



**HAL**  
open science

## Drift-Alfvén waves in space plasmas -theory and mode identification

O. G. Onishchenko, O. A. Pokhotelov, Vladimir Krasnoselskikh, Shatalov S.

► **To cite this version:**

O. G. Onishchenko, O. A. Pokhotelov, Vladimir Krasnoselskikh, Shatalov S.. Drift-Alfvén waves in space plasmas -theory and mode identification. *Annales Geophysicae*, 2009, 27 (2), pp.639-644. 10.5194/angeo-27-639-2009 . insu-02517493

**HAL Id: insu-02517493**

**<https://insu.hal.science/insu-02517493>**

Submitted on 24 Mar 2020

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Drift-Alfvén waves in space plasmas – theory and mode identification

O. G. Onishchenko<sup>1,\*</sup>, O. A. Pokhotelov<sup>2,\*</sup>, V. V. Krasnoselskikh<sup>1</sup>, and S. I. Shatalov<sup>3</sup>

<sup>1</sup>LPCE/CNRS, 3A, Avenue de la Recherche Scientifique, 45071 Orléans Cedex 2, France

<sup>2</sup>Department of Automatic Control and Systems Engineering, University of Sheffield, Sheffield S1 3JD, UK

<sup>3</sup>Institute of Physics of the Earth, 123995, 10 B. Gruzinskaya, Moscow, Russia

\* on leave from: Institute of Physics of the Earth, 123995, 10 B. Gruzinskaya, Moscow, Russia

Received: 28 May 2008 – Revised: 20 November 2008 – Accepted: 16 January 2009 – Published: 11 February 2009

**Abstract.** The theory of drift-Alfvén waves with the spatial scales comparable to the ion Larmor radius is developed. The dispersion relation, the wave impedance and variations of the plasma density perturbations versus the wave frequency are investigated. The relevance of theoretical results obtained to the Cluster observations in the cusp and near a reconnection X line in the Earth’s magnetopause is discussed.

**Keywords.** Electromagnetics (Plasmas) – Interplanetary physics (Plasma waves and turbulence) – Ionosphere (Plasma waves and instabilities)

## 1 Introduction

The spacecraft observations (e.g. Chmyrev et al., 1988; Chaston et al., 2005; Sundkvist et al., 2005a,b) provide the evidence that large- and small-amplitude perturbations of the drift- and kinetic Alfvén waves are permanently present in the near Earth’s plasma environment. Quite often the waves observed by Cluster imply the spatial scales of the order of the ion Larmor radius and have the wave impedance of the order of the local Alfvén speed. The kinetic Alfvén waves (KAWs) as well as the drift-Alfvén waves (DAWs) whose perpendicular wavelengths often determine the fine spatial structure of many auroral processes play an important role in the electrodynamic coupling of the ionosphere and magnetosphere. These waves essentially control the formation of the discrete fluxes of low energy electrons and suprathermal ions. The Cluster observations on 18 March 2002 in

the vicinity of a reconnection X-line of the Earth’s magnetopause (Chaston et al., 2005) reveal small amplitude electromagnetic wave perturbations that have been identified as kinetic Alfvén and drift-Alfvén waves with perpendicular wavelengths of the order of the ion Larmor radius.

Recently Onishchenko et al. (2008) developed a comprehensive nonlinear theory of DAWs that accounts for the arbitrary perpendicular spatial scales. A particular attention in this paper has been paid to the vortex structures with spatial scales of the order of the ion Larmor radius. The results of such an analysis were applied to the interpretation of the Cluster observations in the Earth’s cusp, where nonlinear vortex structures have recently been observed by Sundkvist et al. (2005a).

The main purpose of the present study is focused on obtaining compact relations for the description of small amplitude drift-Alfvén wave perturbation that can be used for the wave identification in space plasmas and on illustration of the wave properties in such key regions as cusp and magnetopause.

The paper is organized as follows: In Sect. 2 the linear DAW dispersion relation with arbitrary perpendicular wave spatial scales is discussed and comparison with the results of Chaston et al. (2005) is provided. In Sect. 3 the wave impedance and other important wave parameters are analyzed. Our discussion and conclusions are found in Sect. 4.

