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# Multilook Intensity Ratio Distribution for 3D Layered Structures with Slightly Rough Interfaces

Richard Dusséaux and Saddek Afifi

**Abstract**— We derive the statistics for the multilook intensity ratio for a multilayered medium bounded by randomly rough surfaces. Calculations are carried out in the context of the first-order small perturbation method and assume slightly rough surfaces of infinite extent and centered Gaussian height distributions. We show that the probability distributions for the co-polar and cross-polarized intensity ratios for  $n$ -look data are functions of three parameters and that the mean exists for  $n > 1$  and the variance for  $n > 2$ . The obtained theoretical expressions are verified by comparison with Monte-Carlo results.

**Index Terms**— Multilook intensity ratio, layered rough surfaces, scattering amplitudes, small perturbation method.

## I. INTRODUCTION

SCATTERING of electromagnetic waves from rough surfaces or multilayered media bounded by randomly rough interfaces is encountered in a large number of physical problems, for remote sensing, civil engineering, geophysics or optics applications. The Small Perturbation Method (SPM) is often used for the analysis of these scattering phenomena. This model is based on Taylor series of the scattering amplitudes and of the boundary value problem and gives closed-form expressions for the coherent and incoherent scattered intensities [1]-[9]. The rigorous methods associated with numerical techniques have the advantage that they do not rely on simplifying approximations [10]-[15]. But, they require the computation of solutions for a large number of realizations of rough surfaces or multilayered media. Within its domain of validity [6]-[8], the SPM allows a fast analysis unlike to the high computational burden of numerical solutions.

Polarimetric synthetic aperture radar has proven to be an efficient tool for geophysical remote sensing [16]. The co- and cross-polarized intensity ratios are widely used indicators for the analysis of polarimetric data [17]. To reduce statistical variations, polarimetric radars use the multilook processing by averaging spatially the backscattered intensities. The statistics of single-look and multilook polarimetric SAR data have been

investigated under the hypothesis of multivariate Gaussian and K-distribution models for the underlying complex scattering amplitudes [18]-[21]. The derivations presented in [18]-[21] are relevant but the connection with an electromagnetic model is not made [22]-[24].

In [25], we derive the closed-form expression for the probability distribution for the scattered intensity ratio for a stratified medium under a monochromatic plane wave illumination. The scattered intensities are derived from the first-order small perturbation method and we assumed slightly rough interfaces of infinite extent and centered Gaussian height distributions. For single-look signatures, the co- and cross-polarized intensity ratios follow heavy-tailed distributions which don't have finite mean and variance. In [26], we showed that the statistics of the co- and cross-polarized intensity ratios allow differentiating a stratification air/clayey soil/rock with or without snow cover. In [27], we derive the statistics for the Normalized Difference Polarization Index (NDPI) which is defined as the ratio of the difference between two intensities under different polarizations on the sum of these two intensities. The NDPI therefore takes values between  $-1$  and  $1$ . Contrary to the intensity ratio, the first and second order statistical moment values of this discriminator are finite.

In the present paper, we are interested in the statistical properties of the co- and cross-polarized intensity ratios but for multilook configurations. We derive the theoretical expressions for the probability density function (PDF) and the cumulative distribution function (CDF) for a stratified medium bounded by random slightly rough interfaces under a plane wave illumination. The scattered intensities are given by the first-order SPM [3]-[4]. For electromagnetic signatures based on more than two looks, we show that the probability distribution has finite mean and variance. To our knowledge, it is the first time that the analytical expressions for the mean and variance and the CDF are found.

This paper is organized as follows. Section II is dedicated to the statistical properties of the stratified medium. Section III is devoted to the analytical expressions derived from the first-order SPM for the scattering amplitudes and the scattered intensities. In section IV and appendix, we derive closed-form expressions for the PDF and CDF and for the mean and variance. In Section V, the theory is verified by comparison with Monte-Carlo results.

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## II. ANALYZED STRUCTURE

As shown in Fig. 1, the structure we consider is a stack of two surfaces separating three media. The rough surfaces are randomly deformed over an area  $L \times L$ . The quantity  $u_0$  designates the layer average thickness. The two boundaries are located at the height  $z = a_1(x, y)$  and  $z = a_2(x, y) - u_0$ . They are realizations of second order stationary, centered Gaussian stochastic processes with Gaussian spectrum  $\hat{R}_{ii}(\alpha, \beta)$ :

$$\hat{R}_{ii}(\alpha, \beta) = \sigma_i^2 l_{xi} l_{yi} \pi \exp\left(-\frac{\alpha^2 l_{xi}^2 + \beta^2 l_{yi}^2}{4}\right) \quad (1)$$

There are two distinct correlation lengths,  $l_{xi}$  and  $l_{yi}$ . The  $i$ -th interface is isotropic if  $l_{xi} = l_{yi}$  and anisotropic if  $l_{xi} \neq l_{yi}$ .

The rms-heights  $\sigma_i$  are small numbers compared to the incident wavelength and the gradients of the surface height functions  $a_i(x, y)$  are much smaller than 1 so that the perturbation method may be used to study wave scattering [6]-[8].

The cross-spectrum  $\hat{R}_{12}(\alpha, \beta)$  is given in the following form:

$$\hat{R}_{12}(\alpha, \beta) = q \sigma_1 \sigma_2 \pi \sqrt{l_{x1} l_{x2} l_{y1} l_{y2}} \times \exp\left[-\alpha^2 \frac{(l_{x1}^2 + l_{x2}^2)}{8} - \beta^2 \frac{(l_{y1}^2 + l_{y2}^2)}{8}\right] \quad (2)$$

The coefficient  $q$  is a mixing parameter with  $|q| \leq 1$  [3], [26]. If  $q = 0$ , both rough interfaces are uncorrelated, otherwise, they are fully or partially correlated.

A monochromatic h- or v-polarized plane wave of wavelength  $\lambda$  impinges on the structure. The incident direction is defined by the wave vector  $\mathbf{k}_0$  and the angles  $\theta_0$  and  $\varphi_0$ . The structure is composed of three different regions characterized by an isotropic and homogeneous permittivity. The top and bottom regions are half-spaces. The permeability of all regions is assumed to be  $\mu_0$ .

