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The theory of nonlinear drift-Alfvén waves with the spatial scales comparable to the ion Larmor radius is developed. It is shown that the set of equations describing the nonlinear dynamics of drift-Alfvén waves in a quasistationary regime admits a solution in the form of a solitary dipole vortex. The vortex structures propagating perpendicular to the ambient magnetic field faster than the diamagnetic ion drift velocity possess spatial scales larger than the ion Larmor radius, and vice versa. The variation of the vortex impedance and spatial scale as the function of the vortex velocity is analyzed. It is shown that incorporation of the finite electron temperature effects results in the appearance of a minimum in the dependence of the vortex impedance on the vortex velocity. This leads to the existence of the vortex structures with the smallest impedance. These structures are probably the most favorable energetically and can easily be excited in space plasmas. The relevance of theoretical results obtained to the Cluster observations in the magnetospheric cusp and magnetosheath is stressed. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2844744]

I. INTRODUCTION

The drift-Alfvén waves, whose perpendicular wavelengths often determine the fine structure of many auroral processes, play an important role in the electrodynamic coupling between the ionosphere and magnetosphere. They substantially contribute to the formation of the discrete fluxes of low-energy electrons and suprathermal ions. Occasionally these waves have been observed in the highly nonlinear regime, e.g., in the form of a two-dimensional solitary vortex or vortex street, in which the perturbed electric and magnetic fields exhibit regular rotation together with the particles trapped inside the structure. Two decades ago the spatial structures of that form were registered on board the Intercosmos-Bulgaria-1300 (ICB-1300) satellite in the auroral ionosphere.[1] Unfortunately, the single satellite measurements available during that time do not allow the separation of spatial variations from temporal. Recently, interest in the drift-Alfvén vortex structures has been considerably reinforced by the possibility of multisatellite wave measurements during the four Cluster spacecraft mission. Contrary to ICB-1300 measurements, this new experiment provides a unique opportunity to detect the coherent vortex structures *in situ* by separating the temporal and spatial perturbations.[2,3] It has been found that vortex structures have been observed occasionally both in the magnetospheric cusp and in the Earth's magnetosheath.[2–4] The observed waves were of an electromagnetic nature and their impedance was of the order of the local Alfvén velocity. The structures possess rather small spatial scales of the order of the ion Larmor radius.[2,3] The perpendicular vortex velocity was substantially smaller than the local Alfvén speed. They have been identified as vortices of the drift-Alfvén waves. The existence of the vortex structures points out on the intermittent nonlocal turbulence of the Alfvén and drift-Alfvén wave interaction with nonlocal fluxes of the wave energy and generalized enstrophy in the inertial region.[5]

The previous theories[6–14] of nonlinear vortex structures in a magnetized collisionless plasma were restricted to consideration of the relatively long wavelength limit when the perpendicular spatial scale L is larger than the ion Larmor radius ρ_i, i.e., $L/\rho_i \gg 1$. The aim of the present paper is to investigate the nonlinear dynamics of drift-Alfvén waves and reveal at which conditions the vortex structures with spatial scales comparable to the ion Larmor radius can exist in the near Earth environment.

The paper is organized as follows. In Sec. II, the general dispersion relation for the drift-Alfvén waves is discussed. Two-fluid hydrodynamic equations describing the nonlinear dynamics of small-scale drift-Alfvén waves are obtained in Sec. III. The quasistationary solutions of the nonlinear equations are found and analyzed in Sec. IV. It is shown that they admit a solution in the form of a solitary vortex with spatial scales comparable to the ion Larmor radius. The typical velocities and impedance of such vortex structures are investigated. Our discussion and conclusions are found in Sec. V.

II. DRIFT-ALFVÉN-WAVE DISPERSION RELATION

Let us consider an inhomogeneous low-β plasma immersed in a uniform external magnetic field $\mathbf{B}_0$. We use a local Cartesian coordinate system whose z axis is parallel to $\mathbf{B}_0$, the x axis is along the plasma density gradient, and the y axis completes the triad. The linear dispersion relation for the drift-Alfvén waves in such a plasma was discussed in Refs. 15–17. The linear dispersion relation of the drift-Alfvén waves with arbitrary wavelength $\lambda_\perp$ compared to the ion Larmor radius is[16]

