dv_\parallel \exp\left(-\frac{v_\parallel^2}{2v_{T_e}^2}\right) \times \left[1 - \frac{\omega}{\omega - k_\parallel v_\parallel} I_0(k_\perp^2 \rho_{L_e}^2) \exp(-k_\perp^2 \rho_{L_e}^2)\right]. \tag{16}$$

For the parallel phase velocities that exceed the electron thermal speed, $\omega/k_\parallel \gg v_{T_e}$, our Eq. (16) yields the standard expression for the density perturbation, e.g., the one used in Ref. 12, in the context of drift-wave electron holes. For practical purposes, Eq. (16) can be further simplified by the Padé approximation, viz., $e^{-\zeta} I_0(\zeta) \approx 1/(1+\zeta)$. Finally, for the perturbations whose parallel phase velocity is smaller than the electron thermal speed $u \ll v_{T_e}$ and with the leading order accuracy, the perturbation of the electron density $\delta n_e = n_e - n_0$ can be written as

$$\delta n_e = n_0 \frac{e\phi}{T_e}\left(1 + \frac{u^2}{v_{T_e}^2}\frac{1}{1+\zeta}\right), \tag{17}$$

where $\zeta = k_\perp^2 \rho_{L_e}^2$. Using the inverse Laplace transform and for the modes propagating in the acoustic range of velocities $\sqrt{m_e/m_i}v_{T_e} \lesssim u \ll v_{T_e}$, the perturbation of the electron density can finally be written in the form

$$\frac{\delta n_e}{n_0} = (1 + \delta\hat{a}_{e_1})\frac{e\phi}{T_e}, \tag{18}$$

where the operator $\delta\hat{a}_{e_1}$ is given by

$$\delta\hat{a}_{e_1} = \frac{u^2}{v_{T_e}^2}(1 - \rho_{L_e}^2 \nabla_\perp^2)^{-1}. \tag{19}$$

B. Large Larmor radius and oblique propagation

As already mentioned, the plasma at the foot of the bow shock is relatively weakly magnetized. Coherent nonlinear structures observed in this region have the typical size perpendicular to the magnetic field of a few hundred meters, which scales as $\sim 20\lambda_D \sim 0.5\rho_{L_e}$,[5] where λ_i is the ion Debye length, and ρ_{L_e} is the electron gyroradius. Their parallel characteristic size may be somewhat larger (within one order of magnitude). Such aspect ratio corresponds to an oblique propagation, $k_\perp / k_\parallel \lesssim 10$, as suggested in Ref. 6, where the propagation at the angle $\sim 75°$ was found, which corresponds to the aspect ratio 0.26. Thus, the approximative expression (16) or its Padé version, Eq. (18), may not be applicable.

In the series expansion (14), for the Bessel functions with relatively small orders and large arguments, $l \ll \zeta$, we use an asymptotic form similar to Eq. (9.2.1), p. 364, of Ref. 13, improved by including the higher order terms in ζ^{-1} that are relevant for the spatial dispersion (the detailed derivation is given in the Appendix Sec. 1)

$$I_l(\zeta) = \frac{\exp(\zeta - l^2/2\zeta)}{\sqrt{2\pi\zeta}}\left[1 + \frac{1}{8\zeta}\left(1 - \frac{2l^2}{\zeta} + \frac{l^4}{3\zeta^2}\right)\right]. \tag{20}$$

The above differs by the factor $\exp(-l^2/2\zeta)$ from the standard handbook expression, see, e.g., Eq. (9.7.1), p. 377, Ref. 13. This factor arises from the term $\Sigma_{n=0}^{K}(1/n!)(-l^2/2\zeta)^n$ in our Eq. (A14), which is often neglected for $\zeta \gg 1$. However, it is important in the study of the linear waves in weakly magnetized plasmas, since it produces the appropriate contribution of perpendicular particle dynamics to the electron density, that is well known in the case of an unmagnetized plasma, see Eq. (22).

Conversely, in the expansion (14), for the Bessel functions with large orders, $l \gg \zeta \gg 1$, we use the principal asymptotic expression (9.3.1), p. 365 of Ref. 13,

$$I_l(\zeta) = \frac{1}{\sqrt{2\pi l}}\left(\frac{e\zeta}{2l}\right)^l \exp\left(\frac{\zeta^2}{4l}\right). \tag{21}$$

The derivation of Eq. (21) from the more general *uniform asymptotic expansion for large orders* and the outline of its validity region are given in the Appendix Sec. 2.