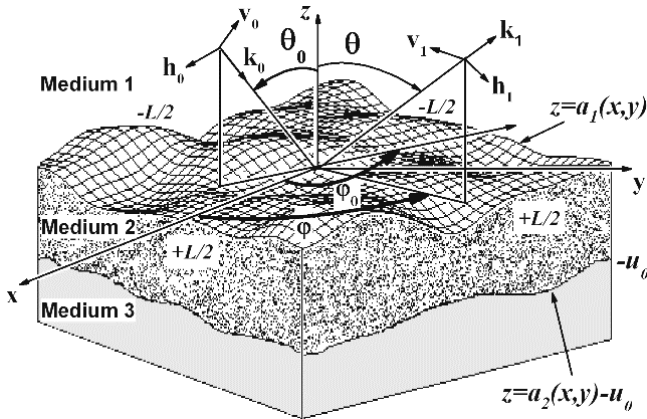


Fig. 1. Structure with two nonparallel interfaces

## III. SCATTERED INTENSITY AS A RANDOM PROCESS

For an observation direction defined by the angles  $\theta$  and  $\varphi$  (see Fig. 1), the first-order amplitudes of the co- and cross-polarized contributions of the field scattered in the upper region are derived from the SPM as follows [3]:

$$A_{1,(ba)}^{(1)}(\theta, \varphi) = K_{1,(ba)}(\alpha, \beta) \hat{a}_1(\alpha - \alpha_0, \beta - \beta_0) + K_{2,(ba)}(\alpha, \beta) \hat{a}_2(\alpha - \alpha_0, \beta - \beta_0) \quad (3)$$

with  $\alpha = k \sin \theta \cos \varphi$ ,  $\beta = k \sin \theta \sin \varphi$ ,  $\alpha_0 = k \sin \theta_0 \cos \varphi_0$ ,  $\beta_0 = k \sin \theta_0 \sin \varphi_0$  and  $k = 2\pi / \lambda$ . The subscript ( $a$ ) gives the impinging plane wave polarization ( $h$  or  $v$ ) and the subscript ( $b$ ), the scattered wave polarization ( $h$  or  $v$ ), respectively. The function  $\hat{a}_i(\alpha, \beta)$  is the Fourier transform of the function  $a_i(x, y)$ . The quantities  $K_{i,(ba)}(\alpha, \beta)$  are the first-order SPM kernels [3].

The normalized scattered intensity  $I_{(ba)}$  is defined as the power scattered per unit of solid angle divided by the incident power. In the direction  $(\theta, \varphi)$ , the first-order perturbation theory gives  $I_{(ba)}$  as follows [3],

$$I_{(ba)}(\theta, \varphi) = \frac{\lambda^2 \cos \theta_0}{\cos^2 \theta} \left| \tilde{A}_{(ba)}^{(1)}(\theta, \varphi) \right|^2 \quad (4)$$

where  $\tilde{A}_{(ba)}^{(1)}(\theta, \varphi) = A_{(ba)}^{(1)}(\theta, \varphi) / L$ . The scattered intensity  $I_{(ba)}(\theta, \varphi)$  depends on rough boundary height profile realizations and for a given direction, it is a random variable. When  $L \rightarrow \infty$ , the mean of  $I_{(ba)}(\theta, \varphi)$  is given by,

$$\langle I_{(ba)}(\theta, \varphi) \rangle = \frac{\lambda^2 \cos \theta_0}{\cos^2 \theta} \sum_{i=1}^2 \sum_{j=1}^2 \text{Re}[K_{i,(ba)}^* K_{j,(ba)} \hat{R}_{ij}(\alpha - \alpha_0, \beta - \beta_0)] \quad (5)$$

where the brackets denote a statistical average over the ensemble of realizations of  $I_{(ba)}(\theta, \varphi)$ .

## IV. MULTILOOK INTENSITY RATIO STATISTICS

### A. Scattering amplitudes as complex random variables

Let  $I_{(ba)}(\theta, \varphi)$  and  $I_{(b'a')}(\theta, \varphi)$  be the (b)- and (b')-polarized contributions of the scattered intensity under the (a)- and (a')-polarized incident waves, respectively. In a first stage, we determine the joint PDF  $p_{I_{(ba)}, I_{(b'a')}}(w, w')$  for the single-look data  $I_{(ba)}(\theta, \varphi)$  and  $I_{(b'a')}(\theta, \varphi)$ . In a second stage, we derive from the closed-form formula of  $p_{I_{(ba)}, I_{(b'a')}}(w, w')$  the joint probability law for the multilook intensities and finally, the probability law for the multilook intensity ratios.

Let  $R_{(ba)}(\theta, \varphi)$  and  $J_{(ba)}(\theta, \varphi)$  be the real and imaginary parts of the scattering amplitude  $\tilde{A}_{(ba)}^{(1)}(\theta, \varphi)$  and let  $R_{(b'a')}(\theta, \varphi)$  and  $J_{(b'a')}(\theta, \varphi)$  be the real and imaginary ones of  $\tilde{A}_{(b'a')}^{(1)}(\theta, \varphi)$ , respectively. We assume that the height distributions of both interfaces are centered and Gaussian.