## 2 Drift -Alfvén wave dispersion relation

The DAW linear dispersion relation in low- $\beta$  plasma ( $1 \gg \beta \gg m_e/m_i$ ,  $m_{e(i)}$  is the electron (ion) mass) accounting



Correspondence to: O. G. Onishchenko  
(onish@ifz.ru)

for the arbitrary wavelengths has been obtained by Mikhailovskii (1992) and reads

$$(\omega - \omega_{i*}) (\omega - \omega_{e*}) = \omega_A^2 \left[ \left( 1 - \frac{\omega_{e*}}{\omega} e^{-z_i} I_0(z_i) \right) z_i [1 - e^{-z_i} I_0(z_i)]^{-1} + z_s \right], \quad (1)$$

where all wave perturbations vary as  $\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$ ,  $\omega$  is the wave frequency and  $\mathbf{k}$  is the wave vector. We make use of SI units and a Cartesian coordinate system in which the unperturbed magnetic field  $\mathbf{B}_0$  is directed along the  $z$ -axis, the  $x$ -axis is along the plasma density gradient and the  $y$ -axis completes the triad. Furthermore,  $\omega_{i*} = k_y v_{iD} = k_y T_i \kappa_n / e B_0$  and  $\omega_{e*} = k_y v_{eD} = -k_y T_e \kappa_n / e B_0$  the ion and electron drift frequencies,  $v_{iD}$  and  $v_{eD}$  the ion and electron drift velocities,  $T_{i(e)}$  the ion (electron) temperature,  $k_\perp^2 = k_x^2 + k_y^2$ ,  $k_x$ ,  $k_y$  and  $k_z$  the  $x$ ,  $y$  and  $z$  components of the wave vector  $\mathbf{k}$ ,  $\omega_A^2 = k_z^2 v_A^2$ ,  $v_A = B_0 / (\mu_0 n_0 m_i)^{1/2}$  the Alfvén velocity,  $\mu_0$  the permeability of free space,  $n_0$  the equilibrium plasma number density,  $m_i$  the ion mass,  $\kappa_n = d \ln(n_0) / dx$ ,  $n_0$  the equilibrium proton or electron number density,  $z_i = k_\perp^2 \rho_i^2$ ,  $\rho_i = (T_i / m_i)^{1/2} / \omega_{ci}$  the ion Larmor radius,  $z_s = z_i T_e / T_i = k_\perp^2 \rho_s^2$  and  $\rho_s = (T_e / m_i)^{1/2} / \omega_{ci}$  is the ion-acoustic Larmor radius,  $\omega_{ci} = e B_0 / m_i$  the ion gyrofrequency,  $e$  the magnitude of the electron charge and  $I_0$  the modified Bessel function of the first kind. Equation (1) represents the reduced version of dispersion relation (6) of Chaston et al. (2005) that has been originally derived by Mikhailovskii (1967). In Eq. (1) the effects solely due to DAWs are retained. The other mode, e.g. magnetostatic electron-drift mode with the wave frequency  $\omega \simeq \omega_{e*}$ , is eliminated.

When plasma inhomogeneity is neglected,  $\omega_{i*} \rightarrow 0$  and  $\omega_{e*} \rightarrow 0$ , Eq. (1) recovers the dispersion relation for the kinetic Alfvén waves

$$\omega^2 = \omega_A^2 \left[ z_i [1 - e^{-z_i} I_0(z_i)]^{-1} + z_s \right], \quad (2)$$

The dispersion relation (1) can be regarded as a generalization of the well known dispersion relation for the KAW in the presence of the plasma inhomogeneity. Making use of an expansion  $1 - e^{-z_i} I_0(z_i) \simeq z_i [1 - (3/4)z_i]$  in the limit  $z_i = k_\perp^2 \rho_i^2 \ll 1$  one can obtain Eq. (2) the well-known dispersion of the kinetic Alfvén waves (Hasegawa and Uberoi, 1982)

$$\omega^2 = \omega_A^2 [1 + z_s (1 + 3T_i / 4T_e)]. \quad (3)$$

In the case of most importance when  $z_i$  takes the finite values of the order of unity or larger (corresponding to Cluster observations) to obtain an appropriate description one can use the so-called Padé approximation  $1 - e^{-z_i} I_0(z_i) \simeq z_i / (1 + z_i)$ . It has been shown by Streltsov et al. (1998) and Stasiewicz et al. (2000) that such an approximation of the term  $1 - e^{-z_i} I_0(z_i)$  is suitable for the entire range of  $z_i$ , and it is almost exact as the approximation when

$z_i > 1$ . With the use of this relation the dispersion relation (1) reduces to the form

$$\omega^2 \left( 1 - \frac{\omega_{*i}}{\omega} \right) = \omega_A^2 \left[ 1 + (z_i + z_s) \left( 1 - \frac{\omega_{*e}}{\omega} \right)^{-1} \right], \quad (4)$$

that allows us to incorporate the full ion Larmor radius effects in the wide range of parameters.