In a further approximation, we will use these asymptotic forms on the entire range of the variable l, switching between the expressions (20) and (21) at $l=\zeta$. As the discontinuity of two asymptotic forms at this point does not exceed 30% in the region of interest, $3 < \zeta \lesssim 10$, see Eq. (27), such simple approximation may be used to obtain a reasonable order-of-magnitude estimate for the corrections to the density in the limit of large FLR. Using Eq. (13), the electron density can now be written as $\delta n_e = \delta n_e^{(0)} + \delta n_e^{(0)}$, where $\delta n_e^{(0)}$ is the standard result for an unmagnetized plasma ($\rho_{L_e} \to \infty$), viz.,

$$\delta n_e^{(0)} = \frac{e\phi}{T_e}\frac{n_0}{2\pi v_{T_e}\sqrt{\zeta}}\sum_{l=-\infty}^{\infty}\int_{-\infty}^{\infty} dv_\parallel \times \frac{l\Omega_e - k_\parallel v_\parallel}{\omega + l\Omega_e - k_\parallel v_\parallel} \times \exp\left(-\frac{v_\parallel^2}{2v_{T_e}^2} - \frac{l^2}{2\zeta}\right). \tag{22}$$

For the phase velocities comparable with the ion beam speed (i.e., smaller that the electron thermal speed), $u = \omega/\sqrt{k_\perp^2 + k_\parallel^2} \ll v_{T_e}$, and for $\zeta > 1$, $\delta n_e^{(0)}$ reduces to

$$\delta n_e^{(0)} = n_0 \frac{e\phi}{T_e}\left(1 + \frac{u^2}{v_{T_e}^2}\right). \tag{23}$$

It is worth noting that, if we replace the infinite sum by the integral $\Sigma_{l=-\infty}^{\infty} \to \int_{-\infty}^{\infty} dl$ and introduce the new variable $v_\perp = -l\Omega_e / k_\perp$, Eq. (22) exactly recovers the standard linear expression for the density perturbation in an unmagnetized plasma.

Conversely, $\delta n_e^{(1)}$ is the correction due to a *large Larmor radius*. In the limit $\omega \ll k v_{T_e}$, it has the form

$$\delta n_e^{(1)} = \frac{e\phi}{T_e}\frac{n_0}{\sqrt{2\pi v_{T_e}^2}}\int_{-\infty}^{\infty} dv_\parallel \exp\left(-\frac{v_\parallel^2}{2v_{T_e}^2}\right) \times \left\{\sum_{l=-\zeta}^{\zeta}\frac{\exp(-l^2/2\zeta)}{\sqrt{2\zeta}8\zeta}\left(-\frac{2l^2}{\zeta} + \frac{l^4}{3\zeta^2}\right) + 2\sum_{l=\zeta}^{\infty}\left[\frac{\exp(-\zeta + \zeta^2/4l)}{\sqrt{2l}}\left(\frac{e\zeta}{2l}\right)^l - \frac{\exp(-l^2/2\zeta)}{\sqrt{2\pi\zeta}}\right]\right\}. \tag{24}$$

Approximating in Eq. (24) the infinite sum by an integral, it is simplified to

$$\delta n_e^{(1)} = n_0 \frac{e\phi}{T_e}\left[G(\zeta) - \frac{\zeta - 3}{12\sqrt{2\pi\zeta}}\exp\left(-\frac{\zeta}{2}\right) - \operatorname{erfc}\sqrt{\frac{\zeta}{2}}\right], \tag{25}$$

where erfc is the complementary error function $\operatorname{erfc}(z) = 2\pi^{-1/2}\int_z^\infty dt \exp(-t^2)$ with $\operatorname{erfc}(z) \approx \exp(-z^2)/(z\sqrt{\pi})$ for $z \gg 1$, while the function $G(\zeta)$ is given by

$$G(\zeta) = 2e^{-\zeta}\int_\zeta^\infty \frac{dl}{\sqrt{2\pi l}}\left(\frac{e\zeta}{2l}\right)^l \exp\left(\frac{\zeta^2}{4l}\right) \approx \frac{0.653\,277}{\zeta^{0.424\,381}}e^{-0.444\,262\zeta}. \tag{26}$$