Since any linear operator transforms a Gaussian process into another Gaussian process, we deduce that the first-order scattering amplitudes given by (3) are Gaussian processes and that the joint PDF of the random variables  $R_{(ba)}(\theta, \varphi)$ ,  $J_{(ba)}(\theta, \varphi)$ ,  $R_{(b'a')}(\theta, \varphi)$  and  $J_{(b'a')}(\theta, \varphi)$  is a four-dimensional Gaussian function. So, it is necessary to determine the covariance matrix characterizing this Gaussian law. Insofar as the rough surface height distributions are centered, the random variables  $R_{(ba)}(\theta, \varphi)$ ,  $J_{(ba)}(\theta, \varphi)$ ,  $R_{(b'a')}(\theta, \varphi)$  and  $J_{(b'a')}(\theta, \varphi)$  are also centered. We showed in [3] that the random variables  $R_{(ba)}(\theta, \varphi)$  and  $J_{(ba)}(\theta, \varphi)$  (respectively,  $R_{(b'a')}(\theta, \varphi)$  and  $J_{(b'a')}(\theta, \varphi)$ ) are asymptotically uncorrelated (when  $L \rightarrow \infty$ ) with equal variances  $\sigma_{R_{ba}}^2$ . The random variables  $R_{(ba)}(\theta, \varphi)$  and  $R_{(b'a')}(\theta, \varphi)$  are correlated with a covariance  $\Gamma_{R_{ba}R_{b'a'}}$  and likewise, the random variables  $R_{(ba)}(\theta, \varphi)$  and  $J_{(b'a')}(\theta, \varphi)$  are also correlated with a covariance  $\Gamma_{R_{ba}J_{b'a'}}$ . These statistical moments are defined as follows [3], [25]:

$$\sigma_{R_{ba}}^2 = \frac{1}{2} \sum_{i=1}^2 \sum_{j=1}^2 \text{Re}[K_{i,(ba)}^* K_{j,(ba)} \hat{R}_{ij}(\alpha - \alpha_0, \beta - \beta_0)] \quad (6)$$

$$\Gamma_{R_{ba}R_{b'a'}} = \frac{1}{2} \sum_{i=1}^2 \sum_{j=1}^2 \text{Re}[K_{i,(ba)}^* K_{j,(b'a')} \hat{R}_{ij}(\alpha - \alpha_0, \beta - \beta_0)] \quad (7)$$

$$\Gamma_{R_{ba}J_{b'a'}} = \frac{1}{2} \sum_{i=1}^2 \sum_{j=1}^2 \text{Im}[K_{i,(ba)}^* K_{j,(b'a')} \hat{R}_{ij}(\alpha - \alpha_0, \beta - \beta_0)] \quad (8)$$

We find from (5) and (6) that:

$$\langle I_{(ba)}(\theta, \varphi) \rangle = 2 \frac{\lambda^2 \cos \theta_0}{\cos^2 \theta} \sigma_{R_{ba}}^2 \quad (9)$$

### B. Probability density function of the multilook intensity ratio

By using polar coordinates, we derive from the joint PDF of  $R_{(ba)}$ ,  $J_{(ba)}$ ,  $R_{(b'a')}$  and  $J_{(b'a')}$  the 4D-PDF of modulus  $M_{(ba)}$  and  $M_{(b'a')}$  and phases  $\psi_{(ba)}$  and  $\psi_{(b'a')}$  of scattering amplitudes  $\tilde{A}_{(ba)}^{(1)}(\theta, \varphi)$  and  $\tilde{A}_{(b'a')}^{(1)}(\theta, \varphi)$ . By integrating over the phases, we obtain the joint PDF  $p_{M_{(ba)}, M_{(b'a')}}(m, m')$  of the modulus  $M_{(ba)}$  and  $M_{(b'a')}$  and we find from  $p_{M_{(ba)}, M_{(b'a')}}(m, m')$  the joint probability law  $p_{I_{(ba)}, I_{(b'a')}}(w, w')$  for the intensities  $I_{(ba)}$  and  $I_{(b'a')}$  [25],

$$p_{I_{(ba)}, I_{(b'a')}}(w, w') = \frac{\kappa_0^2 p_0}{1-r^2} \exp\left[-\frac{\kappa_0}{1-r^2}(w + p_0 w')\right] \times I_0\left[\frac{2r\kappa_0 \sqrt{p_0 w w'}}{1-r^2}\right] \quad (10)$$

where

$$p_0 = \frac{\sigma_{R_{ba}}^2}{\sigma_{R_{b'a'}}^2} = \frac{\langle I_{(ba)}(\theta, \varphi) \rangle}{\langle I_{(b'a')}(\theta, \varphi) \rangle} \quad (11)$$

and

$$\kappa_0 = \frac{\lambda^2 \cos \theta_0}{2\sigma_{R_{ba}}^2 \cos^2 \theta} \quad (12)$$

The function  $I_0$  denotes the zeroth-order modified Bessel function. The quantity  $r$  is the modulus of the correlation coefficient between the two complex random variables  $\tilde{A}_{(ba)}^{(1)}(\theta, \varphi)$  and  $\tilde{A}_{(b'a')}^{(1)}(\theta, \varphi)$  with  $0 \leq r \leq 1$ :

$$r = \sqrt{\frac{\Gamma_{R_{ba}R_{b'a'}}^2 + \Gamma_{R_{ba}J_{b'a'}}^2}{\sigma_{R_{ba}}^2 \sigma_{R_{b'a'}}^2}} \quad (13)$$

The  $n$ -look averaged intensity is defined as follows:

$$\bar{I}_{(ba)} = \frac{1}{n} \sum_{i=1}^n I_{(ba),i} \quad (14)$$

We assume that the  $n$ -look intensities are means of  $n$  independent identically distributed random variables. Because the  $n$ -look intensities are identically distributed, all the random variables  $I_{(ba),i} / n$  ( $1 \leq i \leq n$ ) follows the same probability law

$$p_{I_{(ba)}, I_{(b'a')}}(w_{0ba}, w_{0b'a'}) \text{ with:} \\ p_{I_{(ba)}, I_{(b'a')}}(w_{0ba}, w_{0b'a'}) = p_{\frac{I_{(ba)}}{n}, \frac{I_{(b'a')}}{n}}\left(\frac{w_{ba}}{n}, \frac{w_{b'a'}}{n}\right) \\ = n^2 p_{I_{(ba)}, I_{(b'a')}}(nw_{ba}, nw_{b'a'}) \quad (15)$$