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$$\omega(\omega-\omega_{e*})(\omega-\omega_{i*})[1-e^{-z_i}I_0(z_i)] = k_z^2 v_A^2 z_i[\omega[1+(T_e/T_i)(1-e^{-z_i}I_0(z_i))] - \omega_{e*}e^{-z_i}I_0(z_i)], \quad (1)$$

where ω is the wave frequency, $\omega_{i*}=k_yT_i\kappa_n/eB_0$ and $\omega_{e*}=-k_yT_e\kappa_n/eB_0$ are the ion and electron drift frequencies, respectively, $T_{i(e)}$ is the ion (electron) temperature, $k_\perp^2=k_x^2+k_y^2$, k_x, k_y, and k_z are the x, y, and z components of the wave vector $\mathbf{k}$, respectively, $v_A=B_0/(\mu_0 n_0 m_i)^{1/2}$ is the Alfvén velocity, μ_0 is the permeability of free space, m_i is the ion mass, $\kappa_n=d\ln(n_0)/dx$, $z_i=k_\perp^2\rho_i^2$, $\rho_i=(T_i/m_i)^{1/2}/\omega_{ci}$ is the ion Larmor radius, $\omega_{ci}=eB_0/m_i$ is the ion gyrofrequency, e is the magnitude of the electron charge, and I_0 is the modified Bessel function of the first kind.

In the small ion Larmor radius approximation, $z_i=k_\perp^2\rho_i^2 \ll 1$, one can make use of an expansion $1-e^{-z_i}I_0(z_i)\simeq z_i[1-(3/4)z_i]$. Then the dispersion relation (1) reduces to

$$\omega^2(1-k_\perp^2\rho_i^2\Lambda)-\omega\omega_{i*}=k_z^2v_A^2, \quad (2)$$

where

$$\Lambda=\frac{3}{4}\left(1-\frac{\omega_{i*}}{\omega}\right)+\frac{T_e}{T_i}\frac{k_z^2v_A^2}{\omega^2}\left(1-\frac{\omega_{e*}}{\omega}\right)^{-1}. \quad (3)$$

The nonlinear drift-Alfvén waves, which in the linear approximation are described by the dispersion relation (2), were studied extensively in Refs. 10 and 11. It was shown that they can exist in the form of the dipole vortex with the perpendicular spatial scales larger than the ion Larmor radius, i.e., $L/\rho_i \gg 1$.

When plasma inhomogeneity is neglected, $\omega_{i*}\to 0$ and $\omega_{e*}\to 0$, Eq. (2) recovers the ordinary dispersion relation for the kinetic Alfvén waves (KAWs),[18]

$$\omega^2=k_z^2v_A^2[1+k_\perp^2\rho_s^2(1+3T_i/4T_e)], \quad (4)$$

where $\rho_s=c_s/\omega_{ci}$ and $c_s=(T_e/m_i)^{1/2}$ are the ion acoustic gyroradius and the ion acoustic speed, respectively. The dispersion relation (2) can be regarded as a variety of the well known dispersion relation for the KAWs modified by the presence of the plasma inhomogeneity.

In the case of most importance when z_i takes the finite values of the order of unity or larger (corresponding to Cluster observations), one can employ the so-called Padé approximation $1-e^{-z_i}I_0(z_i)\simeq z_i/(1+z_i)$. It has been shown[18,19] that such an approximation of the term $1-e^{-z_i}I_0(z_i)$ is suitable for the entire range of z_i. It is almost as accurate as the approximation used in Ref. 20 when $z_i<1$. With the use of this relation, the dispersion relation (1) reduces to the form that allows us to incorporate the full ion Larmor radius effects,

$$\omega(\omega-\omega_{*i})=k_z^2v_A^2[1+k_\perp^2\rho_i^2(1+T_e/T_i)(1-\omega_{*e}/\omega)^{-1}]. \quad (5)$$

In what follows, we will derive a set of nonlinear equations describing the dynamics of the drift-Alfvén waves that in the linear approximation are described by dispersion relation (5).