### 3 Hydrodynamic description of drift-Alfvén waves

For description of the wave electromagnetic fields we use the two-potential representation,  $E_z \equiv \mathbf{E} \cdot \hat{\mathbf{z}} = -\partial_t A - \partial_z \phi$ ,  $\mathbf{E}_\perp = -\nabla_\perp \phi$ , and  $\mathbf{B}_\perp = \nabla A \times \hat{\mathbf{z}}$ , where  $\mathbf{E}$  and  $\mathbf{B}_\perp$  are perturbations of the electric and magnetic fields, respectively,  $\hat{\mathbf{z}}$  the unit vector along the ambient magnetic field  $\mathbf{B}_0$ ,  $\partial_t \equiv \partial / \partial t$  and  $\partial_z \equiv \partial / \partial z$ , and the subscripts  $z$  and  $\perp$  denote the components along and perpendicular to  $\hat{\mathbf{z}}$ , respectively. Furthermore,  $\phi$  is the scalar potential of the electric field and  $A$  is the  $z$ -component of the vector potential. We consider that  $\partial_y \gg \partial_x$  and in some cases use the differential operator to represent formulae in more elegant form. Since we consider a low- $\beta$  plasma the parallel component of the magnetic field perturbations can be neglected.

Generally the description of drift-Alfvén waves with arbitrary wavelengths demands fully kinetic treatment. To obtain relevant relations that may be used for identification of DAW perturbations in spacecraft observations we make use of hydrodynamic approach (Kuvshinov and Mikhailovskii, 1996; Onishchenko et al., 2008) neglecting the nonlinear terms.

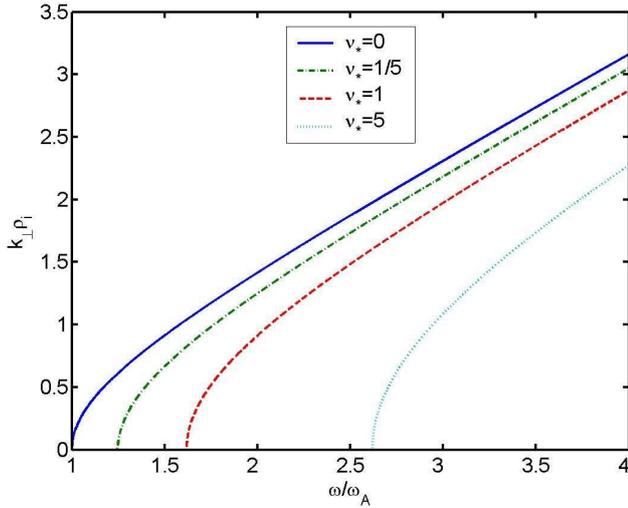
In the low-frequency approximation we decompose the electron velocity  $\mathbf{v}_e$  as  $\mathbf{v}_e = \mathbf{v}_E + \mathbf{v}_{eD} + v_{ze} \hat{\mathbf{z}}$ , where  $\mathbf{v}_E = c B_0^{-1} \mathbf{E}_\perp \times \hat{\mathbf{z}}$  is the  $\mathbf{E} \times \mathbf{B}$  drift velocity,  $\mathbf{v}_{eD} = -T_e (m_i n_0 \omega_{ci})^{-1} (\hat{\mathbf{z}} \times \nabla_\perp n_0)$  is the electron diamagnetic drift velocity and  $v_{ze}$  is the parallel electron speed. The  $z$ -component of the electric current can be found from the Ampère law and is  $j_z = -\mu_0^{-1} \nabla_\perp^2 A$ . We assume that the ion field-aligned velocity is small in considered here low- $\beta$  plasmas and thus the parallel electric current  $j_z$  is driven only by the electrons, i.e.  $j_z = -en_0 v_{ze}$  and  $v_{ze} = \nabla_\perp^2 A / \mu_0 en_0$ . Taking into account that  $\mathbf{E} \times \mathbf{B}$  velocity is divergence free  $\nabla \cdot \mathbf{v}_E = 0$  and  $\nabla \cdot (n_e \mathbf{v}_{eD}) = 0$  the electron continuity equation,  $\partial_t n_e + \nabla_\perp \cdot (n_e \mathbf{v}_e) + n_0 \partial_z v_{ze} = 0$ , reduces to

$$n_0^{-1} \partial_t \tilde{n}_e + v_{eD} \partial_y \Phi_e + v_A^2 \rho_s^2 \partial_z \nabla_\perp^2 A_e = 0. \quad (5)$$

Here  $n_e = n_0 + \tilde{n}_e$ ,  $\tilde{n}_e$  is the perturbed electron number density,  $\Phi_e \equiv e\phi / T_e$ ,  $A_e \equiv eA / T_e$ , and  $\partial_y \equiv \partial / \partial y$ .

