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# Quantum Discrete Levels of the Universe from the Early Trans-Planckian Vacuum to the Late Dark Energy

Norma G. SANCHEZ

CNRS LERMA Observatoire de Paris PSL University,  
Sorbonne University, 61, Avenue de l'Observatoire, 75014 Paris, France

(Dated: May 3, 2020)

**Abstract:** We go forward in completing the standard model of the universe back in time with planckian and trans-planckian physics before inflation in agreement with observations, classical-quantum gravity duality and quantum space-time. The quantum vacuum energy bends the space-time and produces a constant curvature de Sitter background. We link de Sitter universe and the cosmological constant to the (classical and quantum) harmonic oscillator. We find the quantum discrete cosmological levels: size, time, vacuum energy, Hubble constant and gravitational (Gibbons-Hawking) entropy and temperature from the very early trans-planckian vacuum to the classical today vacuum energy. For each level  $n = 0, 1, 2, \dots$  the two: post and pre (trans)-planckian phases are covered: In the post-planckian universe:  $t_{planck} \equiv t_P \leq t \leq 10^{61}t_P$  the levels (in planck units) are: Hubble constant  $H_n = 1/\sqrt{(2n+1)}$ , vacuum energy  $\Lambda_n = 1/(2n+1)$ , entropy  $S_n = (2n+1)$ . As  $n$  increases, radius, mass and  $S_n$  increase,  $H_n$  and  $\Lambda_n$  decrease and *consistently* the universe *classicalizes*. In the pre-planckian (trans-planckian) phase  $10^{-61}t_P \leq t \leq t_P$  the quantum levels are:  $H_{Qn} = \sqrt{(2n+1)}$ ,  $\Lambda_{Qn} = (2n+1)$ ,  $S_{Qn} = 1/(2n+1)$ ,  $Q$  denoting quantum. The  $n$ -levels cover *all* scales from the far past highest excited trans-planckian level  $n = 10^{122}$  with finite curvature,  $\Lambda_Q = 10^{122}$  and minimum entropy  $S_Q = 10^{-122}$ ,  $n$  decreases till the planck level ( $n = 0$ ) with  $H_{planck} = 1 = \Lambda_{planck} = S_{planck}$  and enters the post-planckian phase e.g.  $n = 1, 2, \dots, n_{inflation} = 10^{12}, \dots, n_{cmb} = 10^{114}, \dots, n_{reoin} = 10^{118}, \dots, n_{today} = 10^{122}$  with the most classical value  $H_{today} = 10^{-61}$ ,  $\Lambda_{today} = 10^{-122}$ ,  $S_{today} = 10^{122}$ . We implement the Snyder-Yang algebra in this context yielding a consistent group-theory realization of quantum discrete de Sitter space-time, classical-quantum gravity duality symmetry and a clarifying unifying picture.

Norma.Sanchez@obspm.fr, <https://chalonge-devega.fr/sanchez>

## CONTENTS

I. Introduction and Results	3
II. The Standard Model of the Universe before Inflation	7
III. Classical, Semiclassical and Quantum Vacuum Energy of the Universe	10
IV. Classical and Quantum Dual de Sitter Universes	13
V. Total de Sitter Universe and its Duality Symmetry	14
VI. de Sitter Universe and the Harmonic Oscillator	18
VII. The Harmonic Oscillator and the Cosmological Constant	19
VIII. Quantum Discrete Levels of the Universe	22
IX. Quantum Discrete Levels of the Hubble Constant	24
X. The Snyder-Yang Algebra and Quantum de Sitter Space-Time	28
XI. Conclusions	34
References	38

## I. INTRODUCTION AND RESULTS

Planckian and trans-planckian energies are theoretically allowed, physically motivated too, the universe and its very early stages have all the quantum conditions for such extreme quantum gravitational regimes and energies, the black hole interiors too. The *truly* quantum gravity *domain* is not reduced to be fixed at the planck scale or the neighborhoods of it, but extends deep beyond the planck scale in the highly quantum trans-planckian range.

**In this paper** we go forward in completing the standard model of the universe back in time with planckian and trans-planckian physics before inflation in agreement with observations, classical-quantum gravity duality and quantum space-time in this context.

Quantum theory is more complete than classical theory and tells us what value a classical observable should have. The classical-quantum (or wave-particle) duality is a robust and universal concept (it does not depend on the nature or number of space-time dimensions, compactified or not, nor on particular space-time geometries, topologies, symmetries, nor on other *at priori* condition). Moreover, the quantum trans-planckian eras in the far past universe determine the observables of the post-planckian eras, e.g. the inflation observables, CMB and the cosmological vacuum energy until today dark energy, namely the evolution from the quantum very early phases to the semi-classical and classical phases and the arrow of time as determined by the gravitational entropy.

The complete universe is composed of two main phases, the planck scale being the *transition scale*: the quantum pre-planckian or trans-planckian phase  $0 < 10^{-61}t_P \leq t \leq t_P$  and the semiclassical and mostly classical post-planckian universe  $t_P \leq t \leq t_{today} = 10^{61}t_P$ ,  $t_P$  being the planck time. The pre-planckian era can be tested indirectly through its post-planckian observables, e.g. primordial graviton signals, inflation and the CMB till today dark energy. This framework provides in particular the gravitational entropy and temperature (classical, semiclassical and quantum) in the different cosmological regimes and eras [1],[2], in particular the Gibbons-Hawking entropy and temperature. Interesting too (and related with) are the classical and quantum cosmological vacuum energy values ( $\Lambda, \Lambda_Q$ ) dual of each other: For instance, the quantum  $\Lambda_Q$  obtained from the classical-quantum (or wave-particle) duality approach turns out to be the saddle point obtained from the quantum gravity path integral euclidean approach which action is the well-known Gibbons-Hawking de Sitter entropy, showing the consistency of the results [1],[2].

The huge difference between the observed value of the cosmological *classical* vacuum energy  $\Lambda$  *today* and the *theoretically* evaluated value of the *quantum* particle physics vacuum  $\Lambda_Q$ , must correctly and physically be like that, because the two values correspond to two huge different physical vacua and eras. The observed  $\Lambda$  value today corresponds to the classical, large and dilute (mostly *empty*) universe today, (termed voids and supervoids in cosmological observations, termed vacuum space-time in classical gravitation), and this is consistent with the very low  $\Lambda$  vacuum value, ( $10^{-122}$  in planck units), while the computed quantum value  $\Lambda_Q$  corresponds to the quantum, small and highly dense energetic universe in its far (trans-planckian) past, and this is consistent with its extremely high, trans-planckian,

value ( $10^{122}$  in planck units). As is well known, the theoretical value  $\Lambda_Q \simeq 10^{122}$  is clearly trans-planckian, this value corresponds and fits correctly the value of  $\Lambda_Q$  in the far past trans-planckian era and its physical properties: quantum size and time  $10^{-61}$ , quantum (Gibbons-Hawking) temperature  $10^{61}$  and entropy  $10^{-122}$ . Consistently too, the trans-planckian era provides the quantum precursor of inflation from which the known classical/semiclassical inflation era, its CMB observables and quantum corrections are recovered in agreement with the set of well established cosmological observations.

Starting from quantum theory to reach the planck scale and trans-planckian domain (instead of starting from classical gravity by quantizing general relativity) reveals successful with novel results, "*quantum relativity*" and quantum space-time structure [1],[3],[2]. Beyond the classical-quantum duality of the space-time, the space-time coordinates can be promoted to quantum non-commuting operators: comparison to the harmonic oscillator and global phase space is enlighting, the hyperbolic quantum space-time structure generates the *quantum light cone*: The classical space-time null generators  $X = \pm T$  disappear at the quantum level due to the relevant  $[X, T]$  commutator which is *always* non-zero, a *new* quantum vacuum region beyond the planck scale emerges.

**In this paper** we analyze the new vacuum quantum region inside the planck scale hyperbolae which delimitate the quantum light cone. The effect of the zero point (vacuum) quantum energy bends the space-time and produces a constant curvature de Sitter background. We find the quantum discrete levels in the cosmological vacuum trans-planckian region and in the post-planckian one. The quantum light cone is generated by the quantum planck hyperbolae  $X^2 - T^2 = \pm [X, T]$  due to the quantum uncertainty  $\Delta X \Delta T$  or commutator  $[X, T] = 1$ , (in planck units), the classical light cone generators  $X = \pm T$  being a particular case of it. This generalizes the classical known space-time structure and reduces to it in the classical case (zero quantum commutators). In higher  $D$  space-time dimensions, the quantum non-commuting coordinates  $(X, T)$  and the transverse commuting spatial coordinates  $X_{\perp j}$  generate the quantum two-sheet hyperboloid  $X^2 - T^2 + X_{\perp j} X_{\perp}^j = \pm 1$ ;  $j = 2, \dots, (D-2)$ .