III. NONLINEAR EQUATIONS

To study the nonlinear dynamics of drift-Alfvén waves, we make use of the two-potential representation, $E_z\equiv\mathbf{E}\cdot\hat{\mathbf{z}}=-\partial_tA-\partial_z\phi$, $\mathbf{E}_\perp=-\nabla_\perp\phi$, and $\mathbf{B}_\perp=\nabla A\times\hat{\mathbf{z}}$, where $\mathbf{E}$ and $\mathbf{B}$ are perturbations of the electric and magnetic fields, respectively, $\hat{\mathbf{z}}$ is the unit vector along the ambient magnetic field $\mathbf{B}_0$, $\partial_t\equiv\partial/\partial t$ and $\partial_z\equiv\partial/\partial z$, and the subscripts z and $\perp$ denote the components along and perpendicular to $\hat{\mathbf{z}}$, respectively. Furthermore, ϕ is the scalar potential of the electric field and A is the z component of the vector potential. Since we consider a low-β plasma $(1\gg\beta\gg m_e/m_i)$, the compressional component of the magnetic field can be disregarded.

In the low-frequency approximation, we decompose the electron velocity $\mathbf{v}_e$ as $\mathbf{v}_e=\mathbf{v}_E+\mathbf{v}_{eD}+v_{ze}(\hat{\mathbf{z}}+\mathbf{B}_\perp/B_0)$, where $\mathbf{v}_E$ is the $\mathbf{E}\times\mathbf{B}$ velocity, $\mathbf{v}_{eD}=-T_e(m_in_0\omega_{ci})^{-1}(\hat{\mathbf{z}}\times\nabla_\perp n_0)$ is the electron diamagnetic drift velocity, and v_{ze} is the parallel electron speed. The z component of the electric current can be found from the Ampére law and is $j_z=-\mu_0^{-1}\nabla_\perp^2A$. We assume that the ion velocity parallel to the magnetic field is small and thus the parallel electric current j_z is driven only by the electrons, i.e., $j_z=-en_0v_{ze}$ and $v_{ze}=\nabla_\perp^2A/\mu_0en_0$. Taking into account that $\nabla\cdot\mathbf{v}_E=0$ and $\nabla\cdot(n_e\mathbf{v}_{eD})=0$, the electron continuity equation, $\partial_tn_e+\nabla_\perp\cdot(n_e\mathbf{v}_e)+n_0d_zv_{ze}=0$, reduces to

$$n_0^{-1}d_t^0\tilde{n}_e+v_{eD}\partial_y\Phi_e+v_A^2\rho_s^2d_z\nabla_\perp^2A_e=0. \quad (6)$$

Here $n_e=n_0+\tilde{n}_e$, $\tilde{n}_e$ is the perturbed electron number density, $\Phi_e\equiv e\phi/T_e$, $A_e\equiv eA/T_e$, $d_t^0\equiv\partial_t+\mathbf{v}_E\cdot\nabla$, $d_z\equiv\partial_z+B_0^{-1}\mathbf{B}_\perp\cdot\nabla$, $\partial_t\equiv\partial/\partial t$, and $\partial_y\equiv\partial/\partial y$.

The equation for the electron momentum balance along the total magnetic field $\mathbf{B}_0+\mathbf{B}_\perp$ is $en_0E_\parallel+d_z\tilde{p}_e=0$, where $E_\parallel=E_z+B_0^{-1}\mathbf{B}_\perp\cdot\mathbf{E}_\perp$ is the electric field component along the total magnetic field and $\tilde{p}_e=T_e\tilde{n}_e$ is the electron pressure perturbation. We note that the term due to the electron inertia is neglected since $\beta\gg m_e/m_i$. Then we have

$$(\partial_t+v_{eD}\partial_y)A_e+d_z(\Phi_e-\tilde{n}_e/n_0)=0. \quad (7)$$

In order to close the system of nonlinear equations (6) and (7), it is necessary to supplement it by the equation for the ion motion. It can be considered as two-dimensional, $\mathbf{v}_i=\mathbf{v}_{\perp i}$. To calculate the ion velocity, we make use of the ion momentum equation

$$\frac{e}{m_i}(\mathbf{v}_i\times\mathbf{B}+\mathbf{E})-\frac{\nabla_\perp p_i}{m_in_i}=d_t\mathbf{v}_i, \quad (8)$$

where $p_i=n_iT_i$ is the ion pressure and $d_t\equiv\partial_t+\mathbf{v}_i\cdot\nabla$ is the Lagrangian derivative.