The equation for the electron momentum balance along the ambient magnetic field  $\mathbf{B}_0$  reads  $en_0 (E_z - v_{eD} B_x) + \partial_z \tilde{p}_e = 0$ , where  $\tilde{p}_e = T_e \tilde{n}_e$  is the electron pressure perturbation. We note that since  $\beta \gg m_e / m_i$  the term due to the electron inertia is small and thus neglected. Then we have

$$(\partial_t + v_{eD} \partial_y) A_e + \partial_z (\Phi_e - \tilde{n}_e / n_0) = 0. \quad (6)$$



**Fig. 1.** The dependence of the perpendicular wave number normalized on the ion Larmor radius upon the wave frequency normalized to the Alfvén frequency. Calculations are carried out for the electron to ion temperature ratio  $T_e/T_i=0.5$ . Solid, dash-dotted, dashed and dotted lines correspond to  $v_* = 0; 1/5; 1; \text{ and } 5$ , respectively. With the increase of the plasma inhomogeneity (corresponding to the increase of  $v_*$ ) the wave frequency increases.

To close the system of nonlinear Eqs. (5–6) it is necessary to supplement it by equation for the ion motion. Following Kuvshinov and Mikhailovskii (1996) and Onishchenko et al. (2008) we decompose the ion velocity in the small frequency approximation  $\omega_{ci}^{-1}d_t \ll 1$  as

$$\mathbf{v}_i \simeq \mathbf{v}_E + \mathbf{v}_{iD} + \mathbf{v}_E^P + \mathbf{v}_{iD}^P. \quad (7)$$

Here  $v_{iD} = (m_i n_i \omega_{ci})^{-1} (\hat{\mathbf{z}} \times \nabla_{\perp} p_i)$  is the ion diamagnetic drift velocity,  $\mathbf{v}_E^P$  and  $\mathbf{v}_{iD}^P$  stand for the polarization parts of the ion velocity connected to the drift velocities  $\mathbf{v}_E$  and  $\mathbf{v}_{iD}$  through the relations

$$\mathbf{v}_E^P = \frac{1}{\omega_{ci}} (\hat{\mathbf{z}} \times \partial_t \mathbf{v}_E) = -\rho_i^2 \partial_t \nabla_{\perp} \Phi_i, \quad (8)$$

and

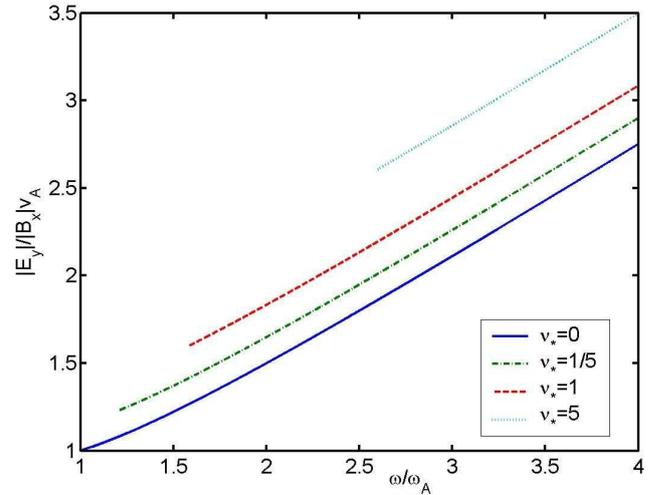
$$\mathbf{v}_{iD}^P = \frac{1}{\omega_{ci}} (\hat{\mathbf{z}} \times \partial_t \mathbf{v}_{iD}) = -\frac{\rho_i^2}{n_0} \partial_t \nabla_{\perp} n_i, \quad (9)$$

where  $\Phi_i = e\phi/T_i$ .