Because the  $n$ -look intensities are independent random variables, the joint probability law  $p_{\bar{I}_{(ba)}, \bar{I}_{(b'a')}}(\bar{w}_{ba}, \bar{w}_{b'a'})$  is found by convolving  $p_{I_{(ba)}, I_{(b'a')}}(w_{0ba}, w_{0b'a'})$  with itself  $n-1$  times [28, chap. 5]. For determining this joint PDF, we also use, as in [19], the properties of the two-dimensional Laplace transform of  $p_{I_{(ba)}, I_{(b'a')}}(w_{0ba}, w_{0b'a'})$ . The mathematical derivation is included in the appendix and gives the following analytical expression,

$$p_{\bar{I}_{(ba)}, \bar{I}_{(b'a')}}(\bar{w}, \bar{w}') = \frac{p_0 n^{n+1} \kappa_0^{n+1} (p_0 \bar{w} \bar{w}')^{(n-1)/2}}{r^{n-1} (1-r^2)^n (n-1)!} \\ \times \exp\left(-\frac{n\kappa_0}{1-r^2}(\bar{w} + p_0 \bar{w}')\right) I_{n-1}\left(\frac{2rn\kappa_0}{1-r^2} \sqrt{p_0 \bar{w} \bar{w}'}\right) \quad (16)$$

where  $I_{n-1}$  is the  $(n-1)$ -th order modified Bessel function.

Let  $V_{n,(ba,b'a')} = \bar{I}_{(ba)} / \bar{I}_{(b'a')}$  be the  $n$ -look intensity ratio.

Finally, we obtain the PDF for the ratio  $V_{n,(ba,b'a')}$  as follows:

$$p_{V_{n,(ba,b'a')}}(v) = \int_0^{+\infty} \bar{w}' p_{\bar{I}_{(ba)}, \bar{I}_{(b'a')}}(v\bar{w}, \bar{w}') d\bar{w}' \quad (17)$$

By substituting (16) into (17), we find:

$$p_{V_{n,(ba,b'a')}}(v) = \frac{n^{n+1} \kappa_0^{n+1} v^{(n-1)/2}}{r^{n-1} (1-r^2)^n (n-1)!} \int_0^{+\infty} \bar{w}'^{n-1} \\ \times I_{n-1}\left(\frac{2rn\kappa_0}{1-r^2} \sqrt{p_0 v \bar{w}'}\right) \exp\left(-\frac{n\kappa_0}{1-r^2}(v + p_0) \bar{w}'\right) d\bar{w}' \quad (18)$$

By using the following relation found in the Laplace transform table [29, p. 73, Eq. 6],

$$\int_0^{+\infty} x^n I_{n-1}(ax) \exp(-sx) dx = \frac{2^n \Gamma(n+1/2) a^{n-1} s}{\sqrt{\pi} (s^2 - a^2)^{n+1/2}} \quad (19)$$

where the letter  $\Gamma$  designates the Gamma function, we get the probability distribution  $p_{V_{n,(ba'b'a')}}(v)$  for the multilook intensity ratio:

$$p_{V_{n,(ba'b'a')}}(v) = \frac{\Gamma(2n)}{\Gamma(n)\Gamma(n)} \frac{(1-r^2)^n p_0^n v^{n-1} (v+p_0)}{[v^2 + 2vp_0(1-2r^2) + p_0^2]^{n+1/2}} \quad (20)$$

The PDF only depends on the three parameters  $n$ ,  $r$  and  $p_0$ . For  $n=1$ , we get the PDF for the single-look intensity ratio. It's a heavy-tailed distribution with infinite mean and variance.

The closed-form expression (20) has been established in [19] and [20]. The derivation begins with a multivariate Gaussian model for the underlying complex scattering amplitudes. But, our approach is not based on the *a priori* assumption that the complex scattering amplitudes follows a multivariate Gaussian distribution. For slightly rough interfaces with Gaussian height distributions, we establish this property in the context of the first-order SPM.

### C. Cumulative density function of the multilook intensity ratio

We find from (20) the recurrence relation between  $p_{V_{n,(ba'b'a')}}(v)$  and  $p_{V_{n+1,(ba'b'a')}}(v)$ :

$$\left[ v + 2p_0(1-2r^2) + \frac{p_0^2}{v} \right] p_{V_{n,(ba'b'a')}}(v) = \quad (21)$$

$$2(1-r^2)p_0 \frac{2n+1}{n} p_{V_{n+1,(ba'b'a')}}(v)$$

By integration, we find from (21):

$$2p_0(1-2r^2)F_{V_{n+1,(ba'b'a')}}(v) + \int_0^v x p_{V_{n+1,(ba'b'a')}}(x) dx \quad (22)$$

$$+ p_0^2 \int_0^v \frac{p_{V_{n+1,(ba'b'a')}}(x)}{x} dx = 2(1-r^2)p_0 \frac{2n+1}{n} F_{V_{n,(ba'b'a')}}(v)$$

where the function  $F_{V_{n,(ba'b'a')}}(v)$  designates the CDF for the  $n$ -look intensity ratio. We write the  $(n+1)$ -look PDF as follows:

$$p_{V_{n+1,(ba'b'a')}}(v) = q_{n+1} \frac{v^n (v+p_0)}{R(v)^{n+1+1/2}} \quad (23)$$

where  $R(v) = cv^2 + bv + a$  with,

$$\begin{cases} a = p_0^2 \\ b = 2p_0(1-2r^2) \\ c = 1 \end{cases} \quad (24)$$

and,

$$q_{n+1} = \frac{\Gamma(2n+2)}{\Gamma(n+1)\Gamma(n+1)} (1-r^2)^{n+1} p_0^{n+1} \quad (25)$$

In accordance with the following recursion formula [30, Eq. (2.263), p.95] with  $m \neq 2n$ ,

$$K_{m,n} = \left[ \frac{x^{m-1}}{(m-2n)cR^{n-1/2}} \right] - \frac{(2m-2n-1)b}{2(m-2n)c} K_{m-1,n} \quad (26)$$

$$- \frac{(m-1)a}{(m-2n)c} K_{m-2,n}$$

where

$$K_{m,n} = \int \frac{x^m}{R^{n+1/2}} dx \quad (27)$$

we find from (23),

$$F_{n+1}(v) = q_{n+1} K_{n+1,n+1} + q_{n+1} p_0 K_{n,n+1} \quad (28)$$

and we also establish that,

$$\int_0^v x p_{n+1}(x) dx + a \int_0^v \frac{p_{n+1}(x)}{x} dx = \quad (29)$$

$$q_{n+1} (K_{n+2,n+1} + p_0 K_{n+1,n+1} + a K_{n,n+1} + a p_0 K_{n-1,n+1})$$

By substituting (29) into (22) and using the relation (26), after some algebraic manipulations, we obtain a mathematical recurrence formula for the CDF:

$$F_{V_{n+1,(ba'b'a')}}(v) = F_{V_{n,(ba'b'a')}}(v) + \frac{v-p_0}{\sqrt{R}} q_n \left( \frac{v}{R} \right)^n \quad (30)$$