A power series expansion of Eq. (8) on small parameter $\varepsilon=\omega_{ci}^{-1}d_t\ll 1$ yields

$$\mathbf{v}_i\simeq\mathbf{v}_E+\mathbf{v}_{iD}+\omega_{ci}^{-1}[\hat{\mathbf{z}}\times(\partial_t+(\mathbf{v}_E+\mathbf{v}_{iD})\cdot\nabla)(\mathbf{v}_E+\mathbf{v}_{iD})]. \quad (9)$$

Here $v_{iD}=(m_in_i\omega_{ci})^{-1}(\hat{\mathbf{z}}\times\nabla_\perp p_i)$ is the ion diamagnetic drift velocity. The last (polarization) term on the right-hand side of Eq. (9) can be decomposed as $\omega_{ci}^{-1}[\hat{\mathbf{z}}\times(\partial_t+(\mathbf{v}_E+\mathbf{v}_{iD})\cdot\nabla)\times(\mathbf{v}_E+\mathbf{v}_{iD})]\equiv\mathbf{v}_E^P+\mathbf{v}_{iD}^P$, where $\mathbf{v}_E^P$ and $\mathbf{v}_{iD}^P$ stand for the polarization parts of the ion velocity connected to the drift velocities $\mathbf{v}_E$ and $\mathbf{v}_{iD}$ through the relations

$$\mathbf{v}_E^P = \frac{1}{\omega_{ci}}(\hat{\mathbf{z}} \times d_t^0 \mathbf{v}_E) = -\rho_i^2 d_t^0 \nabla_\perp \Phi_i \tag{10}$$

and

$$\begin{aligned}\mathbf{v}_{iD}^P &= \frac{1}{\omega_{ci}}(\hat{\mathbf{z}} \times (d_t^0 \mathbf{v}_{iD} + \mathbf{v}_{iD} \cdot \nabla(\mathbf{v}_E + \mathbf{v}_{iD}))) \\ &= -\frac{\rho_i^2}{n_0} d_t^0 \nabla_\perp n_i - \frac{\rho_i^3 v_{Ti}}{n_0}\{n_i, \nabla_\perp \Phi_i\}.\end{aligned} \tag{11}$$

Here $v_{Ti}=(T_i/m_i)^{1/2}$, $\Phi_i=e\phi/T_i$, $d_t^0=\partial_t+\mathbf{v}_E\cdot\nabla=\partial_t+\rho_i v_{Ti}\{\Phi_i,\dots\}$, and $\{A,B\}\equiv(\partial_x A)\partial_y B-(\partial_y A)\partial_x B$ denotes the Poisson bracket. With the help of Eqs. (10) and (11), the ion continuity equation $\partial_t n_i+\nabla_\perp\cdot(n_i\mathbf{v}_i)=0$ reduces to[21]

$$d_t^0 n_i + n_i \nabla \cdot (\mathbf{v}_E^P + \mathbf{v}_{iD}^P) = 0, \tag{12}$$

where the terms $\mathbf{v}_{iD}\cdot\nabla\mathbf{v}_{iD}$ have been neglected in the considered WKB (Wentzel–Kramers–Brillouin) approximation as small of the order of $\lambda_\perp/L$, where $L=\kappa_n^{-1}$ is the spatial scale of the plasma inhomogeneity. Decomposing $n_i=n_0+\tilde{n}_i$, where $\tilde{n}_i(\ll n_0)$ is the wave perturbations of the ion number density, and accounting for the polarization parts of the ion velocity Eqs. (10) and (11), from Eq. (12) one finds that

$$\begin{aligned}&d_t^0(1-\rho_i^2\nabla_\perp^2)\tilde{n}_i - u_{*i} n_0 \partial_y \Phi_i - n_0 \rho_i^2 d_t^0 \nabla_\perp^2 \Phi_i \\ &\quad = \rho_i^3 v_{Ti}\{\nabla_\perp \tilde{n}_i, \nabla_\perp \Phi_i\}.\end{aligned} \tag{13}$$

Here the ion temperature perturbations have been neglected as small corrections of the higher order. Equations (6), (7), and (13) together with the charge neutrality condition, $\tilde{n}_i=\tilde{n}_e$, constitute a closed set of equations describing the nonlinear dynamics of drift-Alfvén waves in a plasma with nonzero ion temperature.