For simplicity, we express the large Larmor radius correction to the electron density $\delta n_e^{(1)}$, Eqs. (25) and (26) in a Padé approximation. A simple expression, which provides a reasonable accuracy if $\zeta > 3.5$, has the form

$$\delta n_e^{(1)} \sim n_0 \frac{e\phi}{T_e} \frac{-0.005}{1-(2/3)^2\zeta + (1/2)^4\zeta^2}. \tag{27}$$

Using $\zeta = k_\perp^2 \rho_{L_e}^2$ and the inverse Laplace transform, for the modes propagating in the acoustic range of velocities $\sqrt{m_e/m_i} v_{T_e} \lesssim u \ll v_{T_e}$, the electron density can finally be written in the form

$$\frac{\delta n_e}{n_0} = (1 + \delta \hat{a}_{e_2}) \frac{e\phi}{T_e}, \tag{28}$$

where the operator $\delta \hat{a}_{e_2}$ is given by

$$\delta \hat{a}_{e_2} = -0.005 \left[1 + \left(\frac{2\rho_{L_e}}{3} \right)^2 \nabla_\perp^2 + \left(\frac{\rho_{L_e}}{2} \right)^4 \nabla_\perp^4 \right]^{-1}. \tag{29}$$

IV. LINEAR SOLUTION AND ITS INSTABILITIES

In the regime when either the finite Larmor radius (FLR) corrections are small, or the propagation is in the quasiperpendicular direction, using Eqs. (10), (18), and (28) in the Poisson's equation, $(\delta n_i - \delta n_e)/n_0 = -\lambda_i^2 \nabla^2 e\phi / T_{iB}$, where $\lambda_i = v_{T_{iB}} (\epsilon_0 m_i / n_0 e^2)^{1/2}$ is the Debye length calculated with the temperature of the beam ions, we have

$$(1 - \rho_{L_e}^2 \nabla_\perp^2) \left(a\varphi - \frac{4}{3} b_B \varphi^{3/2} - \lambda_i^2 \nabla^2 \varphi \right) - \frac{u^2}{v_{T_e}^2} \frac{T_{iB}}{T_e} \varphi = 0, \tag{30}$$

while for the large FLR corrections and oblique propagation, we obtain

$$\left[1 + \left(\frac{2\rho_{L_e}}{3} \right)^2 \nabla_\perp^2 + \left(\frac{\rho_{L_e}}{2} \right)^4 \nabla_\perp^4 \right] \times \left(a\varphi - \frac{4}{3} b_B \varphi^{3/2} - \lambda_i^2 \nabla^2 \varphi \right) - 0.005 \frac{T_{iB}}{T_e} \varphi = 0. \tag{31}$$

Here $\varphi = -e\phi / T_{iB}$ and we consider the regime $\varphi > 0$ $(\phi < 0)$ in which the ion trapping takes place. Using Eq. (9), the parameter a is defined as

$$a = a_B - (1-\beta) \frac{v_{T_{iB}}^2}{u^2} + \frac{T_{iB}}{T_e}. \tag{32}$$

In the linear case, $\varphi^{3/2} \to 0$, Eqs. (30) and (31) describe acoustic waves that are dispersive due to the thermal motion of ions and electrons, if the wavelength is close to the Debye length or to the electron Larmor radius. In an unmagnetized plasma, $\rho_{L_e} \to \infty$, on the scale much longer than the Debye length, $\lambda_i \to 0$, and for modes propagating in the acoustic range of velocities, $v_{T_e} \gtrsim u \gtrsim \sqrt{m_e/m_i} v_{T_e}$, Eqs. (30) and (31) describe nondispersive waves, whose phase velocity is determined from