Finally, we obtain from (30) the analytical expression of  $F_{V_{n,(ba'b'a')}}(v)$ :

$$F_{V_{n,(ba'b'a')}}(v) = F_{V_{1,(ba'b'a')}}(v) + \frac{v-p_0}{\sqrt{v^2 + 2vp_0(1-2r^2) + p_0^2}} \quad (31)$$

$$\times \sum_{m=1}^{n-1} \frac{\Gamma(2m)}{\Gamma(m)\Gamma(m)} \left[ \frac{(1-r^2)p_0 v}{v^2 + 2vp_0(1-2r^2) + p_0^2} \right]^m$$

where the function  $F_{V_{1,(ba'b'a')}}(v)$  is the CDF for a single-look intensity ratio. We established in [25] that

$$F_{V_{1,(ba'b'a')}}(v) = \frac{1}{2} + \frac{v-p_0}{2\sqrt{v^2 + 2vp_0(1-2r^2) + p_0^2}} \quad (32)$$

This is new compared to previous works on the  $n$ -look intensity ratio statistics. We find from (31) that for any  $n$ ,  $F_n(p_0) = F_1(p_0) = 0.5$ . As a result, the median of  $V_{n,(ba,b'a')}$  is  $p_0$  for any value of  $n$ .

### D. Mean and variance of the multilook intensity ratio

Using the recurrence formula (26), we show that the mean  $\langle V_{n,(ba,b'a')} \rangle$  of  $V_{n,(ba,b'a')}$  exists if  $n > 1$  with,

$$\langle V_{n,(ba,b'a')} \rangle = \frac{(n-r^2)p_0}{(n-1)} \quad (33)$$

and that the variance  $\sigma_{V_{n,(ba,b'a')}}^2$  is finite only for  $n > 2$  with,

$$\sigma_{V_{n,(ba,b'a')}}^2 = \frac{p_0^2}{(n-1)^2(n-2)} \times \left\{ n(2n-1) - r^2 \left[ 4(n-1)(1-r^2) + n(2n-r^2) \right] \right\} \quad (34)$$

The mean  $\langle V_{n,(ba,b'a')} \rangle$  and the standard deviation  $\sigma_{V_{n,(ba,b'a'')}}$  increase linearly with  $p_0$ . The limit of the mean is  $p_0$  as  $n$  tends to infinity and, the limit of the variance is zero. When  $n$  goes to infinity, the intensity ratio value is no longer random.

For uncorrelated scattered amplitudes  $\tilde{A}_{(ba)}^{(1)}$  and  $\tilde{A}_{(b'a')}^{(1)}$ ,  $r=0$ . For this case, the multilook intensity ratio follows a Fisher distribution for which the closed-form expressions for the mean and variance are well known [31]. To the best of our knowledge, it is the first time that the analytical expressions for the mean and variance for correlated scattered amplitudes are found.

## V. NUMERICAL RESULTS

We consider a two-layer rough ground. The wavelength  $\lambda$  is equal to 24 cm. The relative dielectric constants of the ground layers are  $\epsilon_{r_2} = 4.66 - 0.29j$  and  $\epsilon_{r_3} = 8.75 - 0.85j$  at 1.25 GHz [32]. The rms-heights are  $\sigma_1 = 0.5$  cm and  $\sigma_2 = 0.4$  cm for the upper and lower surfaces, respectively. A 5cm-thickness ( $u_0$  in Fig. 1) is assumed for the middle layer [33]. The Gaussian spectrum of the upper interface is anisotropic with the correlation lengths  $l_{x1} = 5$  cm and  $l_{y1} = 4$  cm. The lower interface spectrum is isotropic with  $l_{x2} = l_{y2} = 6$  cm. Both random surfaces are partially correlated with a correlation coefficient  $\rho_{12}$  equal to 0.2. The incident wave vector angles  $\theta_0$  and  $\phi_0$  are equal to  $30^\circ$  and  $0^\circ$ .

Fig. 2 shows the PDFs of the co-polarized intensity ratio  $V_{n,(hh,vv)}$  in the backscattering direction. The theoretical probability law obtained for surfaces of infinite extent is given by (20). The histograms are estimated over results associated with  $2^{13}$  surface realizations where  $L = 20\lambda$  [26]. The probability law is representative of the values which the intensity ratio can take and does not lose some details as for the statistical averages. The agreement between the theoretical probability law with the histogram obtained from Monte-Carlo simulations is very good for each value of  $n$  and validates the theory for the co-polarized intensity ratio in the backscattering direction.

Given the simulation parameter values, the correlation coefficient  $r$  is equal to 0.994 and therefore, the co-polarized scattering amplitudes  $A_{(hh)}^{(1)}$  and  $A_{(vv)}^{(1)}$  are strongly correlated. The parameter  $p_0$  is equal to 0.542, which means that  $\langle I_{(hh)} \rangle = 0.542 \langle I_{(vv)} \rangle$ . As shown by Fig. 2, as the number of looks increases, the PDF curve becomes narrower and concentrated near  $p_0$  because multilook processing reduces the statistical variation and this causes an increase of the maximum value of the PDF.