In the linear approximation, a Fourier transform of Eq. (13) gives

$$\frac{\tilde{n}_i}{n_0} = -\left[\frac{\omega_{*i}}{\omega} + \frac{z_i}{1+z_i}\left(1-\frac{\omega_{*i}}{\omega}\right)\right]\Phi_i. \tag{14}$$

Let us compare relation (14) with that of the fully kinetic treatment[15]

$$\frac{\tilde{n}_i}{n_0} = -\left\{\frac{\omega_{*i}}{\omega} + [1-e^{-z_i}I_0(z_i)]\left(1-\frac{\omega_{*i}}{\omega}\right)\right\}\Phi_i. \tag{15}$$

One sees that in the Padé approximation, relation (15) coincides with Eq. (14). This confirms that Eq. (14) adequately describes the ion density perturbations. In the linear approximation, Eqs. (6), (7), and (13) yield the local dispersion relation (5). This justifies the relevance of our hydrodynamic approach to the exact kinetic theory.

IV. VORTEX SOLUTION

Now let us analyze the quasistationary solutions. We assume that all perturbed values depend solely on x and $\eta=y-ut+\alpha z$, where u is the translation speed of the wave along the y axis and α is the angle between the wave front normal and the (x,y) plane. In the stationary (x,η) frame, Eqs. (6) and (7) take the form

$$\hat{D}_\phi\left(\frac{\tilde{n}_e}{n_0} - \frac{v_{eD}}{u}\Phi_e\right) = \frac{\alpha v_A^2 \rho_s^2}{u}\hat{D}_A \nabla_\perp^2 A_e \tag{16}$$

and

$$\hat{D}_A\left[-\frac{u}{\alpha}\left(1-\frac{v_{eD}}{u}\right)A_e + \Phi_e - \frac{\tilde{n}_e}{n_0}\right] = 0, \tag{17}$$

where $\hat{D}_\phi=\partial_\eta-(\rho_s c_s/u)(\nabla\Phi_e\times\nabla)_z$ and $\hat{D}_A=\partial_\eta-(\rho_s c_s/\alpha)\times(\nabla A_e\times\nabla)_z$, respectively. A particular solution of Eq. (17) is

$$\frac{\tilde{n}_e}{n_0} = \Phi_e - \frac{u}{\alpha}\left(1-\frac{v_{eD}}{u}\right)A_e. \tag{18}$$

Substituting Eq. (18) into Eq. (17), one obtains

$$\hat{D}_\phi\left[\left(1-\frac{v_{eD}}{u}\right)\left(\Phi_e - \frac{u}{\alpha}A_e\right)\right] = \frac{\alpha v_A^2 \rho_s^2}{u}\hat{D}_A \nabla_\perp^2 A_e. \tag{19}$$

With the help of the equality

$$\hat{D}_A\left(\Phi_e - \frac{u}{\alpha}A_e\right) = \hat{D}_\phi\left(\Phi_e - \frac{u}{\alpha}A_e\right), \tag{20}$$

Eq. (19) reduces to

$$\hat{D}_A\left[\left(1-\frac{v_{eD}}{u}\right)\left(\Phi_e - \frac{u}{\alpha}A_e\right) - \frac{\alpha v_A^2}{u}\rho_s^2 \nabla_\perp^2 A_e\right] = 0. \tag{21}$$

A particular solution of Eq. (21) is

$$\Phi_e = \frac{u}{\alpha}\left[1 + \frac{\alpha^2 v_A^2}{u^2}\left(1-\frac{v_{eD}}{u}\right)^{-1}\rho_s^2\nabla_\perp^2\right]A_e. \tag{22}$$

With the help of relations (18) and (22) and the condition of the electroneutrality perturbations $\tilde{n}_e=\tilde{n}_i=\tilde{n}$, Eq. (13) reduces to

$$\begin{aligned}&\hat{D}_\phi\{[u(v_{iD}-u)-\alpha^2 v_A^2]\nabla_\perp^2 + \alpha^2 v_A^2 \rho_i^2(1+T_e/T_i) \\ &\quad \times(1-v_{eD}/u)^{-1}\nabla_\perp^4\}\Phi_i = 0.\end{aligned} \tag{23}$$

A particular solution of Eq. (23) is

$$\kappa^{-2}\nabla_\perp^4\Phi_i - \nabla_\perp^2\Phi_i + C(\Phi_i - uB_0 x) = 0, \tag{24}$$

where

$$\kappa^2 = -\frac{(u^2 - uv_{iD} - \alpha^2 v_A^2)(1-v_{eD}/u)}{\alpha^2 v_A^2 \rho_i^2(1+T_e/T_i)}, \tag{25}$$

and C is the constant value. Equation (24) is of the fourth order instead of the second order in the case of the Charney–Hasegawa–Mima equation (cf. Refs. 6, 8, and 10 and references therein). Such an equation and its solution were investigated in Refs. 11–13.