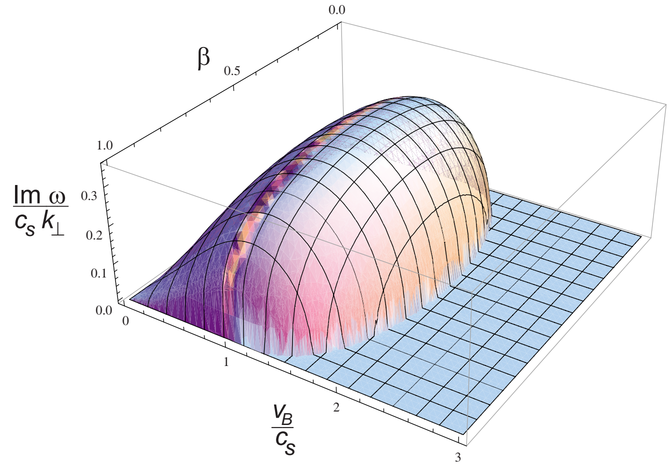

FIG. 1. (Color online) The dimensionless growth rate Im $\omega / c_s k_\perp$ of the acoustic mode driven by the ion beam.

$$a_B - (1-\beta) \frac{v_{T_{iB}}^2}{u^2} + \frac{T_{iB}}{T_e} = 0. \tag{33}$$

For linear waves which propagate with the beam $u \approx v_B$, we have $a_B = \beta$ and consequently

$$u^2 = v_{T_{iB}}^2 \frac{1-\beta}{\beta + T_{iB}/T_e}, \tag{34}$$

which is positive because $0 < \beta < 1$. Such wave is stable. It can be identified as the acoustic wave, whose beam ions are thermalized and do not participate in the wave dynamics.

Conversely, when the phase velocity is sufficiently different from that of the beam $|u - v_B| \gg v_{T_{iB}}$, we may regard the ion beam as cold, $T_{iB} \to 0$, and we have $a_B = -\beta v_{T_{iB}}^2 / (u - v_B)^2$. Thus, Eq. (33) reduces to

$$1 - (1-\beta) \frac{c_s^2}{u^2} - \frac{\beta c_s^2}{(u - v_B)^2} = 0, \tag{35}$$

or equivalently

$$(u^2 - c_s^2)(u - v_B)^2 + \beta c_s^2 v_B (v_B - 2u) = 0, \tag{36}$$

where $c_s = \sqrt{T_e / m_i}$ is the acoustic speed. In the absence of a beam, $\beta \to 0$, this dispersion relation describes the standard acoustic waves. In the presence of an ion beam, Eq. (36) is of the fourth order and may have complex roots, which correspond to the linearly unstable modes. The growth rate of such beam driven acoustic instability is displayed in Fig. 1. The boundary of the unstable region is found as the locus, in the $\beta - v_B$ plane, of the double root of the dispersion relation, yielding

$$v_B^6 - 3v_B^4 + 3v_B^2 [9\beta(\beta - 1) + 1] - 1 = 0.$$

Furthermore, modes that are stable with respect to such hydrodynamic beam instability, can be destabilized by the inverse Landau damping (kinetic bump-on-tail instability). Using

$$\mathrm{Re}\, a_B = \frac{\beta}{\sqrt{2\pi v_{T_{iB}}^2}} P \int_{-\infty}^{\infty} \frac{v\,dv}{v - u + v_B} \exp\left(-\frac{v^2}{2v_{T_{iB}}^2} \right), \tag{37}$$

$$\mathrm{Im}\ a_B = \frac{\beta\pi(u-v_B)}{\sqrt{2\pi v_{T_{iB}}^2}} \exp\left[-\frac{(u-v_B)^2}{2v_{T_{iB}}^2}\right], \tag{38}$$

where P denotes the principal value of the integral, and assuming that the number of resonant ions is relatively small, $|\mathrm{Re}\ a_B| \gg |\mathrm{Im}\ a_B|$, setting $u \to u+iw$, where u and w are real quantities, we calculate the real part u from the dispersion relation (33)) in which we substituted a_B by $\mathrm{Re}\ a_B$, and the imaginary part w is found as

$$w = \frac{\mathrm{Im}\ a_B}{(d/du)[\mathrm{Re}\ a_B - (1-\beta)(v_{T_{iB}}^2/u^2)]}. \tag{39}$$

Obviously, under the condition $|\mathrm{Re}\ a_B| \gg |\mathrm{Im}\ a_B|$, the growth rate of the bump-on-tail instability, Eq. (39), is much smaller than that of the hydrodynamic beam-driven instability.