Interestingly enough, the quantum space-time structure turns out to be *discretized* in quantum hyperbolic levels. For times and lengths larger than the planck time and length  $(t_P, l_P)$ , the levels are  $(X_n^2 - T_n^2) = \pm(2n + 1)$ ,  $n = 0, 1, 2, \dots$ ,  $(X_n, T_n)$  and the *mass levels* being  $\sqrt{(2n + 1)}$ . The discrete allowed levels from the quantum planck scale hyperbolae

$(X_n^2 - T_n^2) = \pm 1$ , ( $n = 0$ ) and the quantum levels (low  $n$ ) until the quasi-classical and classical ones (intermediate and large  $n$ ), tend asymptotically (very large  $n$ ) to a continuum classical space-time. In the trans-planckian domain: times and lengths smaller than the planck scale, the  $(X_n, T_n)$  levels are  $1/(2n + 1)$ , the most higher  $n$  being the more excited quantum and transplanckian ones.

For each level  $n = 0, 1, 2, \dots$ , the two: post and pre (trans) - planckian phases are covered: In the post-planckian universe  $t_P \equiv t_{planck} < t \leq t_{today} = 10^{61}t_P$  the levels (in planck units) for the Hubble constant  $H_n$ , vacuum energy  $\Lambda_n$ , and gravitational (Gibbons-Hawking) entropy  $S_n$  are

$$H_n = 1/\sqrt{(2n + 1)}, \quad \Lambda_n = 1/(2n + 1), \quad S_n = (2n + 1), \quad n = 0, 1, 2, \dots \quad (1.1)$$

As  $n$  increases, radius and mass increase,  $H_n$  and  $\Lambda_n$  decrease,  $S_n$  increases and *consistently* the universe *classicalizes*. In the pre-planckian (trans-planckian) phase  $10^{-61}t_P \leq t \leq t_P$ , the quantum trans-planckian levels ( $Q$  denoting quantum) are:

$$H_{Qn} = \sqrt{(2n + 1)}, \quad \Lambda_{Qn} = (2n + 1), \quad S_{Qn} = 1/(2n + 1), \quad n = 0, 1, 2, \dots \quad (1.2)$$

The scalar curvature levels in the respective phases being  $R_{Qn} = (2n+1)$  and  $R_n = 1/(2n+1)$ . The  $n$ -levels cover *all* scales from the remote past highly excited trans-planckian level  $n = 10^{122}$  with maximum curvature  $R_Q = 10^{122}$ , vacuum  $\Lambda_Q = 10^{122}$  and minimum entropy  $S_Q = 10^{-122}$ ,  $n$  decreases passing the planck level  $n = 0$ :  $H_{planck} = 1 = \Lambda_{planck} = S_{planck}$  and enters the post-planckian phase:  $n = 1, 2, \dots n_{infl} = 10^{12}, \dots n_{cmb} = 10^{114}, \dots n_{reion} = 10^{118}, \dots n_{today} = 122$  with the most classical value  $H_{today} = 10^{-61}$ ,  $\Lambda_{today} = 10^{-122}$ ,  $S_{today} = 10^{122}$ .

The space-time (the arena of events) in the quantum domain is described by a *quantum algebra* of space-time position and momenta: We implement the *Snyder-Yang* algebra in the cosmological context thus yielding a consistent group-theory realization of quantum discrete de Sitter space-time, classical-quantum gravity duality and its symmetry with a clarifying unifying picture: Our complete (classical and quantum) length  $L_{QH}(l_P, L_H) = L_Q + L_H = l_P(L_H/l_P + l_P/L_H)$ ,  $L_H$  being the classical universe radius,  $L_Q = l_P^2/L_H$  being its quantum size (the compton length), turns out to be the appropriate length for the two-parameter Snyder-Yang algebra, thus providing a quantum operator realization of the complete de Sitter universe including the quantum trans-planckian and classical late de Sitter phases.

**This paper is organized as follows:** In section II we describe the standard model of the universe extended back in time before inflation, thus covering its different phases: classical, semiclassical and quantum -planckian and transplanckian- domains and their properties including the gravitational entropy and temperature. In Section III we describe the classical, semiclassical and quantum cosmological vacuum and its observed and computed values. In Sections IV and V we describe the classical, quantum dual and complete de Sitter universe covering the different de Sitter regimes. Sections VI and VII show the link of de Sitter universe and the cosmological constant to the harmonic oscillator. Section VIII shows the link of the space-time structure to the phase space (classical and quantum) harmonic oscillator and describes the quantum space-time discrete levels. We find in Sections VIII and IX the quantum discrete levels of the universe: size, time, vacuum energy, Hubble constant, entropy and their properties from the very early trans-planckian phase to today dark energy. Section X describes the Snyder-Yang algebra as a group-theory realization of quantum discrete de Sitter space-time and of classical-quantum gravity duality symmetry. Section XI summarizes remarks and conclusions and the clarifying unifying picture we obtained.

## II. THE STANDARD MODEL OF THE UNIVERSE BEFORE INFLATION

The set of robust cosmological data (cosmic microwave background, large scale structure and deep galaxy surveys, supernovae observations, measurements of the Hubble-Lemaitre constant and other data) support the standard (concordance) model of the universe and place de Sitter (and quasi-de Sitter) stages as a real part of it [4],[5],[6],[7],[8][9],[10]. Moreover, the physical classical, semiclassical and quantum planckian and trans-planckian de Sitter regimes are particularly important for several reasons:

(i) The classical, present time accelerated expansion of the Universe and its associated dark energy or cosmological constant in the today era: classical cosmological de Sitter regime.

(ii) The semiclassical early accelerated expansion of the Universe and its associated Inflation era: semiclassical cosmological de Sitter (or quasi de Sitter) regime (classical general relativity plus quantum field fluctuations.)

(iii) The quantum, very early stage preceding the Inflation era: Planckian and super-Planckian quantum era. Besides its high conceptual and fundamental physics interest, this



era could be of realistic cosmological interest for the test of quantum theory itself at such extreme scales, as well as for the search of gravitational wave signals from quantum gravity for e-LISA [11] for instance, after the success of LIGO [12],[13]. In addition, this quantum stage should be relevant in providing quantum precursors and consistent initial states for the semiclassical (fast-roll and slow roll) inflation, and their imprint on the observable primordial fluctuation spectra for instance. Moreover, a novel result is that this quantum era allows a clarification of dark energy as the vacuum cosmological energy or cosmological constant.

(iv) de Sitter is a simple and smooth constant curvature vacuum background without any physical singularity, it is maximally symmetric and can be described as a hyperboloid embedded in Minkowski space- time with one more spatial dimension. Its radius, curvature and equivalent density are described in terms of only one physical parameter: the cosmological constant.

The lack of a complete theory of quantum gravity (in field and in string theory) does not preclude to explore and describe quantum planckian and transplanckian regimes. Instead of going from classical gravity to quantum gravity by quantizing general relativity, (is not our aim here to review it), we start from quantum physics and its foundational milestone: the classical-quantum (wave-particle) duality, and extend it to include gravity and the planck scale domain, namely, wave-particle-gravity duality, (or classical-quantum gravity duality), [1], [16]. As a consequence, the different gravity regimes are covered: classical, semiclassical and quantum, together with the planckian and trans-planckian domain and the elementary particle mass range as well. This duality is *universal*, as the wave-particle duality, this does not rely on the number of space-time dimensions (compactified or not), nor on any symmetry, isometry nor on any other *at priori* condition. It includes the known classical-quantum duality as a special case and allows a general clarification from which physical understanding and cosmological results can be extracted. This is not an assumed or conjectured duality.

**The standard model of the universe extended to earlier trans-planckian eras.**  
The gravitational history of the universe before the Inflation era and the current picture can be extended by including the quantum precursor phase within the standard model of the universe in agreement with observations. Quantum physics is more complete than classical physics and contains it as a particular case: It adds a new quantum planckian and transplanckian phase of the Universe from the planck time  $t_P$  until the extreme past

$10^{-61}t_P$ , which is an upper bound for the origin of the Universe, with energy  $H_Q = 10^{61}h_P$ , in a similar manner the present age is a lower bound to the (unknown) future age.

The classical large dilute Universe today and the highly dense very early quantum transplanckian Universe are classical-quantum duals of each other in the precise meaning of the classical-quantum duality. This means the following: The classical Universe today  $U_\Lambda$  is clearly characterized by the set of physical gravitational magnitudes or observables (age or size, mass, density, temperature, entropy)  $\equiv (L_\Lambda, M_\Lambda, \rho_\Lambda, T_\Lambda, S_\Lambda)$ :

$$U_\Lambda = (L_\Lambda, M_\Lambda, \rho_\Lambda, T_\Lambda, S_\Lambda) \quad (2.1)$$

The highly dense very early quantum Universe  $U_Q$  is characterized by the corresponding set of quantum dual physical quantities  $(L_Q, M_Q, \rho_Q, T_Q, S_Q)$  in the precise meaning of the classical-quantum duality:

$$U_Q = (L_Q, M_Q, \rho_Q, T_Q, S_Q) \quad (2.2)$$

$$U_Q = \frac{u_P^2}{U_\Lambda}, \quad u_P = (l_P, m_P, \rho_P, t_P, s_P) \quad (2.3)$$

$u_P$  standing for the corresponding quantities at the fundamental constant planck scale, the *crossing scale* between the two main (classical and quantum) gravity domains. The classical  $U_\Lambda$  and quantum  $U_Q$  Universe eras or regimes (classical/semiclassical eras of the known Universe and its quantum planckian and transplanckian very early phases), satisfy Eqs.(2.1)-(2.3). The *total* Universe  $U_{Q\Lambda}$  is composed by their classical/semiclassical and quantum phases:

$$U_{Q\Lambda} = ( U_Q + U_\Lambda + u_P ) \quad (2.4)$$

Subscript  $\Lambda$  -or equivalently  $H$  for Hubble Lemaitre- stands for the classical magnitudes,  $Q$  stands for Quantum, and  $P$  for the fundamental planck scale constant values.