With the help of Eq. (7) the ion continuity equation  $\partial_t n_i + \nabla_{\perp} \cdot (n_i \mathbf{v}_i) = 0$  reduces to (Kuvshinov and Mikhailovskii, 1996; Onishchenko et al., 2001, 2008)

$$\partial_t n_i + \mathbf{v}_E \cdot \nabla n_i + n_i \nabla \cdot (\mathbf{v}_E^P + \mathbf{v}_{iD}^P) = 0, \quad (10)$$

Decomposing  $n_i = n_0 + \tilde{n}_i$ , where  $\tilde{n}_i (\ll n_0)$  is the wave perturbation of the ion number density, and accounting for the



**Fig. 2.** The wave impedance normalized to the wave frequency. Other parameters are the same as in Fig. 1.

polarization parts of the ion velocity (Eqs. 8 and 9), from Eq. (10) one finds

$$\partial_t (1 - \rho_i^2 \nabla_{\perp}^2) \tilde{n}_i - u_{*i} n_0 \partial_y \Phi_i - n_0 \rho_i^2 \partial_t \nabla_{\perp}^2 \Phi_i = 0. \quad (11)$$

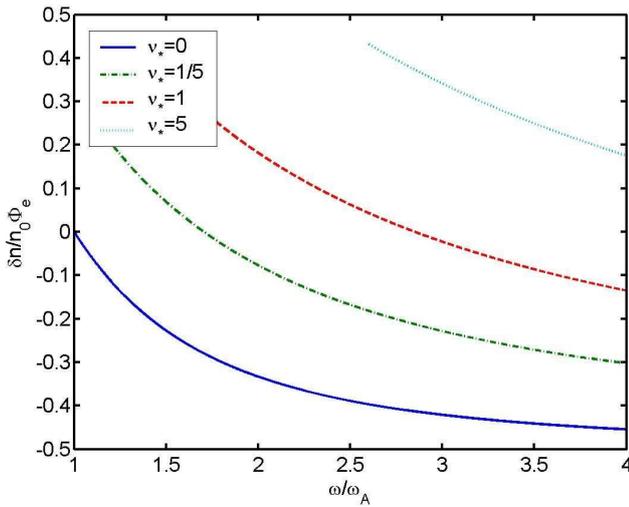
Here the ion temperature perturbations have been neglected as small corrections of the higher order. Equations (5), (6) and (11) together with the charge neutrality condition,  $\tilde{n}_i = \tilde{n}_e \equiv \delta n$ , constitute a closed set of equations describing the drift-Alfvén waves in a plasma with nonzero ion temperature. Considering that all wave perturbations vary as  $\exp(-i\omega t + i\mathbf{k} \cdot \mathbf{r})$  from these equations one obtains the following dispersion relation

$$k_{\perp} \rho_i = \left[ \frac{(\omega^{*2} - \omega^* v_*^{1/2} - 1)[1 + (T_e/T_i)v_*^{1/2}/\omega^*]}{1 + T_e/T_i} \right]^{1/2}, \quad (12)$$

where  $\omega^* \equiv \omega/\omega_A$  is the normalized wave frequency and  $v_* \equiv (\omega_{i*}/\omega_A)^2$  is the parameter characterizing the degree of a plasma inhomogeneity. The limiting case  $v_* \rightarrow 0$  corresponds to the KAWs. From Eq. (12) follows that the DAWs exist for the wave frequencies  $\omega^* \geq \omega_c^*$  where  $\omega_c^* \equiv [v_*^{1/2} + (v_* + 4)^{1/2}]/2$ . In the case of the KAWs  $\omega_c^* = 1$ . Figure 1 shows the dependence of the normalized wave frequency upon the perpendicular wave number  $k_{\perp} \rho_i$  for the electron to ion temperature ratio equals to  $T_e/T_i = 0.5$  (this ratio is typical for the Earth’s cusp) and for  $v_* = 0; 1/5; 1; \text{ and } 5$ . With the increase of the plasma inhomogeneity (corresponding to the increase of  $v_*$ ) the wave frequency also increases.

The wave impedance normalized to the Alfvén velocity evaluated from Eqs. (5), (6) and (11) is

$$\frac{E_y}{B_x v_A} = -\frac{\omega^{*2} \frac{T_i}{T_e} + \omega^* v_*^{1/2} + 1}{\omega^* (1 + \frac{T_i}{T_e})}. \quad (13)$$



**Fig. 3.** The particle number density perturbations as the function of the wave frequency. Other parameters are the same as in Fig. 1.

Figure 2 illustrates the dependence of the normalized wave impedance as a function of the wave frequency for  $\omega^* \geq \omega_c^*$ . One sees that with the increase of the wave frequency and plasma inhomogeneity the wave impedance increases. The Fourier transform of Eq. (11) gives

$$\frac{\tilde{n}_i}{n_0} = - \left[ \frac{\omega_{*i}}{\omega} + \frac{z_i}{1+z_i} \left( 1 - \frac{\omega_{*i}}{\omega} \right) \right] \Phi_i. \quad (14)$$