Now we introduce the polar coordinates $r\equiv(x^2+\eta^2)^{1/2}$ and $\theta\equiv\tan^{-1}(\eta/x)$ and then seek the solution of Eq. (24) in the dipolar form $\Phi_i(r,\theta)=\Phi(r)\cos\theta$. We suppose that at a certain point $r=a$, the solution of Eq. (24) is singular, so when $r>a$ (in the vortex exterior), the constant value C

vanishes and $C \neq 0$ when $r < a$ (in the vortex interior) and consider that $\Phi(a) = uB_0 a/c$. In the external region, the solution is

$$\Phi(r) = \Phi(a)\left[c_1 \frac{K_1(\beta r/a)}{K_1(\beta)} + c_2 \frac{a}{r}\right], \tag{26}$$

and in the internal region

$$\Phi(r) = \Phi(a)\left[\frac{r}{a} + c_3 \frac{J_1(\gamma r/a)}{J_1(\gamma)} + c_4 \frac{I_1(\nu r/a)}{I_1(\nu)}\right], \tag{27}$$

where $\beta = a\kappa$, c_1, c_2, c_3, and c_4 are constants, and J_1, I_1, and K_1 are the Bessel function and the modified Bessel functions of the first and second kind, respectively. The coefficients γ and ν are

$$\gamma = a^2[(\kappa^4 + 4\delta)^{1/2} - \kappa^2] \tag{28}$$

and

$$\nu^2 = a^2[(\kappa^4 + 4\delta)^{1/2} + \kappa^2], \tag{29}$$

where $\delta = -C\kappa^2$. The coefficients c_1, c_2, c_3, and c_4 can be obtained from continuity of $\Phi(r)$, $d\Phi(r)/dr$, $d^2\Phi(r)/dr^2$, and $d^3\Phi(r)/dr^3$ at $r = a$. The additional relation connecting the vortex parameters κ, γ, and ν, usually called the matching condition, is obtained from a continuity of $d^4\Phi(r)/dr^4$ at $r = a$ (e.g., Refs. 11–13 for further details). It should be noted that $c_1 \gg c_2$. Using this fact from Eq. (26), one concludes that in the nearby external region where the first term on the right is dominant, $\Phi(r) \propto K_1(\kappa r)$, whereas $\Phi(r)$ scales as $\propto r^{-1}$ at larger r.

We note that Eq. (25) can be obtained from the linear dispersion relation (5) with the use of substitutions $\omega/k_y \to u$ and $k_z/k_y \to \alpha$ and taking into account that in the external region for large r the asymptotics of $\Phi(r)$ scales as $\propto \exp(-\kappa r)$ so that $k_\perp^2 \to -\kappa^2$. Therefore, Eq. (25) can be considered as a modified dispersion relation for the drift-Alfvén vortices. Vortices propagating with velocity $u < 0$ or $u > 0$ correspond to the electron-drift or the ion-drift modes, respectively. Below, we will restrict our consideration to drift-Alfvén vortices related to the ion-drift and kinetic Alfvén modes. The impedance $E_\perp/B_\perp$ of these waves is of the order of v_A, whereas the impedance of the electron-drift waves substantially exceeds v_A. The multiplier $1 - v_{eD}/u = 1 + (T_e/T_i)v_{iD}/u$ in the denominator of Eq. (25) accounts for the electron contribution. When the vortex velocity becomes close to the electron diamagnetic drift velocity $u \simeq |v_{eD}|$, the ion- and electron-drift modes are coupled and thus this term can play an important role. From Eq. (25) one sees that localized drift Alfvén vortices exist, $\kappa^2 > 0$, i.e.,

$$u^2 - uv_{iD} - \alpha^2 v_A^2 < 0, \tag{30}$$

or in the range of the vortex velocities $0 \leq u \leq u_{\max}$ with the maximum vortex velocity given by $u_{\max} = v_{iD}[1 + (1 + 4\mu_*)^{1/2}]/2$, where $\mu_* \equiv (\alpha v_A/v_{iD})^2$. From Eq. (25) one can obtain the relation that connects the vortex scale L, $L = \kappa^{-1}$, with vortex velocity u normalized to the ion diamagnetic velocity, $\bar{u} \equiv u/v_{iD}$,

$$\frac{L}{\rho_i} = \left[\frac{\bar{u}\mu_*(1 + T_e/T_i)}{(\bar{u} + T_e/T_i)(\mu_* + \bar{u} - \bar{u}^2)}\right]^{1/2}. \tag{31}$$