In particular, the quantum dual de Sitter universe  $U_Q$  is generated from the classical de Sitter universe  $U_\Lambda$  through Eqs.(2.1)-(2.4): *classical-quantum de Sitter duality*. The *total* (classical plus quantum dual) de Sitter universe  $U_{Q\Lambda}$  endowes automatically a *classical-quantum de Sitter symmetry*. This includes in particular the classical, quantum and total de Sitter temperatures and entropies and allows to characterize in a complete and precise way the different classical, semiclassical, quantum planckian and superplanckian de Sitter regimes.  $H$  stands for the classical Hubble-Lemaitre constant, or its equivalent  $\Lambda = 3(H/c)^2$ .

$H_Q$  (or  $\Lambda_Q$ ) stands for quantum dual, and  $Q\Lambda$  (or  $QH$ ) for the total or complete quantities including the both ones.

The size of the Universe is the gravitational length  $L_\Lambda = \sqrt{3/\Lambda}$  in the classical regime, it is the quantum compton length  $L_Q$  in the quantum dual regime (which is the full quantum planckian and superplanckian regime), and it is the planck length  $l_P$  at the fundamental planck scale: the *crossing scale*. The *total* (or complete) size  $L_{Q\Lambda}$  is the sum of the two components. Similarly, the horizon acceleration (surface gravity)  $K_\Lambda$  of the universe in its classical gravity regime becomes the quantum acceleration  $K_Q$  in the quantum dual gravity regime. The temperature  $T_\Lambda$ , measure of the classical gravitational length or mass becomes the quantum temperature  $T_Q$  (measure of the quantum size or compton length) in the quantum regime. Consistently, the Gibbons-Hawking temperature is *precisely* the quantum temperature  $T_Q$ . Similarly, the classical/semiclassical gravitational area or entropy  $S_\Lambda$  (Gibbons-Hawking entropy) has its quantum dual  $S_Q$  in the quantum gravity (planckian and trans-planckian) regime. The concept of gravitational entropy is *the same* for any of the gravity regimes:  $Area/4l_P^2$  in units of  $k_B$ . For a classical object of size  $L_\Lambda$ , this is the classical area  $A_\Lambda = 4\pi L_\Lambda^2$ . For a quantum object of quantum size  $L_Q$ , this is the area  $A_Q = 4\pi L_Q^2$ :

$$A_\Lambda = a_P \left( \frac{L_\Lambda}{\lambda_P} \right)^2, \quad A_Q = a_P \left( \frac{\lambda_P}{L_\Lambda} \right)^2 = \frac{a_P^2}{A_\Lambda}, \quad a_P = 4\pi l_P^2 \quad (2.5)$$

$a_P$  being the planck area. The corresponding gravitational entropies  $S_\Lambda, S_Q$  are

$$S_\Lambda = \frac{\kappa_B}{4} \frac{A_\Lambda}{l_P^2}, \quad S_Q = \frac{\kappa_B}{4} \frac{A_Q}{l_P^2} \quad (2.6)$$

And the total (classical and quantum) gravitational entropy  $S_{Q\Lambda}$  being given by

$$S_{QH} = 2 \left[ s_P + \frac{1}{2} (S_H + S_Q) \right], \quad s_P = \frac{\kappa_B a_P}{4 l_P^2} = \pi \kappa_B, \quad (2.7)$$

$s_P$  being the planck entropy.

### III. CLASSICAL, SEMICLASSICAL AND QUANTUM VACUUM ENERGY OF THE UNIVERSE

The classical universe today  $U_\Lambda$  is precisely a *classical dilute gravity vacuum dominated by voids and supervoids* as shown by observations [21], [22], [23] whose observed  $\rho_\Lambda$  or  $\Lambda$  value today [6],[7],[8],[9],[10] is *precisely* the classical dual of its quantum precursor values  $\rho_Q, \Lambda_Q$

in the quantum very early precursor vacuum  $U_Q$  as determined by Eqs.(2.1)-(2.2). The high density  $\rho_Q$  and cosmological constant  $\Lambda_Q$  are precisely the quantum particle physics transplanckian value  $10^{122}$ . This is precisely expressed by Eqs.(2.1)-(2.2) applied to this case:

$$\Lambda = 3H^2 = \lambda_P \left( \frac{H}{h_P} \right)^2 = \lambda_P \left( \frac{l_P}{L_H} \right)^2 = (2.846 \pm 0.076) 10^{-122} m_P^2 \quad (3.1)$$

$$\Lambda_Q = 3H_Q^2 = \lambda_P \left( \frac{h_P}{H} \right)^2 = \lambda_P \left( \frac{L_H}{l_P} \right)^2 = (0.3516 \pm 0.094) 10^{122} h_P^2 \quad (3.2)$$

$$\Lambda_Q = \frac{\lambda_P^2}{\Lambda}, \quad \lambda_P = 3h_P^2 \quad (3.3)$$

The quantum dual value  $\Lambda_Q$  is *precisely* the quantum vacuum value  $\rho_Q = 10^{122} \rho_P$  obtained from particle physics:

$$\rho_Q = \rho_P \left( \frac{\Lambda_Q}{\lambda_P} \right) = \frac{\rho_P^2}{\rho_\Lambda} = 10^{122} \rho_P \quad (3.4)$$

Eqs.(3.1)-(3.4) are consistently supported by the data [6],[7],[8],[9],[10] which we also *link to the gravitational entropy and temperature of the universe*. The *complete* total vacuum energy density  $\rho_{Q\Lambda}$  or  $\Lambda_{Q\Lambda}$  is the sum of its classical and quantum components (corresponding to the classical today era and its quantum planckian and trans-planckian precursor):

$$\Lambda_{Q\Lambda} = \lambda_P \left( \frac{\Lambda}{\lambda_P} + \frac{\lambda_P}{\Lambda} + 1 \right) = \lambda_P ( 10^{-122} + 10^{+122} + 1 ) \quad (3.5)$$

The observed  $\Lambda$  or  $\rho_\Lambda$  today is the *classical gravity vacuum* value in the classical universe  $U_\Lambda$  *today*. Such observed value must be consistently in such way because of the *large classical* size of the universe today  $L_\Lambda = \sqrt{3/\Lambda}$ , and of the empty or vacuum dilute state today dominated by *voids and supervoids* as shown by the set of large structure observations [21], [22], [23]. This is one main physical reason for such a *low*  $\Lambda$  value at the present age today  $10^{61}t_P$ . Its precursor value and density  $\Lambda_Q, \rho_Q$  is a high superplanckian value precisely because this is a high density very early *quantum cosmological vacuum* in the extreme past  $10^{-61}t_P$  of the quantum trans-planckian precursor phase  $U_Q$ .

The quantum vacuum density  $\Lambda_Q = \rho_Q = 10^{122}$  (in planck units) in the precursor trans-planckian phase  $U_Q$  at  $10^{-61}t_P$ , (the extreme past), became the classical vacuum density  $\Lambda = \rho_\Lambda = 10^{-122}$  in the classical universe  $U_\Lambda$  today at  $10^{61}t_P$ . The transplanckian value is consistently in such way because is a extreme quantum gravity (transplanckian) vacuum in the extreme quantum past  $10^{-61}t_P$  with minimal entropy  $S_Q = 10^{-122} = \Lambda = \rho_\Lambda$ . Eqs.(3.1) to (3.4),(3.5) concisely *explain why* the classical gravitational vacuum  $\Lambda$  or  $\rho_\Lambda$  *coincides*

with such observed *low value*  $10^{-122}$  in planck units, and *why* their corresponding quantum gravity precursor vacuum has such extremely *high* trans-planckian *value*  $10^{122}$ . The classical gravitational entropy  $S_\Lambda$  today has *precisely* such high value:

$$S_\Lambda = s_P \left( \frac{\rho_Q}{\rho_P} \right) = s_P \left( \frac{\lambda_P}{\Lambda} \right) = s_P 10^{+122} \quad (3.6)$$

$$S_Q = s_P \left( \frac{\rho_\Lambda}{\rho_P} \right) = s_P \left( \frac{\Lambda}{\lambda_P} \right) = s_P 10^{-122} \quad (3.7)$$

The *total*  $Q\Lambda$  gravitational entropy turns out the sum of the three components as it must be: classical (subscript  $\Lambda$ ), quantum (subscript  $Q$ ) and planck value (subscript  $P$ ) corresponding to the tree gravity regimes:

$$S_{Q\Lambda} = 2 [s_P + \frac{1}{2}(S_\Lambda + S_Q)] = 2 s_P [ 1 + \frac{1}{2}(10^{+122} + 10^{-122}) ] \quad (3.8)$$

The gravitational entropy  $S_\Lambda$  of the present time large *classical universe* is a very *huge number*, consistent with the fact that the universe today contains a very huge amount of information. Moreover, to reach such a huge size and entropy today  $10^{+122}$ , the universe in its very beginning should have been in a hugely energetic initial vacuum  $10^{+122}$ .