V. COHERENT NONLINEAR SOLUTION

We seek a solution whose spatial scale is comparable or smaller that the electron Larmor radius ρ_{Le} and much larger than the Debye length λ_i. First, in the regime of small FLR corrections or quasiperpendicular propagation, setting $\lambda_i \to 0$, we rewrite Eq. (30) as

$$(1-\rho_{L_e}^2\nabla_\perp^2)\left(a\varphi - \tfrac{4}{3}b_B\varphi^{3/2}\right) - (u^2/v_{T_e}^2)(T_{iB}/T_e)\varphi = 0. \tag{40}$$

Using the variables $\varphi' = \varphi(4b_B/3a)^2$ and $\vec{r}' = \vec{r}/\rho_{L_e}$, Eq. (40) obtains a dimensionless form

$$(1-\nabla_\perp^2)(\varphi - s\varphi^{3/2}) - \xi\varphi = 0, \quad s = \mathrm{sign}\ b_B, \tag{41}$$

which a relatively simple equation of second order with one physical parameter, $\xi = (u^2/av_{T_e}^2)(T_{i_B}/T_e)$. However, we did not find any localized nonlinear solutions.

Localized solutions are found in the regime of large FLR corrections and oblique propagation. We use Eq. (31), which is rewritten as

$$\left(\frac{\rho_{L_e}^2}{4}\nabla_\perp^2 + q\right)\left(\frac{\rho_{L_e}^2}{4}\nabla_\perp^2 + \frac{1-\gamma}{q}\right)\left(a\varphi - \frac{4}{3}b_B\varphi^{3/2}\right) - \gamma\frac{4}{3}b_B\varphi^{3/2} = 0, \tag{42}$$

where the notations are

$$q = 8/9 + \sqrt{(8/9)^2 + \gamma - 1}, \qquad \gamma = 0.005T_{iB}/aT_e. \tag{43}$$

We note that the parameter q is complex if $\gamma < 1-(8/9)^2$, when in the linear case ($\varphi^{3/2} \to 0$) Eq. (42) describes two wave modes with complex wave numbers.

Conversely, for $\gamma > 1$, we have $q > 0$ and $(1-\gamma)/q < 0$, i.e., the first mode is sinusoidal and the second is evanescent. In the nonlinear regime, $\varphi = \mathcal{O}(1)$, the latter may give rise to a localized coherent structure, whose tails are exponentially decreasing in space, and smoothly connected in the center by the variation in the effective (nonlinear) wave number, and/or by the coupling with the sinusoidal mode q. Obviously, such coherent structure may be possible only if we have the scaling $0 < a \sim 0.005T_{i_B}/T_e \sim |b_B\varphi^{1/2}| \ll 1$, i.e., if its phase velocity is very close to that of the nondispersive linear mode, determined by $a=0$, Eqs. (33)–(35).

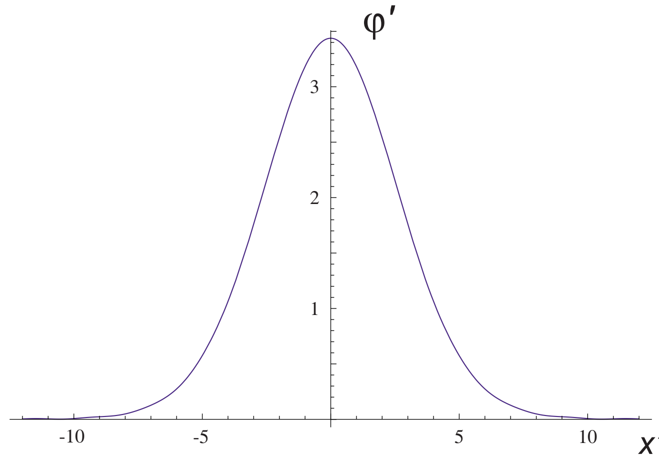

FIG. 2. (Color online) Dimensionless electrostatic potential of a slab 1D ion hump, obtained as the solution of Eq. (44) when $\gamma=1.5$ and $s=-1$. The normalizations are defined by $\phi=(-e\Phi/T_{i_B})\varphi'$ and $\vec{r}=\vec{e}_x Lx'$, where $\Phi=[3a(\gamma-1)/4b\gamma]^2$, $L=(\rho_{L_e}/2)\sqrt{q/(\gamma-1)}$, and a, γ, and q are defined in Eqs. (32) and (43).