It is worth noting that fully kinetic treatment provided by Mikhailovskii (1992) reads

$$\frac{\tilde{n}_i}{n_0} = - \left\{ \frac{\omega_{*i}}{\omega} + [1 - e^{-z_i} I_0(z_i)] \left( 1 - \frac{\omega_{*i}}{\omega} \right) \right\} \Phi_i. \quad (15)$$

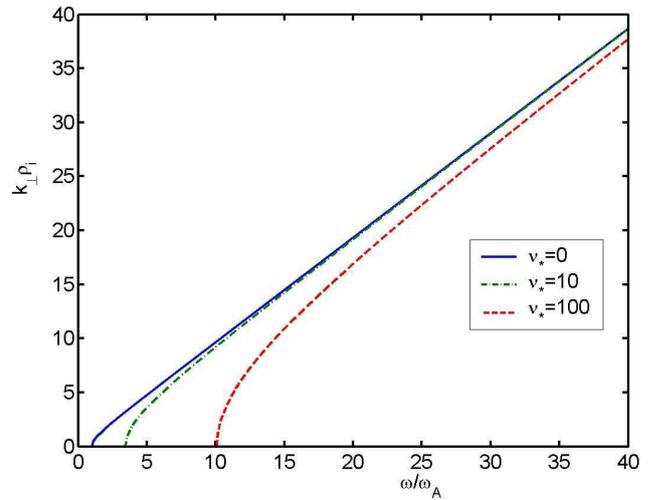
The comparison of Eq. (14) with Eq. (15) shows that in the Padé approximation they are identical. This confirms that Eq. (14) adequately describes the ion density perturbations. Making use of Eq. (14) and dispersion relation (12) one can represent the particle number density perturbation as the function of the wave frequency and perturbed electrostatic potential

$$\frac{\delta n}{n_0} = - \frac{\omega^{*2} - \omega^*(2 + T_e/T_i)v_*^{1/2} - 1}{\omega^{*2}T_i/T_e + \omega^*v_*^{1/2} + 1} \Phi_e. \quad (16)$$

Figure 3 shows the dependence of the particle number density perturbations as a function of the wave frequency for  $\omega^* \geq \omega_c^*$ .

#### 4 Summary

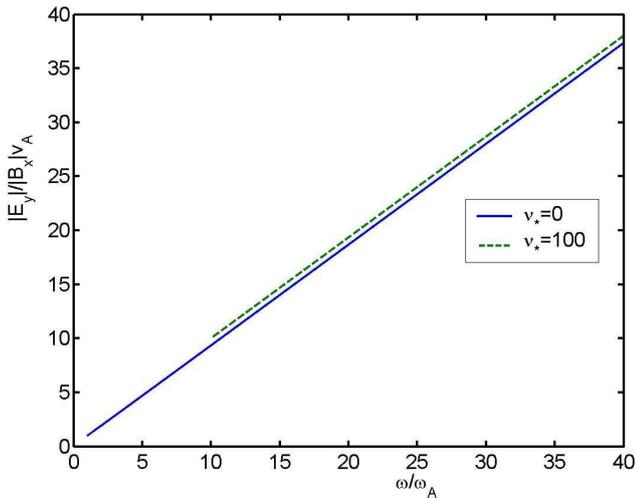
In this paper we have investigated in the linear approximation the drift-Alfvén waves with arbitrary  $k_{\perp}\rho_i$  in the so-called Padé approximation. A special attention has been paid



**Fig. 4.** The dependence of the perpendicular wave number normalized to the ion Larmor radius as a function of the normalized frequency. Calculations are carried out for the electron to ion temperature ratio  $T_i/T_e=14$ . Solid, dash-dot, dashed lines correspond to  $\nu^*=0, 10$ , and  $100$ , respectively.

to the waves with spatial scales of the order of the ion Larmor radius. The present analysis can be considered as an extension of our previous study (Onishchenko et al., 2008) of drift-Alfvén waves, which was limited to the investigation of quasi-stationary nonlinear vortex structures. We have studied the dependence of the wave characteristic parameters such as the perpendicular wavelength, the wave impedance and the particle number density perturbations versus the wave frequency. Our compact expressions for characteristic wave parameters (Eqs. 12–13 and 16) may be used for the mode identification of Cluster data in the cusp (Sundkvist et al., 2005a,b) and in the vicinity of a reconnection X-line (Chaston et al., 2005). Figures 1–6 illustrate the correlation between characteristic spatial and temporal scales, the wave impedance, and the particle number perturbations in a plasma with different degrees of plasma inhomogeneity.