**A whole picture.** Overall, a consistent unifying picture of the gravitational cosmic history through *its vacuum energy* does emerge from the extreme past quantum transplanckian, planckian and post-planckian phases: semiclassical (inflation) and classical today phases and their relevant physical magnitudes: size, age, gravitational entropy and temperature, all in terms of the vacuum energy. This sheds light on inflation and dark energy. The whole duration (of the transplanckian plus post-planckians eras) is precisely  $10^{-61} t \leq t \leq 10^{+61}$  (in planck units  $t_P = 10^{-44}$  sec). That is to say, *each* component *naturally* dominates in each phase: classical time component  $10^{+61}$  in the classical era, quantum planck time  $t_P$  in the quantum preceding era. The present time of the universe at  $10^{+61}t_P$ , is a *lower bound* for the future (if any) age of the universe, the remote past quantum precursor equal to  $10^{-61} t_P$ , is an *upper bound* for the origin of the universe. The known classical/semi-classical inflation era which occurred at about  $10^{+6}t_P$ ,  $H = 10^{-6}h_P$  has a preceding quantum era at  $10^{-6}t_P$ ,  $H = 10^6h_P$  which is in fact a semi-quantum era ('low  $H$ ' with respect to the extreme past transplanckian state  $H = 10^{61}h_P$ ), and similarly, for any of the other known eras in the classical post-planckian universe: they have a corresponding quantum precursor era in the transplanckian phase. This appears to be the way in which the universe has evolved.

The **total or complete (classical plus quantum)** physical quantities are invariant under the classical-quantum duality:  $H \leftrightarrow Q$  (or  $\Lambda \leftrightarrow Q$ ) as it must be: This means physically that: (i) what occurred in the quantum phase before  $t_P$  *determines* through Eqs.(2.1)-(2.4) what occurred in the classical phase after  $t_P$ . And: (ii) what occurred in the quantum phase before the planck time  $t_P$  is the *same observable* which occurred after  $t_P$  but in a different physical state in the precise meaning of Eqs.(2.1)-(2.4). That is to say: The quantum quantities in the phase before  $t_P$ , are the *quantum precursors* of the classical/semiclassical quantities after  $t_P$ . As the wave-particle duality at the basis of quantum mechanics, the wave-particle-gravity duality, is reflected in all cosmological eras and its associated quantities, temperatures and entropies. Cosmological evolution goes from a quantum transplanckian vacuum energy phase to a semiclassical accelerated era (de Sitter inflation), then to the classical known eras until the present classical de Sitter phase. The classical-quantum or wave-particle-gravity duality specifically manifests in this evolution, between the different gravity regimes, and could be view as a mapping between asymptotic (in and out) states characterized by sets  $U_Q$  and  $U_\Lambda$  and thus as a Scattering-matrix description.

#### IV. CLASSICAL AND QUANTUM DUAL DE SITTER UNIVERSES

de Sitter space-time in  $D$  space-time dimensions is the hyperboloid embedded in  $(D + 1)$  dimensional Minkowski space-time:

$$X^2 - T^2 + X_j X^j + Z^2 = L_H^2, \quad j = 2, 3, \dots, (D - 2) \quad (4.1)$$

$L_H$  is the classical radius or characteristic length of the de Sitter universe. The scalar curvature  $R$  is constant. Classically:

$$L_H = c/H, \quad R = H^2 D(D - 1) = \frac{2D}{(D - 2)} \Lambda, \quad \Lambda = \frac{H^2}{2}(D - 1)(D - 2)$$

A mass  $M_H$  can be associated to  $L_H$  or  $H$ , such that ( $D = 4$  for simplicity):

$$L_H = GM_H/c^2 \equiv L_G, \quad M_H = c^3/(GH) \quad (4.2)$$

The corresponding quantum dual magnitudes  $L_Q$ ,  $M_Q$  are:

$$L_Q = \frac{\hbar}{M_H c} = \frac{\hbar GH}{c^3} = \frac{l_P^2}{L_H}, \quad M_Q = \frac{\hbar H}{c^2} = \frac{m_P^2}{M_H} \quad (4.3)$$

$$\text{ie, } L_Q = \frac{l_P^2}{L_H}, \quad M_Q = \frac{m_P^2}{M_H} \quad (4.4)$$

$l_P$  and  $m_P$  being the planck length and Planck mass respectively:

$$l_P = \sqrt{\hbar G/c^3}, \quad m_P = \sqrt{c \hbar/G} \quad (4.5)$$

The quantum dual Hubble constant  $H_Q$  and the quantum curvature  $R_Q$  are:

$$H_Q = h_P^2/H, \quad R_Q = r_P^2/R, \quad \Lambda_Q = \lambda_P^2/\Lambda \quad (4.6)$$

where  $h_P, r_P, \lambda_P$  are the planck scale values of the Hubble constant, scalar curvature and cosmological constant respectively:

$$h_P = c/l_P, \quad r_P = h_P^2 D(D-1), \quad \lambda_P = \frac{h_P^2}{2} (D-1)(D-2) \quad (4.7)$$

$$h_P = c^2 \sqrt{c/\hbar G}, \quad r_P = 12 h_P^2 = 4 \lambda_P, \quad \lambda_P = 3 (c^5/\hbar G), \quad (D=4) \quad (4.8)$$

## V. TOTAL DE SITTER UNIVERSE AND ITS DUALITY SYMMETRY

The classical and quantum lengths:  $L_H, L_Q$  can be extended to a more complete length  $L_{QH}$  which contains both: the Q and H lengths):

$$L_{QH} = (L_H + L_Q) = l_P \left( \frac{L_H}{l_P} + \frac{l_P}{L_H} \right) \quad (5.1)$$

and we have then :

$$X^2 - T^2 + X_j X^j + Z^2 = L_{QH}^2 = 2 l_P^2 \left[ 1 + \frac{1}{2} \left[ \left( \frac{L_H}{l_P} \right)^2 + \left( \frac{l_P}{L_H} \right)^2 \right] \right] \quad (5.2)$$

with  $j = 2, 3, \dots, (D-3)$ .

Eq.(5.2) quantum generalize de Sitter space-time including the classical, semiclassical and quantum planckian and transplanckian de Sitter regimes as well. It contains two non-zero lengths ( $L_H, L_Q$ ) or two relevant scales ( $H, l_P$ ) enlarging the possibilities for the space-time phases, thus:

- For  $L_H \gg l_P$ , ie  $L_Q \ll L_H$ , Eq.(5.2) yields the classical de Sitter space-time. For intermediate  $L_H$  values between  $l_P$  and  $L_Q$  it yields the semiclassical de Sitter space-time.

- For  $L_H = l_P$  ie  $L_Q = l_P = L_{QH}$ , Eq.(5.2) yields the planck scale de Sitter hyperboloid.
- For  $L_H \ll l_P$ , ie  $L_Q \gg L_H$  it yields the highly quantum de Sitter regime, deep inside the planck domain.

$H = c/L_H$  is ( $c^{-1}$ ) times the surface gravity (or gravity acceleration) of the classical de Sitter space-time. Similarly,  $H_Q = c/L_Q$  and  $H_{QH} = c/L_{QH}$  are the surface gravity in the quantum and whole QH de Sitter phases respectively. Similarly, Eq. (5.1) and Eqs (4.2)-(4.4), yield for the mass:

$$M_{QH} = (M_H + M_Q) = m_P \left( \frac{M_H}{m_P} + \frac{m_P}{M_H} \right) \quad (5.3)$$

$$\frac{M_{QH}}{m_P} = m_P \left( \frac{L_H}{l_P} + \frac{l_P}{L_H} \right) = \frac{L_{QH}}{l_P} \quad (5.4)$$

$M_{QH}/m_P$  and  $L_{QH}/l_P$  both have the same expression with respect to their respective planck values.