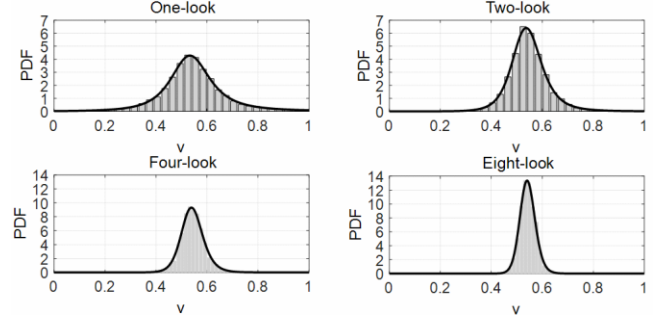


Fig. 2. Theoretical PDF of random variable  $V_{n,(hh,vv)}$  ( $\theta = -30^\circ, \phi = 0^\circ$ ) and normalized histogram for  $n=1, 2, 4$  and  $8$ .

Fig. 3 shows the theoretical CDF and the CDF estimated from the set of realizations of the co-polarized scattered intensities. For each value of  $n$ , the two curves are superimposed. As the number of looks increases, the CDF becomes a staircase function about the value  $p_0$  because the reduction of the statistical variation by the multilook processing.

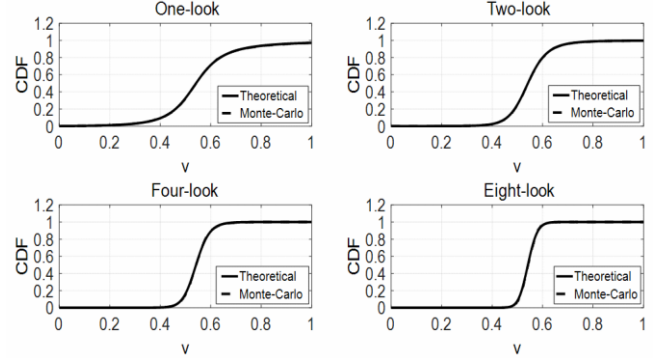


Fig. 3. Theoretical CDF of random variable  $V_{n,(hh,vv)}$  ( $\theta = -30^\circ, \phi = 0^\circ$ ) and CDF estimated from Monte-Carlo simulations for  $n=1, 2, 4$  and  $8$ .

Fig.4 compares the values of the theoretical mean and standard deviation of  $V_{n,(hh,vv)}$  ( $\theta = -30^\circ, \phi = 0^\circ$ ) with the values estimated over Monte-Carlo simulations versus the number of looks. The comparison is conclusive. These curves show that the mean  $\langle V_{n,(hh,vv)} \rangle$  quickly converges towards the median  $p_0$  and for large values of  $n$ , the standard

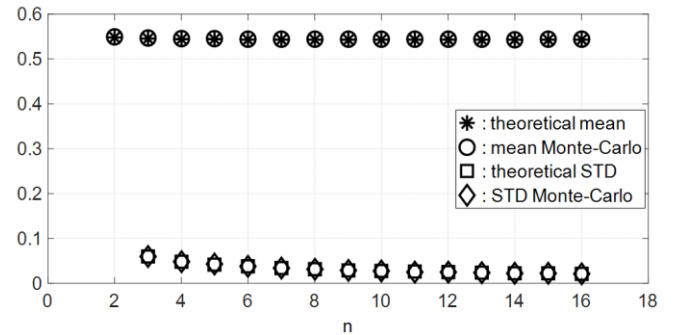


Fig. 4. Values of theoretical means and standard deviation of  $V_{n,(hh,vv)}$  ( $\theta = -30^\circ, \phi = 0^\circ$ ) and values estimated over Monte-Carlo simulations versus number of looks

deviation  $\sigma_{V_{n,(hh,vv)}}$  decreases in  $1/\sqrt{n}$ .

In the plane perpendicular to the incidence plane, there is a depolarization and we established in the context of the first-order perturbation theory that the cross-polarized intensity ratio is only defined for a (v)-polarized impinging plane wave [34]. Fig. 5 shows the theoretical PDF of  $V_{n,(hv,vv)}$  ( $\theta = -30^\circ, \varphi = 90^\circ$ ) and the associated histogram for four values of  $n$ . Given simulation parameter values, the correlation coefficient  $r$  is equal to 0.872 and therefore, the co-polarized scattering amplitude  $A_{(vv)}^{(1)}$  and the cross-polarized amplitude  $A_{(hv)}^{(1)}$  are strongly correlated. The parameter  $p_0$  is equal to 11.2 and  $\langle I_{(hv)} \rangle = 11.2 \langle I_{(vv)} \rangle$ . As a result, for this medium under consideration, the depolarization is strong within the transverse plane. At  $n=1$ , the random variable  $V_{1,(hv,vv)}$  follows a heavy-tailed law and has a large amount of skewness. As  $n$  increases, the PDF starts to become symmetrical about  $v = p_0$  so that the ratio mean approaches  $p_0$  and the maximum value increases. As previously mentioned, the histograms are estimated from  $2^{13}$  realizations over areas of  $400\lambda^2$ . We find a very good agreement between the histograms and the corresponding theoretical probability laws for the four values of the number of looks.

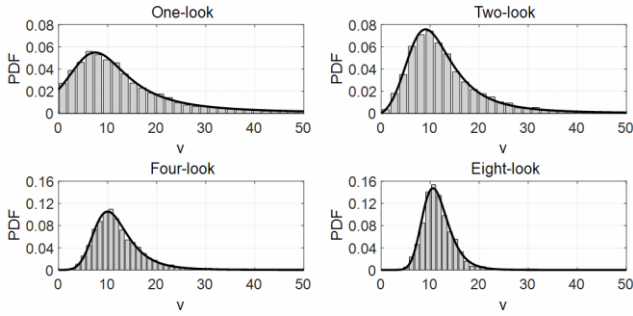


Fig. 5. Theoretical PDF of random variable  $V_{n,(hv,vv)}$  ( $\theta = -30^\circ, \varphi = 90^\circ$ ) and normalized histogram for  $n=1, 2, 4$  and  $8$ .

Fig. 6 shows the theoretical CDF and the CDF estimated from Monte-Carlo simulation data. For each value of  $n$ , the two curves are superimposed and this comparison validates the analytical formulation for the CDF for the cross-polarized ratio. At  $n=1$ , the distribution has a heavy tail and as a result, the CDF converges very slowly towards 1 as the ratio tends to positive infinity. As  $n$  increases, the convergence is faster.