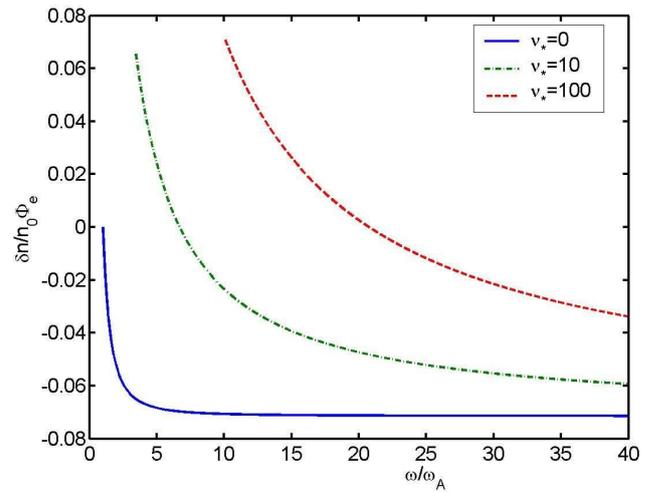
Let us apply the results obtained to the identification of the wave modes in specific satellite observations. For example, in the cusp region during the Cluster satellite crossing (Sundkvist et al., 2005a,b) a typical electron to ion temperature ratio was  $T_e/T_i=0.5$ . Figures 1–3 show the perpendicular wave number, wave impedance and particle number density perturbations as the functions of wave frequency for different values  $\nu_*=(\omega_{*i}/\omega_A)^2$ , i.e. for  $\nu^*=0; 1/5; 1$  and  $5$  characterizing the degree of plasma inhomogeneity. Figure 1 shows that DAWs exist for the frequency range  $\omega^* \geq \omega_c^*$ , where  $\omega_c^* \equiv [v_*^{1/2} + (\nu_* + 4)^{1/2}]/2$ . The wave frequency increases with the increase in the plasma inhomogeneity, with increase of  $\nu_*$ , for constant perpendicular wavelength. Figures 1 and 2 show that the smallest wave impedance normalized to the



**Fig. 5.** The wave impedance as a function of the wave frequency. The parameters are the same as in Fig. 4.

local Alfvén speed is attained for the long-wavelength perturbations, when  $\omega^* \rightarrow \omega_c^*$  or  $\lambda_\perp \rightarrow \infty$ . The smallest impedance which equals to the Alfvén speed in the case of the kinetic Alfvén waves,  $\nu_* = 0$ , increases with  $\nu_*$ . The wave impedance increases with the increase in the wave frequency and decrease of  $\lambda_\perp$ . Figure 3 shows the dependence of  $\delta n/n_0 \Phi_e$  versus wave frequency. One can see that  $\delta n/n_0 \Phi_e$  attains the largest value in the long-wavelength limit, when  $\omega^* \rightarrow \omega_c^*$  or  $\lambda_\perp \rightarrow \infty$ , that is equal to 0 in the case of KAWs, and increases with the increase in the plasma inhomogeneity. With increase in the wave frequency the value  $\delta n/n_0 \Phi_e$  becomes smaller.

In the vicinity of a reconnection X-line where DAWs have been observed by Cluster on 18 March over 14:55–14:56:30 UT the average ion to electron temperature ratio was  $T_i/T_e \simeq 14$ . Figures 4–6 illustrate the dependence of the wave perpendicular spatial length, the wave impedance and the particle number density perturbations as the function of the wave frequency at  $\nu^* = 0; 10$  and  $100$  for  $T_i/T_e = 14$ . Figure 4 shows that perpendicular wave number increases with the wave frequency as it was registered by Cluster, see Fig. 3a and b from Chaston et al. (2005). Figure 5 shows that the wave impedance increases approximately in a linear proportion with the wave frequency from  $|E|/|B| = v_A$  in a homogeneous plasma or from  $|E|/|B| \simeq 10v_A$  in a highly inhomogeneous plasma when  $\nu^* = 100$ . The waves with the perpendicular wavelength of the order of the ion Larmor radius have the wave impedance  $|E|/|B| = 1.4v_A$  or  $10v_A$  for  $\nu^* = 0$  or  $\nu^* = 100$ , respectively. When the Cluster spacecraft moved from homogeneous plasma region with large local Alfvén speed to the region with strong plasma density gradients and small local Alfvén speed, see Fig. 1b from Chaston et al. (2005), the effective value of  $\nu^*$  increased from  $\nu^* = 0$  to the



**Fig. 6.** Same as Fig. 5 but for the particle number density variations.

large values so that the wave impedance increases from the local Alfvén speed to the large values, see Fig. 3c of Chaston et al. (2005). Figure 6 shows that the normalized particle number density perturbation increases with the increase in the plasma inhomogeneity and decreases with the wave frequency.