### The complete QH Hubble constant $H_{QH}$ , curvature $R_{QH}$ and $\Lambda_{QH}$ .

The total (classical and quantum) QH Hubble constant  $H_{QH}$ , curvature  $R_{QH}$  and  $\Lambda_{QH}$  follow from the QH de Sitter length  $L_{QH}$  Eq.(5.1):

$$H_{QH} = \frac{c}{L_{QH}}, \quad R_{QH} = H_{QH}^2 D(D-1), \quad \Lambda_{QH} = \frac{H_{QH}^2}{2} (D-1)(D-2) \quad (5.5)$$

where from Eqs.(5.1) and (4.6):

$$H_{QH} = \frac{H}{[1 + (l_P H/c)^2]}, \quad H_{QH}/h_P = \frac{(H/h_P)}{[1 + (H/h_P)^2]}, \quad h_P = c/l_P \quad (5.6)$$

which exhibit the *symmetry* of  $H_{QH}$  under  $(H/h_P) \rightarrow (h_P/H)$ , ie under  $H \rightarrow H_Q = (h_P^2/H)$  :

$$H_{QH}(H/h_P) = H_{QH}(h_P/H) \quad (5.7)$$

The classical  $H$  and quantum  $H_Q$  are classical-quantum duals of each other through the planck scale  $h_P$ , but the total  $H_{QH}$  is *invariant*. And similarly, for the total quantum curvature  $R_{QH}$  and cosmological constant  $\Lambda_{QH}$  Eq.(5.5):

$$R_{QH}(H/h_P) = R_{QH}(h_P/H), \quad \Lambda_{QH}(H/h_P) = \Lambda_{QH}(h_P/H) \quad (5.8)$$



where:

$$R_{QH} = \frac{R_H}{[1 + R_H/r_P]^2} = \frac{R_Q}{[1 + R_Q/r_P]^2}, \quad r_P = 12 h_P^2 \quad (5.9)$$

$$\Lambda_{QH} = \frac{\Lambda_H}{[1 + \Lambda_H/\lambda_P]^2} = \frac{\Lambda_Q}{[1 + \Lambda_Q/\lambda_P]^2}, \quad \lambda_P = 3 h_P^2 \quad (5.10)$$

The classical  $H/h_P \ll 1$ , quantum  $H/h_P \gg 1$  and planck  $H/h_P = 1$  regimes are clearly exhibited in the QH expressions Eqs (5.5), Eq.(5.6):

$$H_{QH} (H \ll h_P) = H [1 - (H/h_P)^2] + O(H/h_P)^4 \quad (5.11)$$

$$H_{QH} (H = h_P) = \frac{h_P}{2}, \quad h_P = c/l_P \quad (5.12)$$

$$H_{QH} (H \gg h_P) = (h_P^2/H) [1 - (h_P/H)^2] + O(h_P/H)^4 \quad (5.13)$$

The three above equations show respectively the three different de Sitter phases:

- The classical gravity de Sitter universe (with lower curvature than the planck scale  $r_P$ ) *outside* the planck domain ( $l_P < L_H < \infty$ ).
- The planck curvature de Sitter state ( $R_H = r_P$ ,  $L_H = l_P$ )
- The highly quantum or high curvature ( $R_H \gg r_P$ ) de Sitter phase *inside* the quantum gravity planck domain ( $0 < L_H \leq l_P$ ).

Is natural here to define the dimensionless magnitudes:

$$\mathcal{L} \equiv L_{QH}/l_P, \quad \mathcal{M} \equiv M_{QH}/m_P, \quad \mathcal{H} \equiv H_{QH}/h_P, \quad l \equiv L_H/l_P, \quad h \equiv H/h_P = l^{-1} \quad (5.14)$$

in terms of which, Eqs (5.1),(5.3) and (5.6) and their duality symmetry Eqs (5.7), (5.8) simply read:

$$\mathcal{L} = (l + \frac{1}{l}) = \mathcal{M}, \quad \mathcal{H} = \frac{1}{(l + \frac{1}{l})} = \mathcal{L}^{-1} \quad (5.15)$$

$$\mathcal{L}(l^{-1}) = \mathcal{L}(l), \quad \mathcal{M}(l^{-1}) = \mathcal{M}(l) \quad (5.16)$$

$$\mathcal{H}(l^{-1}) = \mathcal{H}(l), \quad \mathcal{R}(l^{-1}) = \mathcal{R}(l), \quad \mathbf{\Lambda}(l^{-1}) = \mathbf{\Lambda}(l) \quad (5.17)$$

The QH magnitudes are complete variables covering both classical and quantum, planckian and transplanckian, domains. Similarly, for the classical, quantum and QH de Sitter densities ( $\rho_H, \rho_Q, \rho_{QH}$ ),  $\rho_P$  being the Planck density scale):

$$\rho_H = \rho_P (H/h_P)^2 = \rho_P (\Lambda/\lambda_P), \quad \rho_P = 3 h_P^2/8\pi G, \quad \lambda_P = 3 h_P^2/c^4 \quad (5.18)$$

$$\rho_Q = \rho_P (H_Q/h_P)^2 = \rho_P (\Lambda_Q/\lambda_P) = \rho_P^2/\rho_H = \rho_P (h_P/H)^2 = \rho_P (\lambda_P/\Lambda) \quad (5.19)$$

$$\rho_{HQ} = \rho_H + \rho_Q = \rho_P (H_{HQ}/h_P)^2 = \rho_P (\Lambda_{HQ}/\lambda_P) \quad (5.20)$$

From which it follows that:

$$\rho_{HQ} = \frac{\rho_H}{[1 + \rho_H/\rho_P]^2} = \frac{\rho_Q}{[1 + \rho_Q/\rho_P]^2}, \quad (5.21)$$

which satisfies

$$\rho_{HQ}(\rho_H) = \rho_{HQ}(\rho_Q) = \rho_{HQ}(\rho_P^2/\rho_H),$$

For small and high densities with respect to the planck density  $\rho_P$ , the QH density  $\rho_{QH}$  behaves:

$$\rho_{QH}(\rho_H \ll \rho_P) = \rho_H [1 - 2(\rho_H/\rho_P)] + O(\rho_H/\rho_P)^2 \quad (5.22)$$

$$\rho_{QH}(\rho_H = \rho_Q = \rho_P) = \frac{1}{4}\rho_P : \text{ (planck scale density)} \quad (5.23)$$

$$\rho_{QH}(\rho_H \gg \rho_P) = \rho_Q [1 - 2(\rho_Q/\rho_P)] + O(\rho_Q/\rho_P)^2, \quad (5.24)$$

corresponding to the classical/semiclassical de Sitter regime (and its quantum corrections), planck scale de Sitter state and highly quantum transplanckian de Sitter density. The complete QH de Sitter magnitudes ( $L_{QH}, H_{QH}, M_{QH}$ ), [and their constant planck scale values ( $l_P, h_P, m_P$ ) only depending on ( $c, \hbar, G$ )], allow to characterize in a precise way the classical, semiclassical, planckian and quantum (super-planckian) de Sitter regimes:

- $L_{QH} = L_{QH}(L_H, L_Q) \equiv L_{QH}(H, \hbar)$  yields the *whole* (classical/semiclassical, planck scale and quantum (super-planckian) de Sitter universe.
- $L_{QH} = L_H = L_Q$  yields the planckian de Sitter state, (planck length de Sitter radius, planckian vacuum density and planckian scalar curvature):  $L_H = l_P, H = h_P, \lambda_P = 3 h_P^2, R = r_P = 4 \lambda_P, l_P = \sqrt{(\hbar G/c^3)}$
- $L_{QH} = L_H \gg L_Q$ , ie  $L_H \gg l_P, H \ll h_P$ , yields the classical de Sitter space-time.
- $L_{QH} = L_Q \gg L_H$ , ie  $L_H \ll l_P, H \gg h_P$ , (high curvature  $R \gg r_P = 4\Lambda_P$ ), yields a full quantum gravity transplanckian de Sitter phase (inside the Planck domain  $0 < L_H \leq l_P$ ).
- $L_{QH} \gg L_Q$  ie  $L_{QH} \rightarrow \infty$  for  $L_H \rightarrow \infty$ , ie  $H \rightarrow 0$  ie  $\Lambda \rightarrow 0$ , (zero curvature) yields consistently the classical Minkowski space-time, equivalent to the limit  $L_Q \rightarrow 0$  ie  $l_P \rightarrow 0$  ( $\hbar \rightarrow 0$ ).

The three de Sitter regimes are characterized in a complete and precise way:

- (i) *classical and semiclassical de Sitter regimes*: (inflation and more generally the whole known -classical and semiclassical- universe is within this regime):  
 $l_p < L_H < \infty$ , ie  $0 < L_Q < l_P$ ,  $0 < H < h_P$ ,  $m_P < M_H < \infty$ .
- (ii) *planck scale de Sitter state with planck curvature and planck radius*:  
 $L_H = l_P$ ,  $L_Q = l_P$ ,  $H = h_P = c/l_P$ ,  $M_H = m_P$ .
- (iii) *quantum planckian and trans-planckian de Sitter regimes*:  $0 < L_H \leq l_P$ ,  
 ie  $l_P \leq L_Q < \infty$ ,  $h_P \leq H < \infty$ ,  $0 < M_H < m_P$ .