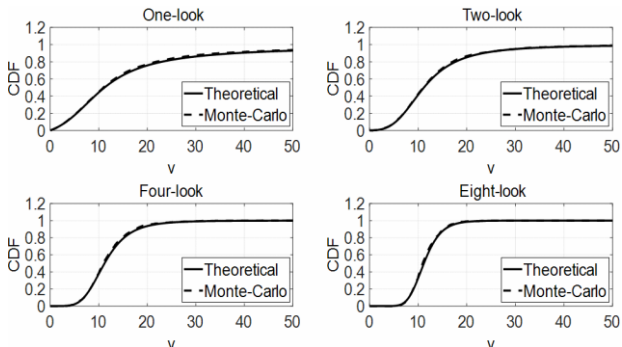


Fig. 6. Theoretical CDF of random variable  $V_{n,(hv,vv)}$  ( $\theta = -30^\circ, \varphi = 90^\circ$ ) and CDF estimated from Monte-Carlo simulations for  $n=1, 2, 4$  and  $8$ .

Fig.7 gives the values of the theoretical mean and standard deviation for the random variable  $V_{n,(hv,vv)}$  ( $\theta = -30^\circ, \varphi = 90^\circ$ ) and the values estimated over Monte-Carlo simulations versus the number of looks. As previously in the backscattered direction, the mean  $\langle V_{n,(hv,vv)} \rangle$  quickly converges towards the median  $p_0$  and for large values of  $n$ , the standard deviation  $\sigma_{V_{n,(hv,vv)}}$  decreases in  $1/\sqrt{n}$ . The comparison is satisfactory and validates the closed-form expressions for the mean and variance for the multilook cross-polarized intensity ratio. For  $n=2$ , there is a slight difference between the estimated and theoretical mean values and for  $n=3$ , a weak difference for the variance values, respectively. This difference decreases when increasing the area  $L \times L$ .

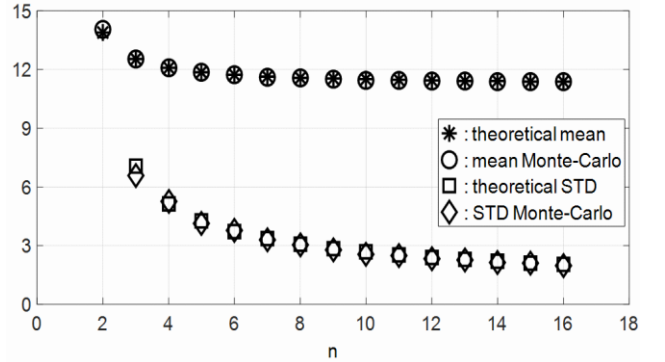


Fig. 7. Values of theoretical means and standard deviation of  $V_{n,(hv,vv)}$  ( $\theta = -30^\circ, \varphi = 90^\circ$ ) and values estimated over Monte-Carlo simulations versus number of looks.

## VI. CONCLUSION

We have derived the theoretical statistics for multilook intensity ratios for 3-D layered structures under a monochromatic plane wave illumination. The derivation begins with a multivariate Gaussian model for the underlying complex scattering amplitudes. In contrast to previous works [19]-[20], our approach is not based on the *a priori* assumption that the complex scattering amplitudes follow a multivariate Gaussian distribution. For slightly rough interfaces with an infinite extent and centered Gaussian height distributions, we establish this property in the context of the first-order small perturbation method.

Assuming that the  $n$ -look intensities are means of  $n$  independent identically distributed single look intensities, we obtain a 3-parameter probability distribution and we show that the PDF and CDF only depend on the number of looks  $n$ , the correlation coefficient  $r$  between the complex scattering amplitudes under study and the ratio  $p_0$  between the associated average single-look intensities. We have shown that the parameter  $p_0$  is the median of the  $n$ -look intensity ratio for any value of  $n$  and that the mean exists for  $n > 1$  and the variance for  $n > 2$ , respectively. The mean and the standard deviation of the  $n$ -look intensity ratio increase linearly with  $p_0$ . The limit of the mean is  $p_0$  as  $n$  tends to infinity and for large values of  $n$ , the standard deviation decreases in  $1/\sqrt{n}$ .

The obtained formulae are derived from the first-order SPM and they are valid for slightly rough interfaces only.

For a two-layer rough ground, the analytical results were compared with those derived from Monte Carlo simulations. We have assumed an upper interface with an anisotropic Gaussian spectrum and a lower interface with an isotropic one. Both random surfaces are partially correlated. The stratified medium is studied in the backscattering direction and in the transverse plane. The analytical formulas assume rough interfaces with infinite extent. Nevertheless, we have shown by Monte-Carlo simulations that these analytical expressions can be used for surfaces of a few hundred wavelength squared. The agreement between simulated data and theory is observed to be very good for the probability density function, the cumulative density function, the mean and the standard deviation for the co- and cross-polar ratios. We have considered random processes with Gaussian spectra and cross-spectra but the analytical results established in the paper can be used for all random processes with finite memory and for an arbitrary number of interfaces.

#### APPENDIX

The two-dimensional Laplace Transform (LT2) of  $p_{I_{0(ba)}, I_{0(b'a)}}(w_{0ba}, w_{0b'a'})$  is defined as follows:

$$\begin{aligned} P_{I_{0(ba)}, I_{0(b'a)}}(s, s') &= LT_2 \left\{ p_{I_{0(ba)}, I_{0(b'a)}}(w_{0ba}, w_{0b'a'}) \right\} \\ &= \int_0^{+\infty} \int_0^{+\infty} \frac{n^2 \kappa_0^2 P_0}{1-r^2} \exp \left[ -\frac{n\kappa_0}{1-r^2} (w + p_0 w') \right] \\ &\quad \times I_0 \left[ \frac{2rn\kappa_0 \sqrt{p_0 w w'}}{1-r^2} \right] \exp[-(sw + s'w')] dw dw' \end{aligned} \quad (A1)$$