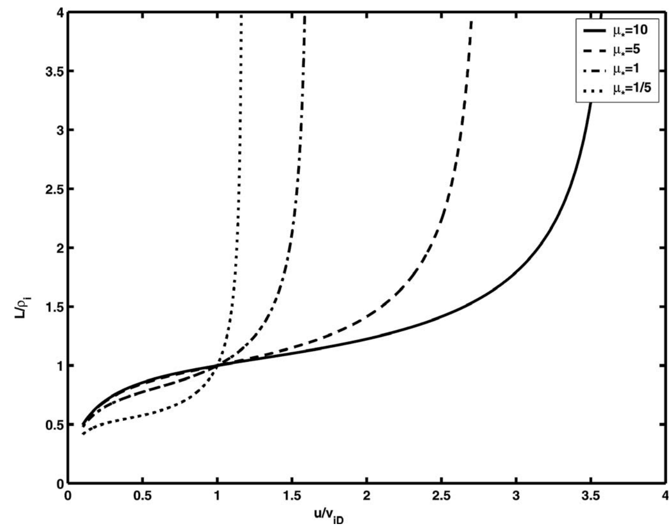

FIG. 1. A plot of the normalized vortex spatial scale as a function of normalized vortex velocity. The electron to ion temperature ratio is $T_e/T_i = 0.5$. Solid, dashed, dashed-dotted, and dotted lines correspond to μ_* equal to 10, 5, 1, and 1/5, respectively. With the increase in the normalized vortex scale L/ρ_i from 1 to 3, the vortex velocity varies from $\bar{u} = 1$ to $\bar{u}_{\max}$, which equals the normalized velocities 3.7, 2.8, 1.6, or 1.2 when $\mu_* = 10$, 5, 1, or 1/5, respectively.

Figure 1 shows the dependence of the normalized vortex spatial scale L/ρ_i as a function of the normalized vortex velocity $\bar{u}$. We note that the waves with finite μ_* correspond to the drift-Alfvén waves, whereas in the limiting cases $\mu_* \to \infty$ or $\mu_* \to 0$, they correspond to the kinetic Alfvén ($L_n \to \infty$) or ion-drift ($\alpha \to 0$) waves, respectively. The vortex structures propagating perpendicular to the ambient magnetic field with velocities smaller than the diamagnetic ion drift velocity, $u \leq v_{iD}$, possess spatial scales smaller than the ion Larmor radius, $L \leq \rho_i$ and vice versa. Equation (25) shows that in the homogeneous plasma, $v_{i,eD} \to 0$, the kinetic Alfvén wave vortices exist when their scale exceeds the ion Larmor radius, $L \geq \rho_i$.

Let us now analyze the vortex impedance. The latter is given by

$$\frac{E_\perp}{B_\perp} = v_A D, \tag{32}$$

where

$$D = \frac{1}{\mu_*^{1/2}\bar{u}} \frac{\bar{u}^2 + (T_e/T_i)(\mu_* + \bar{u})}{1 + T_e/T_i}. \tag{33}$$

In the cold electron temperature limit, the vortex impedance scales as $D = \bar{u}/\mu_*^{1/2}$ and thus increases with the growth in $\bar{u}$. However, the incorporation of the finite electron temperature