Using the variables $\varphi' = \varphi/\Phi$ and $\vec{r}' = \vec{r}/L$, where $\Phi = [3a(\gamma-1)/4\gamma b_B]^2$ and $L = (\rho_{L_e}/2)\sqrt{q/(\gamma-1)}$ from Eq. (42), we obtain the following dimensionless equation for the electrostatic potential which, by virtue of Eq. (43), contains only one physical parameter, viz.,

$$\left(\frac{\gamma-1}{q^2}\nabla_\perp^2 + 1\right)(\nabla_\perp^2 - 1)\left(\varphi - \frac{\gamma-1}{\gamma}s\varphi^{3/2}\right) = s\varphi^{3/2}, \tag{44}$$

where $s = \mathrm{sign}\ b_B = b_B/|b_B|$. Equation (44) possesses localized solutions if $b_B < 0$ (i.e., when there exists the *surplus of trapped ions*, $\alpha_B > 1$), which corresponds to an *ion hump*. The existence of a localized solution is easily verified in the long wavelength limit. Namely, for $\gamma - 1 \ll 1$ ($q \approx 16/9$), Eq. (44) reduces to the standard equation for small amplitude ion or electron holes that has been studied in details, see, e.g., Ref. 10(one needs to be careful here, because in the limit $\gamma \to 1$, the condition of reasonably large Larmor radii in Eq. (27), $\zeta > 3.5$, may be violated), and it is readily solved as

$$\varphi(x) = (25/16)\mathrm{sech}^4(x/4). \tag{45}$$

In the general case, $\gamma = \mathcal{O}(1)$, we have found a 1D ($\nabla_\perp = \vec{e}_y d/dy$) localized solution of Eq. (44) for $\gamma=1.5$ It has a similar shape as the long wavelength solution (45), but with a somewhat larger amplitude, see Fig. 2.

VI. CONCLUSIONS

We have developed the nonlinear kinetic theory of ion acoustic perturbations, in the ion beam-plasma system in the presence of a weak magnetic field, under the conditions that are typical for the region at the foot of the Earth's quasiperpendicular bow shock, whose spatial dispersion comes from the effects of the weak magnetic field on the electrons. In the fully nonlinear regime, we found a time stationary, coherent, localized 1D solution in the form of an ion hump, supported by the nonlinearities coming from the increased population of the beam ions that are in resonance with the structure, and trapped in the self-consistent potential. These ion humps are somewhat smaller that the local electron Larmor radius and much larger that the Debye length. In contrast to the usual ion holes/humps in an unmagnetized plasma, their spatial properties are determined by the large ρ_{L_e} effects and they provide an adequate explanation for the observations of the bipolar pulses upstream of the shock by several satellite missions[2,5,6]

We may envisage two possible scenarios for the creation of the ion humps by the saturation of the linear instabilities of the ion acoustic waves. First, we may imagine that the kinetic, bump-on-tail instability, Eq. (39), associated with the resonant ions, gets saturated by the deformation of the ion distribution function in the resonant region, in the form of a *hump* (or maximum), rather than a plateau. However, the growth rate of such instability is not very large. Conversely, the faster growing, hydrodynamic beam-plasma instability, see Fig. 1, corresponds to the complex values of the phase velocity u found from the dispersion relation $a=0$. As the phase velocity of such linear mode features $\mathrm{Re}\, u \sim \mathrm{Im}\, u$, its nonlinear stabilization by the ion trapping must involve also the acceleration/deceleration, to the velocity of a linearly stable (or a marginally unstable) mode.