## VI. DE SITTER UNIVERSE AND THE HARMONIC OSCILLATOR

As is known, the Einstein Equations in the presence of a constant vacuum energy (cosmological constant) are

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} , \quad (6.1)$$

and the energy-momentum tensor corresponding to the vacuum energy density  $\rho$  and pressure  $p$  is

$$T_{\mu\nu} = p g_{\mu\nu} = -\rho g_{\mu\nu}, \quad (p = w\rho, \quad w \equiv -1) \quad (6.2)$$

the vacuum energy being equivalent to a cosmological constant:  $\rho_\Lambda = \Lambda c^4 / (8\pi G)$ .

As known, de Sitter space-time has *constant* scalar space-time curvature:

$$R = 12 H^2 = 4 \Lambda, \quad \Lambda = 3 H^2, \quad (D = 4)$$

We restrict to  $D = 4$ . Recall the energy-momentum tensor for massive particles of density  $\rho$  plus vacuum constant energy (or cosmological constant)  $\Lambda$  is:

$$T_\nu^\mu = \Lambda \delta_\nu^\mu + \rho \delta^{\mu 0} \delta_{\nu 0}, \quad T \equiv T_\mu^\mu = 4 \Lambda + \rho \quad (6.3)$$

The corresponding Einstein equations are

$$R_\nu^\mu = 8\pi G \left( T_\nu^\mu - \frac{\delta_\nu^\mu}{2} T \right), \quad 0 \leq \mu, \nu \leq 3 \quad (6.4)$$

and for non relativistic matter its pressure is neglected with respect to its rest mass.

In the weak field limit:

$$g_{00} = 1 + 2V \quad , \quad g_{ik} = -\delta_{ik} \quad R_0^0 = \nabla^2 V \quad ,$$

$V$  being the gravitational potential, Einstein's Eqs.(6.4) become

$$\nabla^2 V = 4\pi G \rho - 8\pi G \Lambda \quad (6.5)$$

$$V(\vec{X}) = V_\rho(X) - \frac{4\pi G \Lambda}{3} X^2 \quad , \quad (6.6)$$

For a distribution of rest particles of mass  $m$ ,  $\rho(\vec{X}) = m \sum_i \delta(\vec{X} - \vec{X}_i)$ , the gravitational potential  $V(\vec{X})$ , gravitational field  $\vec{\mathcal{G}}$  and potential energy  $\mathcal{U}$  of the system are:

$$V(\vec{X}) = V(\vec{X})_m - \frac{4\pi G \Lambda}{3} X^2, \quad V(\vec{X})_\rho \equiv V(\vec{X})_m = -G \sum_i \frac{m}{|\vec{X} - \vec{X}_i|}.$$

$$\vec{\mathcal{G}}(\vec{X}) = -\nabla V(\vec{X}) = \vec{\mathcal{G}}_m + \frac{8\pi G \Lambda}{3} \vec{X} \quad (6.7)$$

$$\mathcal{U} = \mathcal{U}_m - \frac{4\pi G \Lambda}{3} m \sum_i X_i^2 \quad (6.8)$$

Therefore, the Hamiltonian is equal to:

$$\frac{P_i P^i}{m^2} + \mathcal{U} = \frac{P_i P^i}{m^2} - \frac{4\pi}{3} G \Lambda m X_i^2 \quad (6.9)$$

The cosmological constant energy contribution to the potential energy  $\mathcal{U}$  decreases for increasing values of the particle distances  $r_i$  to the center of mass. The gravitational effect of the vacuum zero point energy or cosmological constant push particles outwards and equivalently, the last term of the gravitational field Eq.(6.7) points outward (the repulsive cosmological constant effect). The Hamiltonian Eq.(6.9) is like that of a harmonic oscillator for a particle of mass  $m$  and oscillator constant  $\omega^2 m$ . We analyze it below.

## VII. THE HARMONIC OSCILLATOR AND THE COSMOLOGICAL CONSTANT

For simplicity and physical insight we consider the case of just one particle, Eqs.(6.8) and (6.9) yield:

$$\ddot{\vec{X}} = \frac{8\pi G \Lambda}{3} \vec{X} \quad (7.1)$$

This is an *harmonic oscillator* equation with imaginary frequency and *oscillator constant*  $\kappa_{oscill}$ :

$$\ddot{\vec{X}} = -\kappa_{oscill}\vec{X}, \quad \kappa_{oscill} = \omega^2 m, \quad \omega = \sqrt{\frac{8\pi G\Lambda}{3m}}, \quad (7.2)$$

with the solution,

$$\vec{X}(t) = \vec{X}(0) \cosh Ht + \frac{1}{H} \dot{\vec{X}}(0) \sinh Ht, \quad (7.3)$$

where

$$H \equiv \sqrt{(8\pi G\Lambda)/3}$$

The particle runs away exponentially fast in time. The Hubble constant  $H^2$  is the constant of the oscillator

$$\kappa_{osc} = H^2, \quad H = \omega\sqrt{m} \quad (7.4)$$

the oscillator length  $l_{osc}$  being

$$l_{osc} = \sqrt{3/(8\pi G\Lambda)}, \quad H = c/l_{osc} = \kappa \equiv \text{surface gravity}$$

The length of the oscillator is the Hubble radius and the Hubble constant is the surface gravity of the universe (similar to the black hole surface gravity, the inverse of the black hole radius).

The non-relativistic or weak field newtonian results reproduce very well the full space-time relativistic effects in the presence of the cosmological constant. The exact solution of the Einstein equations for the energy-momentum tensor eq.(6.3) with  $\rho = 0$  is the de Sitter universe. It must be stressed that the non-relativistic trajectories Eq.(7.3) exhibit the same exponential runaway behaviour of the exact relativistic geodesics in de Sitter space-time. The non-relativistic approximation keeps the essential features of the particle motion in de Sitter space-time [31], [32], [33].

We summarize in the following our main results allowing to describe de Sitter (and Anti de Sitter) space-time as a classical and quantum harmonic oscillator:

- The motion of a particle in an harmonic oscillator potential corresponds to the particle motion in the non-relativistic limit of a constant curvature space-time. The harmonic oscillator with an imaginary frequency, namely the inverted oscillator for  $\Lambda > 0$  corresponds to de Sitter space-time; the real frequency normal oscillator  $\Lambda < 0$  describes anti-de Sitter space-time, and the free motion is flat Minkowski space-time  $\Lambda = 0$ .

- The constant of the oscillator is the cosmological constant, as shown by Eq.(7.2), which is the Hubble constant  $H^2$  or surface gravity squared Eq.(7.4).
- For the *classical* harmonic oscillator, the phase space is the classical one, and the algebra of the  $(X, P)$  variables or  $(X, T = iP)$  variables is commuting. The classical Hamiltonian is  $2H_{osc} = X^2 + P^2 =$  or in light-cone variables  $2H_{osc} = 2UV = 2VU$ . The light-cone structure  $X^2 - T^2$  is the classical known one, there is no difference with the Minkowski light-cone structure of special relativity. Upon the identification  $P = iT$  the classical commuting  $(X, T)$  variables of Minkowski space-time and its invariant distance  $s^2 = X^2 - T^2$  correspond to a classical phase space  $(X, P)$  and Hamiltonian  $s^2 = 2H_{osc} = X^2 + P^2$  which is the harmonic oscillator hamiltonian.
- The non relativistic approximation describes very well the essential properties of the constant curvature -de Sitter or anti de Sitter- geometries and captures its physics. Thus, the classical non-relativistic de Sitter invariant space-time, or the anti-de Sitter space-time, and the Minkowski Poincare-invariant space-time all three describe special relativity. We see that this reaches from another approach and motivation, the fact that a constant curvature space-time describes special relativity, as refs [34] [35], or the so called "triple relativity"  $\Lambda > 0$ ,  $\Lambda < 0$  and  $\Lambda = 0$ .
- For the *quantum* harmonic oscillator, the quantum zero point energy bends the light cone generators into the planck scale hyperbolae  $X^2 - T^2 = 1$  and therefore the space-time is curved: de Sitter (or anti de Sitter) space-time. And, as it is known, the non-relativistic and relativistic de Sitter space-times are very similar.
- Upon the identification  $T = iP$ , the classical non commuting coordinates  $(X, T)$  of Minkowski space-time and its distance  $s^2 = X^2 - T^2$  are the classical non commuting phase space  $(X, P)$  and classical quadratic form  $2H_{osc} = X^2 + P^2 \rightarrow s^2$ , which is the harmonic oscillator Hamiltonian. And this is too the Hamiltonian of a particle in a constant curvature (cosmological constant) de Sitter or anti de Sitter space (in its non-relativistic limit).
- Explicitely: The  $(a, a^+)$  creation and annihilation operators are the *light-cone* type quantum coordinates of the phase space  $(X, P)$ :  $a = (X + iP)/\sqrt{2}$ ,  $a^+ = (X - iP)/\sqrt{2}$ . The temporal variable  $T$  in the space-time configuration  $(X, T)$  is like the

(imaginary) momentum in phase space  $(X, P)$ . The identification  $P = iT$  yields:

$$X = (a^+ + a)/\sqrt{2}, \quad T = (a^+ - a)/\sqrt{2}, \quad [a, a^+] = 1 \text{ with the algebra:}$$

$$2H_{osc} = (X^2 - T^2) = 2(a^+a + \frac{1}{2}), \quad (X^2 + T^2) = (a^2 + a^{+2}), \quad (7.5)$$

$$[2H_{osc}, T] = X, \quad [2H_{osc}, X] = T, \quad [X, T] = 1, \quad (7.6)$$

$a^+ a = N$  being the number operator.