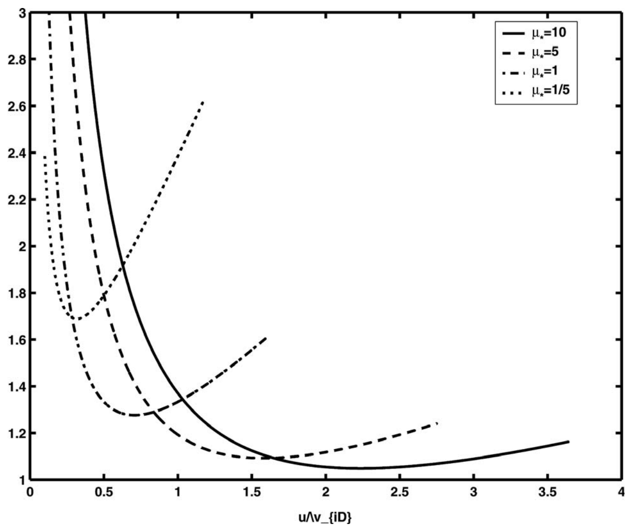

FIG. 2. Same as in Fig. 1 but for the normalized vortex impedance. The vortices propagating with the normalized velocities $\bar{u}$ equal to 2.2, 1.6, 0.7, or 0.3 have minimum of the normalized impedance 1, 1.1, 1.3, or 1.7 when $\mu_*=10$, 5, 1, or 1/5, respectively.

effects results in the appearance of the minimum in variation of D versus $\bar{u}$. This value is attained at $\bar{u}=\bar{u}^* \equiv (\mu_* T_e/T_i)^{1/2}$ and the smallest value of the impedance is $v_A D_{\min}$ with

$$D_{\min} = \frac{(T_e/T_i)^{1/2}}{1+T_e/T_i}\left[2+\left(\frac{T_e/T_i}{\mu_*}\right)^{1/2}\right]. \tag{34}$$

The variation of the normalized vortex impedance versus $\bar{u}$ is depicted in Fig. 2. Figures 1 and 2 show that the small-scale vortices with $L<\rho_i$ and large impedance, $E_\perp/B_\perp \gg v_A$, propagate with small velocities $\bar{u}\ll 1$. We note that the vortex structures of the drift-Alfvén waves with large spatial scales $L\gg\rho_i$ propagating with velocities close to $u\simeq u_{\max}$ have been investigated in Refs. 6–14.

V. SUMMARY

Our analysis represents an extension of the previous study of nonlinear drift-Alfvén waves,[1,6,7,11,12] which was limited to consideration of waves with the spatial scales L larger than the ion Larmor radius ρ_i. We have extended the previous analyses to the case of the arbitrary ratio L/ρ_i using the so-called Padé approximation. Particular attention has been paid to the waves with spatial scales of the order of the ion Larmor radius. In the quasistationary regime, a set of equations describing the nonlinear dynamics of the drift-Alfvén waves has been reduced to a single equation that possesses a solution in the form of the dipolar vortex. The basic results of our analysis can be summarized as follows:

(i) It has been shown that the velocity u of the solitary drift-Alfvén vortex structures is localized in the range $0\leqslant u\leqslant u_{\max}$.

(ii) The variation of the vortex scale L versus vortex velocity u has been analyzed, and the corresponding results have been depicted in Fig. 1. It has been found that the vortex structures propagating faster than the diamagnetic ion velocity, $u\gg v_{iD}$, possess spatial scales greater than the ion Larmor radius, $L\gg\rho_i$, and vice versa.

(iii) The variation of the vortex impedance as the function of the vortex velocity u, shown in Fig. 2, has been investigated. It has been found that when the vortex speed is close to the electron drift velocity, the drift-Alfvén and the electron-drift modes are coupled. Due to this coupling, the vortex impedance can have a minimum value that is attained when $\bar{u}=\bar{u}^* \equiv (\mu_* T_e/T_i)^{1/2}$. The existence of the minimum in the vortex impedance provides the most favorable conditions for the generation of the drift-Alfvén vortex structures.

The Cluster observations reveal the existence of small-scale drift-Alfvén vortex structures with the impedance of the order of the Alfvén speed and characteristic spatial scales $L\simeq(1-3)\rho_i$. The theoretical results obtained allow us to modify the existing interpretation of the vortex structures in the cusp region. Figures 1 and 2 show that when $\mu_*\simeq 5-10$, the vortex structures possess the smallest impedance $\simeq(1-1.1)v_A$, they propagate with velocities $u\simeq(1.6-2.2)v_{iD}$, and they have the spatial scales $L\simeq(1.2-1.3)\rho_i$. The existence of the structures at the ion Larmor radius scale in the cusp can be explained as the result of their preferable generation in the vicinity of the vortex impedance minimum. Our theoretical results are in reasonable agreement with the Cluster observations of the drift-Alfvén vortices.[3,4]

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