ACKNOWLEDGMENTS

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APPENDIX: ASYMPTOTIC EXPRESSIONS FOR BESSEL FUNCTIONS

1. Bessel functions with large arguments

In the limit $\zeta \gg 1$, the asymptotic expansion for Bessel functions has the form given in p. 269 of Ref. 14, viz.,

$$I_l(\zeta) = \frac{e^{\zeta}}{\sqrt{2\pi\zeta}}\left[\sum_{k=0}^{K-1}(-\zeta)^{-k}A_k(l) + \delta_K\right] + \frac{e^{-\zeta}}{\sqrt{2\pi\zeta}}\left[\sum_{k=0}^{K-1}\zeta^{-k}A_k(l) + \gamma_K\right], \tag{A1}$$

where $A_0(l)=1$ and

$$A_k(l) = \frac{1}{k!\,8^k}\prod_{n=1}^{k}[4l^2-(2n-1)^2], \quad k \geq 1. \tag{A2}$$

The remainders $|\delta_K|$ and $|\gamma_K|$ are comparable with the first term in the expansion that has been omitted, $|\delta_K|, |\gamma_K| \sim |\zeta^{-K}A_K(l)|$. The limit in the summation of Eq. (A1) cannot be adopted as $K\to\infty$, because such infinite series would diverge. The limit K is adopted so that the remainders are smaller than the last term retained in the sum, $|\zeta^{-K}A_K(l)| < |\zeta^{1-K}A_{K-1}(l)|$, which is realized if either

$$\zeta > K - 1/2 > |l|, \tag{A3}$$

or

$$|l| > K - 1/2 > \sqrt{l^2-\zeta+\zeta^2} - \zeta. \tag{A4}$$

In the case $|l| \gg \zeta \gg 1$ the condition Eq. (A3) cannot be realized, while the second condition, Eq. (A4), simplifies to $|l| > K > |l| - \zeta$. Conversely, in the opposite limit $|l| \ll \zeta$, these two conditions reduce to $\zeta > K > l^2/2\zeta$. Thus, the summation in Eq. (A1) can be truncated at a relatively small upper limit, $K \sim \mathcal{O}(1)$, only for the Bessel modes whose order is not very high, $|l| \lesssim \sqrt{2\zeta}$.

We proceed by rewriting the coefficients $A_k(l)$ as polynomials in l^2, viz.,

$$A_k(l) = \frac{1}{k!\,8^k}\sum_{m=0}^{k}P_{k,m}(2l)^{2m}, \tag{A5}$$

where

$$P_{k,m} = \lim_{l\to 0}\frac{1}{(2m)!\,2^{2m}}\frac{\partial^{2m}}{\partial l^{2m}}\prod_{n=1}^{k}[4l^2-(2n-1)^2], \tag{A6}$$

which yields

$$P_{k,0} = (-4)^k\left[\frac{\Gamma\left(\frac{1}{2}+k\right)}{\Gamma\left(\frac{1}{2}\right)}\right]^2 = (-1)^k[(2k-1)!!]^2, \tag{A7}$$

$$P_{k,1} = -\frac{P_{k,0}}{2!\,2^2}\left[\pi^2 - 2\psi^{(1)}\left(k+\frac{1}{2}\right)\right], \tag{A8}$$

$$P_{k,2} = \frac{P_{k,0}}{4!\,2^4}\left\{\pi^4 + 12\psi^{(1)}\left(\frac{1}{2}+k\right)\left[\psi^{(1)}\left(\frac{1}{2}+k\right)-\pi^2\right] + 2\psi^{(3)}\left(\frac{1}{2}+k\right)\right\},$$

....

$$\begin{aligned}P_{k,k-3} &= \frac{1}{3!}\left\{\left[-\sum_{n=1}^{k}(2n-1)^2\right]^3 + 3\sum_{n=1}^{k}(2n-1)^2 \times \sum_{n=1}^{k}[(2n-1)^2]^2 - 2\sum_{n=1}^{k}[(2n-1)^2]^3\right\}\\ &= -\frac{k}{945}(2240k^8 - 12\,096k^7 + 15\,600k^6 + 13\,104k^5 - 29\,820k^4 - 4284k^3 + 17\,605k^2 + 441k - 2790),\end{aligned} \tag{A9}$$

$$\begin{aligned}P_{k,k-2} &= \frac{1}{2!}\left\{\left[\sum_{n=1}^{k}(2n-1)^2\right]^2 - \sum_{n=1}^{k}[(2n-1)^2]^2\right\}\\ &= \frac{k}{90}(80k^5 - 144k^4 - 40k^3 + 120k^2 + 5k - 21),\end{aligned} \tag{A10}$$

$$P_{k,k-1} = -\sum_{n=1}^{k}(2n-1)^2 = -\frac{k}{3}(4k^2-1), \tag{A11}$$

$$P_{k,k} = 1 \tag{A12}$$

where $\psi(z)$ is the digamma function, $\psi(z)=\Gamma'(z)/\Gamma(z)$, and $\psi^{(n)}(z)=d^m\psi(z)/dz^m$.