- In other words: The non-relativistic cosmological constant (de Sitter or Anti de Sitter) space-time, the harmonic oscillator phase space and Minkowski space - time are in correspondence one into another. The line element in Minkowski space-time in  $D$  space-time dimensions  $s^2 = X^2 - T^2 + X_j^2$  is equal to the (non relativistic) harmonic oscillator Hamiltonian  $2H_{osc} = X^2 + P^2 + X_j^2$ . Thus, there are the three possibilities for special relativity. The interesting point in our studies is that the *quantum* harmonic oscillator algebra describes the *quantum non-commuting space-time* structure.
- Upon the identification  $T = iP$ , the de Sitter hyperboloid Eq.(4.1) yields :

$$X^2 + P^2 + X_j^2 + Z^2 = L_{QH}^2, \quad j = 2, 3, \dots(D - 2) \quad (7.7)$$

corresponding to a harmonic oscillator  $(X, P)$  embedded in a Minkowski space of  $(D - 2 + 1) = (D - 1)$  spatial dimensions, ie a Minkowski space-time of  $D$  space-time dimensions.

## VIII. QUANTUM DISCRETE LEVELS OF THE UNIVERSE

Let us go beyond the classical-quantum duality of the space-time recently discussed and promote the space-time coordinates to quantum non-commuting operators. As we have seen, comparison to the harmonic oscillator  $(X, P)$  variables and global phase space is enlighting: The phase space instanton  $(X, P = iT)$  describes the hyperbolic quantum space-time structure and generates the quantum light cone. The classical Minkowski space-time null generators  $X = \pm T$  *dissapear* at the quantum level due to the relevant  $[X, T]$  commutator which is *always* non-zero. A new quantum planck scale vacuum region emerges. In the case of the Rindler and Schwarzschild-Kruskal space-time structures, the four Kruskal regions merge inside a single quantum planck scale region [1], [3].

The quantum space-time structure consists of *hyperbolic discrete levels* of odd numbers

$$(X^2 - T^2)_n = H_{oscn}^2 = (2n + 1) \quad (\text{in planck units}), \quad n = 0, 1, 2, \dots \quad (8.1)$$

$(X_n, T_n)$  and the *mass levels* being  $\sqrt{(2n + 1)}$ ,  $n = 0, 1, 2, \dots$

The planck scale hyperbolae  $(T^2 - X^2)(n = 0) = \pm 1$  delimitate the external space-time regions from the new internal ones.  $(T^2 - X^2)(n = 0) = \pm 1$  are the fundamental ( $n = 0$ ) level from which the space-time levels go to the quantum (low  $n$ ) levels and to the semiclassical and classical (large  $n$ ) levels. Asymptotically, for very large  $n$  the space-time becomes continuum.

The internal region to the four quantum Planck scale hyperbolae  $(T^2 - X^2)(n = 0) = \pm 1$  is *totally quantum* and *deep inside* the Planck scale domain: this is the *quantum vacuum* or "zero point" planckian and transplanckian energy region.

In terms of variables  $(x_{n\pm}, t_{n\pm})$ , covering only one: pre-planckian or post planckian phase, the space-time discrete levels read:

$$x_{n\pm} = [ \sqrt{2n + 1} \pm \sqrt{2n} ] \quad (8.2)$$

$$t_{n\pm} = [ \sqrt{2n + 1} \pm \sqrt{(2n + 1) + 1/2} ], \quad (8.3)$$

$$x_{n=0} (+) = x_{n=0} (-) = 1 : \text{planck scale}$$

The low  $n$ , intermediate, and large  $n$  levels describe respectively the quantum, semiclassical and classical behaviours, interestingly enough the  $(\pm)$  branches consistently reflect the classical-quantum duality properties.

$(X_n, T_n), (x_n, t_n)$  are given in planck (length and time) units. In terms of the global quantum gravity dimensionless length  $\mathcal{L} = L_{QH}/l_P$  and mass  $\mathcal{M} = M_{QH}/m_P$ , Eqs. (5.14) or the local ones  $x = m/m_p$ , translate into the discrete mass levels:

$$\mathcal{L}_n = \sqrt{(2n + 1)} = \mathcal{M}_n \quad n = 0, 1, 2, \dots \quad (8.4)$$

$$L_{QHn} \quad n \gg 1 = l_P [ \sqrt{2n} + \frac{1}{2\sqrt{2n}} + O(1/n^{3/2}) ] \quad (8.5)$$

$$M_{QHn} \quad n \gg 1 = m_P [ \sqrt{2n} + \frac{1}{2\sqrt{2n}} + O(1/n^{3/2}) ] \quad (8.6)$$

The above Eqs for  $L_{QHn}, M_{QHn}$  yield the levels for  $L_{Hn\pm}$  and  $M_{Hn\pm}$ :

$$L_{Hn\pm} = [ L_{QHn} \pm \sqrt{L_{QHn}^2 - l_P^2} ] \quad (8.7)$$



$$M_{Hn\pm} = [ M_{QHn} \pm \sqrt{M_{QHn}^2 - m_P^2} ] \quad (8.8)$$

The condition  $L_{QHn} \geq l_P$ ,  $M_{QHn} \geq m_P$  consistently corresponds to the whole spectrum  $n \geq 0$ , the lowest level  $n = 0$  being the planck mass and length:

$$L_{Hn\pm} = l_P [ \sqrt{2n+1} \pm \sqrt{2n} ] \quad \text{for all } n = 0, 1, 2, \dots \quad (8.9)$$

$$M_{Hn\pm} = m_P [ \sqrt{2n+1} \pm \sqrt{2n} ] \quad \text{for all } n = 0, 1, 2, \dots \quad (8.10)$$

The mass and radius of the universe  $M_H$ ,  $L_H$  have discrete levels  $L_{Hn\pm}$ ,  $M_{Hn\pm}$ , from the most fundamental one ( $n = 0$ ), going to the semiclassical (intermediate  $n$ ), to the classical ones (large  $n$ ) which yield a continuum classical universe as it must be. This is clearly seen from the mass level  $M_{Hn\pm}$  expressions (and similarly for the radius levels). Explicitly:

$$M_{H(n=0)+} = M_{H(n=0)-} = M_{QH(n=0)} = m_P, \quad n = 0 : \text{planck mass} \quad (8.11)$$

$$M_{Hn+} = m_P [ 2\sqrt{2n} - \frac{1}{2\sqrt{2n}} + O(1/n^{3/2}) ], \quad \text{large } n : \text{branch (+) : masses } > m_P \quad (8.12)$$

$$M_{Hn-} = \frac{m_P}{2\sqrt{2n}} + O(1/n^{3/2}), \quad \text{large } n : \text{branch (-) : masses } < m_P \quad (8.13)$$

Large  $n$  levels are semiclassical tending towards a classical continuum space-time. Low  $n$  are quantum, the lowest mode ( $n = 0$ ) being the planck scale. Two dual ( $\pm$ ) branches are present in the local variables ( $\sqrt{2n+1} \pm \sqrt{2n}$ ) reflecting the duality of the large and small  $n$  behaviours and covering the *whole* spectrum: from the largest cosmological masses and scales in branch (+) to the quantum smallest masses and scales in branch (-) passing by the planck mass and length.

## IX. QUANTUM DISCRETE LEVELS OF THE HUBBLE CONSTANT

Eqs.(8.4) yields the (dimensionless) quantum levels for the total: Hubble constant, vacuum energy and constant curvature:

$$\mathcal{H}_n = \frac{1}{\sqrt{(2n+1)}}, \quad \Lambda_n = \frac{1}{(2n+1)}, \quad \mathcal{R}_n = \frac{1}{(2n+1)} \quad n = 0, 1, 2, \dots \quad (9.1)$$

$$n = 0 : \quad \mathcal{H}_0 = 1, \quad \Lambda_0 = 1, \quad \mathcal{R}_0 = 1 : \quad \text{planck scale (dimensionless)} \quad (9.2)$$

$$H_{QH(n=0)} = \frac{c}{l_P} = h_P, \quad \Lambda_{QH(n=0)} = \lambda_P \quad R_{QH(n=0)} = 4\lambda_P : \quad \text{planck scale values} \quad (9.3)$$

And for the gravitational entropy:

$$S_n = (2n + 1) \quad \text{in planck units} \quad s_P = 4\pi$$

The lowest  $n = 0$  level corresponds to the fundamental planck scale values  $(h_P, \lambda_P, 4\lambda_P, s_P)$  for the Hubble constant, cosmological constant, constant curvature and gravitational entropy respectively. Let us analyze now the implications of these results and the general picture which they arise.