Our theoretical results are in reasonable agreement with Cluster observations obtained during crossings of the cusp and the vicinity of the reconnection region. This gives us sufficient grounds to support conclusions of Sundkvist et al. (2005a,b) and Chaston et al. (2005) that observed in these regions waves are DAWs.

*Acknowledgements.* This research was supported by the CNRS-Russian academy collaboration through project No. 21291, by PPARC through grant PP/D002087/1, the Russian Fund for Basic Research (grants No. 07-05-00774 and 08-05-00617), by the Program of the Russian academy of Sciences No. 16 and by ISTC through the grant No. 3520. The work of V.V.K. was financially supported by CNES through Space Research Proposal entitled “Cluster Co-I DWP”. We are grateful to V. V. Lobzin for valuable discussions.

Topical Editor R. Nakamura thanks two anonymous referees for their help in evaluating this paper.

## References

- Cheng, C. Z.: A kinetic-magnetohydrodynamic model for low-frequency phenomena, *J. Geophys. Res.*, 96, 21156–21171, 1991.
- Chmyrev, V. M., Bilichenko, S. V., Pokhotelov, O. A., Marchenko, V. A., Lazarev, V. I., Streltsov, A. V., and Stenflo, L.: Alfvén vortices and related phenomena in the magnetosphere, *Physica Scripta*, 38, 841–854, 1988.
- Chaston, C. C., Phan, T. D., Bonnell, J. W., Mozer, F. C., Acuna, M., Goldstein, M. L., Balogh, A., Andre, M., Reme, H., and Fazaker-

- ley, A.: Drift-kinetic Alfvén waves observed near a reconnection  $x$  line in the Earth's magnetopause, *Phys. Rev. Lett.*, 95, 065002, doi:10.1103/PhysRevLett.95.065002, 2005.
- Hasegawa, A. and Uberoi, C.: *The Alfvén wave*, US Dept. of Energy, Washington, 1982.
- Kuvshinov, B. N. and Mikhailovskii, A. B.: MHD model including small-scale perturbations in a plasma with temperature variations, *Plasma Phys. Rep.*, 22, 529–534, 1996.
- Mikhailovskii, A. B.: Oscillations of an inhomogeneous plasma, in: *Rev. of Plasma Physics*, vol. 3, edited by: Leontovich, M. A., Consultants Bureau, New York, 1967.
- Mikhailovskii, A. B.: *Electromagnetic instabilities in an inhomogeneous plasma*, IOP, Bristol, 1992.
- Onishchenko, O., Pokhotelov, O. A., Pavlenko, V., Shukla, P., Farid, T., Stenflo, L., Kamenets, F., and Bogdanov, A.: Nonlinear ion-drift waves in nonuniform plasma with nonzero ion-temperature-gradient effects, *Phys. Plasmas*, 8, 59–66, 2001.
- Onishchenko, O. G., Krasnoselskikh, V. V., and Pokhotelov, O. A.: Drift-Alfvén vortices at the ion Larmor radius scale, *Phys. Plasmas*, 15, 022903, doi:10.1063/1.2844744, 2008.
- Stasiewicz, K., Bellan, P., Chaston, C., Kletzing, C., Lysak, R., Maggs, J., Pokhotelov, O., Seyler, C., Shukla, P., Stenflo, L., Streltsov, A., and Wahlund, J.-E.: Small scale Alfvénic structure in the aurora, *Space Sci. Rev.*, 92, 423–533, 2000.
- Streltsov, A. V., Lotko, W., Johnson, J. R., and Cheng, C. Z.: Small-scale, dispersive field line resonances in the hot magnetospheric plasma, *J. Geophys. Res.*, 103, 26559–26572, 1998.
- Sundkvist, D., Krasnoselskikh, V., Shukla, P. K., Vaivads, A., André, M., Buchert, S., and Reme, H.: In situ multi-satellite detection of coherent vortices as a manifestation of Alfvénic turbulence, *Nature*, 436, 825–828, doi:10.1038/nature03931, 2005a.
- Sundkvist, D., Vaivads, A., André, M., Wahlund, J.-E., Hobara, Y., Joko, S., Krasnoselskikh, V. V., Bogdanova, Y. V., Buchert, S. C., Cornilleau-Wehrlin, N., Fazakerley, A., Hall, J.-O., Rme, H., and Stenberg, G.: Multi-spacecraft determination of wave characteristics near the proton gyrofrequency in high-altitude cusp, *Ann. Geophys.*, 23, 983–995, 2005b, <http://www.ann-geophys.net/23/983/2005/>.