Neglecting exponentially small terms $\sim\exp(-\zeta)$ in Eq. (A1), the asymptotic expansion, for $\zeta \gg 1$, of the Bessel function I_l takes the form

$$I_l(\zeta) = \frac{e^{\zeta}}{\sqrt{2\pi\zeta}}\left[1 + \sum_{k=1}^{K}\frac{(-1)^k}{k!\,(8\zeta)^k}\sum_{m=0}^{k}P_{k,m}(4l^2)^m\right]. \tag{A13}$$

Separating the term with $m=k$ from the second sum, and changing the order of the summations, we can rewrite this expression as

$$I_l(\zeta) = \frac{e^{\zeta}}{\sqrt{2\pi\zeta}}\left[\sum_{n=0}^{K}\frac{1}{n!}\left(\frac{-l^2}{2\zeta}\right)^n + \sum_{m=0}^{K-1}\sum_{k=m+1}^{K}\frac{P_{k,m}(4l^2)^m}{k!\,(-8\zeta)^k}\right]. \tag{A14}$$

For the Bessel functions whose order is not too large, viz., $\pi^2 l^2 \leq 4\zeta$, with the accuracy to leading order in ζ^{-1}, in Eq. (A14) we may keep only one term in the sum with respect to k, i.e., we take $\Sigma_{k=m+1}^{K} \approx \Sigma_{k=m+1}^{m+1}$. Two other sums in Eq. (A14) converge, and we can take their upper limits to be arbitrarily large, viz., $\Sigma_{n=0}^{K} \to \Sigma_{n=0}^{\infty}$ and $\Sigma_{m=0}^{K-1} \to \Sigma_{m=0}^{\infty}$, which finally yields

$$I_l(\zeta) = \frac{\exp(\zeta - l^2/2\zeta)}{\sqrt{2\pi\zeta}}\left[1 + \frac{1}{8\zeta}\left(1 - \frac{2l^2}{\zeta} + \frac{l^4}{3\zeta^2}\right)\right], \tag{A15}$$

which is applicable if $l \ll \zeta$. We note that Eq. (A15) differs by the factor $\exp(-l^2/2\zeta)$ from the often used expression (9.7.1), p. 377, Ref. 13. This factor arises from the first term, $\Sigma_{n=0}^{K}$ in our Eq. (A14), which is usually neglected for $\zeta \gg 1$, but it is of a crucial importance for the waves in plasmas in the limit of weak magnetic fields.

2. Bessel functions with large orders

For the Bessel functions whose order is larger than their argument $l \gg \zeta$, we use the uniform asymptotic expansion for large orders, Eq. (9.7.7), p. 378, Ref. 13

$$I_l(lz) = \frac{1}{\sqrt{2\pi l}}\frac{l\eta}{(1+z^2)^{1/4}}\sum_{k=0}^{K} l^{-k}u_k(t). \tag{A16}$$

Here $\eta=\sqrt{1+z^2}+\log[z/(1+\sqrt{1+z^2})]$, $t=1/\sqrt{1+z^2}$, $u_0(t)=1$, $u_1(t)=(3t-5t^2)/24$, and

$$u_{k+1}(t) = \frac{1}{2}t^2(1-t^2)u_k'(t) + \frac{1}{8}\int_0^t (1-\tau^2)u_k(t)dt.$$

For very large orders and large arguments, $l \gg \zeta \gg 1$ ($z=\zeta/l \ll 1$), with the accuracy to first order, we may truncate the summation in Eq. (A16) using $K=1$ as the upper boundary, which yields Eq. (9.3.1) in Ref. 13, viz.,

$$I_l(\zeta) = \frac{1}{\sqrt{2\pi l}}\left(\frac{e\zeta}{2l}\right)^l \exp\left(\frac{\zeta^2}{4l}\right). \tag{A17}$$

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