The determination of  $P_{I_{0(ba)}, I_{0(b'a)}}(s, s')$  requires two one-dimensional Laplace transforms (LT),

$$\begin{aligned} P_{I_{0(ba)}, I_{0(b'a)}}(s, s') &= LT_1 \left\{ \frac{n^2 \kappa_0^2 P_0}{1-r^2} \exp \left( -\frac{n\kappa_0 p_0}{1-r^2} w' \right) \right. \\ &\quad \left. \times LT_1 \left[ \exp \left( -\frac{np_1}{1-r^2} w \right) I_0 \left( \frac{2rn\kappa_0 \sqrt{p_0 w w'}}{1-r^2} \right) \right] \right\} \end{aligned} \quad (A2)$$

By using the following representation of the zeroth-order modified Bessel function [29, p. 919, eq. 8.447],

$$I_0 \left( \frac{2nr\kappa_0 \sqrt{p_0 w w'}}{1-r^2} \right) = \sum_{m=0}^{+\infty} \left( \frac{rn\kappa_0 \sqrt{p_0 w w'}}{1-r^2} \right)^{2m} \frac{1}{(m!)^2} \quad (A3)$$

and after some mathematical calculations, we find:

$$\begin{aligned} P_{I_{0(ba)}, I_{0(b'a)}}(s, s') &= \frac{n^2 \kappa_0^2 P_0}{1-r^2} \frac{1}{s + \frac{n\kappa_0}{1-r^2}} \\ &\quad \times TL_1 \left\{ \exp \left[ -\left( \frac{n\kappa_0 p_0}{1-r^2} - \frac{r^2 n^2 \kappa_0^2 P_0}{(1-r^2)^2 \left( s + \frac{n\kappa_0}{1-r^2} \right)} \right) w' \right] \right\} \end{aligned} \quad (A4)$$

The analytical calculation of the last one-dimensional Laplace transform yields:

$$\begin{aligned} P_{I_{0(ba)}, I_{0(b'a)}}(s, s') &= \frac{n^2 \kappa_0^2 P_0}{1-r^2} \\ &\quad \times \frac{1}{s \left( s' + \frac{n\kappa_0 p_0}{1-r^2} \right) + \frac{n\kappa_0}{1-r^2} \left( s' + \frac{n\kappa_0 p_0}{1-r^2} \right) - \frac{r^2 n^2 \kappa_0^2 P_0}{(1-r^2)^2}} \end{aligned} \quad (A5)$$

The joint probability law of  $p_{\bar{I}_{(ba)}, \bar{I}_{(b'a)}}(\bar{w}, \bar{w}')$  is found by convolving  $p_{I_{0(ba)}, I_{0(b'a)}}(w_{0ba}, w_{0b'a'})$  with itself  $n-1$  times.

The  $LT_2$  of the convoluted PDFs is  $\left[ P_{I_{0(ba)}, I_{0(b'a)}}(s, s') \right]^n$  and the joint PDF  $p_{\bar{I}_{(ba)}, \bar{I}_{(b'a)}}(\bar{w}, \bar{w}')$  is obtained from a 2D inverse Laplace transform ( $LT_2^{-1}$ ) as follows:

$$\begin{aligned} p_{\bar{I}_{(ba)}, \bar{I}_{(b'a)}}(\bar{w}, \bar{w}') &= LT_2^{-1} \\ &\quad \left\{ \frac{\left( \frac{n^2 \kappa_0^2 P_0}{1-r^2} \right)^n}{\left( s \left( s' + \frac{n\kappa_0 p_0}{1-r^2} \right) + \frac{n\kappa_0}{1-r^2} \left( s' + \frac{n\kappa_0 p_0}{1-r^2} \right) - \frac{r^2 n^2 \kappa_0^2 P_0}{(1-r^2)^2} \right)^n} \right\} \end{aligned} \quad (A6)$$

The two-dimensional inverse Laplace transform (A6) becomes with two consecutive one-dimensional inverse Laplace transforms,

$$\begin{aligned} p_{\bar{I}_{(ba)}, \bar{I}_{(b'a)}}(\bar{w}, \bar{w}') &= \left( \frac{n^2 \kappa_0^2 P_0}{1-r^2} \right)^n TL_1^{-1} \left\{ \frac{1}{\left( s' + \frac{n\kappa_0 p_0}{1-r^2} \right)^n} \right. \\ &\quad \left. \times TL_1^{-1} \left\{ 1 / \left( s + \frac{n\kappa_0}{1-r^2} - \frac{r^2 n^2 \kappa_0^2 P_0}{(1-r^2)^2 \left( s' + \frac{n\kappa_0 p_0}{1-r^2} \right)} \right)^n \right\} \right\} \end{aligned} \quad (A7)$$

Knowing that

$$TL_1 \left\{ w^{n-1} \exp(-aw) \right\} = \frac{(n-1)!}{(s+a)^n} \quad (A8)$$

the relationship (A7) becomes:

$$\begin{aligned} p_{\bar{I}_{(ba)}, \bar{I}_{(b'a)}}(\bar{w}, \bar{w}') &= \left( \frac{n^2 \kappa_0^2 P_0}{1-r^2} \right)^n \frac{\bar{w}^{n-1}}{(n-1)!} \exp \left( -\frac{n\kappa_0}{1-r^2} \bar{w} \right) \\ &\quad \times TL_1^{-1} \left\{ \exp \left( \frac{r^2 n^2 \kappa_0^2 P_0 \bar{w}}{(1-r^2)^2 \left( s' + \frac{n\kappa_0 p_0}{1-r^2} \right)} \right) / \left( s' + \frac{n\kappa_0 p_0}{1-r^2} \right)^n \right\} \end{aligned} \quad (A9)$$

By using the following relationship found in the Laplace transform table [29, pp. 245, Eq. 9],

$$TL_1^{-1} \left\{ \frac{\exp(a/s)}{s^n} \right\} = (w/a)^{(n-1)/2} I_{n-1}(2\sqrt{aw}) \quad (A10)$$

We find the closed-form formula (16) giving the joint PDF for the multilook intensities  $\bar{I}_{(ba)}$  and  $\bar{I}_{(b'a)}$ .



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