In the **post-planckian universe**  $t_P \leq t \leq t_{today} = 10^{61}t_P$ : We see that the physical magnitudes as the Hubble radius, vacuum energy density, constant curvature, entropy start at the planck scale: the zero level ( $n = 0$ ). As  $n$  increases, the universe radius, mass and entropy increase, the Hubble constant, curvature and vacuum energy *consistently decrease* and the universe *classicalizes*. The decreasing with  $n$  of these quantities is given by Eq.(9.1), and for large  $n$ ,  $H_n$ ,  $\Lambda_n$  and  $\mathcal{R}_n$  *classicalize* as:

$$\mathcal{H}_{n \gg 1} = \frac{c}{l_P \sqrt{2n}} \left[ 1 - O\left(\frac{1}{2n}\right) \right] \ll 1 \quad (9.4)$$

$$\Lambda_{n \gg 1} = \frac{3 c^2}{l_P^2 (2n)} \left[ 1 - O\left(\frac{1}{2n}\right) \right] \ll 1 \quad (9.5)$$

$$\mathcal{R}_{n \gg 1} = \frac{12 c^2}{l_P^2 (2n)} \left[ 1 - O\left(\frac{1}{2n}\right) \right] \ll 1, \quad (9.6)$$

*precisely accounting for the low classical values of  $H$  and  $\Lambda$  in the universe today which is a classical, large and dilute universe.* The present universe values  $H_{today} = 10^{-61}$ ,  $\rho_\Lambda = 10^{-122}$  correspond to a large  $n$ -level  $n = 10^{122} \equiv n_{today}$ .

More generally, in the **post-planckian universe**:  $t_P \leq t \leq t_{today} = 10^{61}t_P$ , Eq.(9.1) yields the quantum  $n$ -levels:

$$n = \frac{1}{2} (H_n^{-2} - 1) : \quad t_{(n=0)} = t_P \leq t_n \leq t_{n \text{ today}} = 10^{61}t_P \quad (9.7)$$

Thus, the more characteristic evolution values from the planck time  $t_P$  till today:

$$h_P, \dots, H_{inf}, \dots, H_{cmb}, \dots, H_{reion}, \dots, H_{today} , \quad (9.8)$$

corresponds to the  $n$ -levels:

$$n = 0, 1, 2, \dots, n_{inf} = 10^{12}, \dots, n_{cmb} = 10^{114}, \dots, n_{reoin} = 10^{118}, \dots, n_{today} = 10^{122} \quad (9.9)$$

and the *discrete*  $H_n$ ,  $\Lambda_n$  and  $S_n$  values:

$$H_n = 1, 0.577, \dots H_{n,inf} = 10^{-6}, \dots H_{n,cmb} = 10^{-57}, \dots H_{n,reoin} = 10^{-59}, \dots H_{n,today} = 10^{-61} \quad (9.10)$$

$$\Lambda_n = 1, 0.333, \dots \Lambda_{n,inf} = 10^{-12}, \dots \Lambda_{n,cmb} = 10^{-114}, \dots \Lambda_{n,reoin} = 10^{-118}, \dots \Lambda_{n,today} = 10^{-122} \quad (9.11)$$

$$S_n = 1, 3, \dots S_{n,inf} = 10^{12}, \dots S_{n,cmb} = 10^{114}, \dots S_{n,reoin} = 10^{118}, \dots S_{n,today} = 10^{122} \quad (9.12)$$

**In the pre-planckian or precursor phase**, namely, the trans-planckian phase:

$$10^{-61} t_P \leq t_n \leq t_P \quad (n = 0), \quad (9.13)$$

the quantum  $n$ -levels for  $H_{Qn}$ ,  $\Lambda_{Qn}$ ,  $S_{Qn}$  Eqs (4.6) are:

$$H_{Qn} = \sqrt{2n+1}, \quad \Lambda_{Qn} = (2n+1), \quad S_{Qn} = \frac{1}{(2n+1)}, \quad n = 0, 1, 2, \dots \quad (9.14)$$

Thus:

$$n = \frac{1}{2} (H_{Qn}^2 - 1), \quad 10^{-61} t_P \leq t_n \leq t_P \quad (n = 0) \quad (9.15)$$

and the more characteristic values in this phase, namely:

$$h_P, \dots H_{Qinf}, \dots H_{Qcmb}, \dots H_{Qreoin}, \dots H_{Qtoday} \equiv H_{far\ past}, \quad (9.16)$$

correspond to the  $n$  - level values:

$$n = 0, 1, \dots n_{Qinf} = 10^{12}, \dots n_{Qcmb} = 10^{114}, \dots n_{Qreoin} = 10^{118}, \dots n_{Qtoday} \equiv n_{far\ past} = 10^{122} \quad (9.17)$$

And the  $H_{Qn}$ ,  $\Lambda_{Qn}$  and  $S_{Qn}$  levels have the values:

$$H_{Qn} = 1, 1.732, \dots H_{Qinf} = 10^6, \dots H_{Qcmb} = 10^{57}, \dots H_{Qreoin} = 10^{59}, \dots H_{Qtoday} = 10^{61} \quad (9.18)$$

$$\Lambda_{Qn} = 1, 3, \dots \Lambda_{Qinf} = 10^{12}, \dots \Lambda_{Qcmb} = 10^{114}, \dots \Lambda_{Qreoin} = 10^{118}, \dots \Lambda_{Qtoday} = 10^{122} \quad (9.19)$$

$$S_{Qn} = 1, 0.333, \dots S_{Qinf} = 10^{-12}, \dots S_{Qcmb} = 10^{-114}, \dots S_{Qreoin} = 10^{-118}, \dots S_{Qtoday} = 10^{-122} \quad (9.20)$$

Figure 1 shows the whole picture, including **both the pre-planckian and post-planckian phases**, and the complete discrete spectrum of levels from the far past to today level. The universe pre- planckian phase, namely the quantum precursor phase is the setting of the physically meaningful quantum trans-planckian energies. In the post-planckian

(semiclassical and classical) eras, *no* trans-planckian energies are present: only mathematically or artificially (non physical) trans-planckian energies could be generated in the present universe. However, signals or observables from the quantum precursor phase are present in the classical and semiclassical universe, the most known being inflation and the present dark (vacuum) energy.

Consistently, the pre-planckian phase covering  $10^{-61}t_P \leq t \leq t_P$ , provides too the two dual: (+) and (-) branches, as it must be:

$$H_{n\pm} = h_P [ \sqrt{2n+1} \pm \sqrt{2n} ] \quad n = 0, 1, 2, \dots \quad (9.21)$$

$$H_{n=0} = h_P : \quad \text{planck scale value} \quad (9.22)$$

$$H_{n+, n \gg 1} = h_P [ 2\sqrt{2n} - \frac{1}{2\sqrt{2n}} + O(1/n^{3/2}) ] \gg 1 \quad \text{large } n: \text{ branch (+)} \quad (9.23)$$

$$H_{n-, n \gg 1} = \frac{h_P}{2\sqrt{2n}} + O(1/n^{3/2}) \ll 1 \quad \text{large } n : \text{ branch (-)} \quad (9.24)$$

And for the universe radius levels  $L_{Hn}$ :

$$L_{H(n=0)+} = L_{H(n=0)-} = L_{QH(n=0)} = l_P \quad n = 0 : \text{planck length} \quad (9.25)$$

$$L_{Hn+, n \gg 1} = l_P [ 2\sqrt{2n} - \frac{1}{2\sqrt{2n}} + O(1/n^{3/2}) ] \gg 1 \quad \text{large } n : \text{branch (+)} \quad (9.26)$$

$$L_{Hn-, n \gg 1} = \frac{l_P}{2\sqrt{2n}} + O(1/n^{3/2}) \ll 1 \quad \text{large } n : \text{branch (-)} \quad (9.27)$$

The same expressions hold for the mass levels  $M_{Hn}(\pm)$ ; the vacuum levels  $\Lambda_n(\pm)$  and the gravitational entropy  $S_n(\pm)$  levels follow from them.

The quantum levels cover *all* the range of scales from the largest cosmological scales and time  $10^{61}t_P$  today to the smallest one  $10^{-61}l_P$  in the extreme past  $10^{-61}t_P$  of the precursor or trans-planckian phase, passing through the planck scale  $(l_P, t_P)$ , covering the two phases: post and pre planckian phases respectively. The quantum mass levels are associated to the quantum space-time structure. Quantum mass levels here cover *all* masses  $10^{-61}m_P \leq M_n \leq 10^{61}m_P$  of the universe phases. The two *dual mass* branches ( $\pm$ ) correspond to the larger and smaller masses with respect to the planck mass  $m_P$  respectively, they cover the *whole mass range* from the planck mass in branch (+) until the largest cosmological masses, and from the smallest masses in branch (-), the pre-planckian phase, til near the Planck mass. As  $n$  increases, masses in the branch (+) *increase* (as  $2\sqrt{2n}$ ). Masses in the branch (-),

the very quantum one, *decrease* in the large  $n$  behaviour, precisely as  $1/(2\sqrt{2n})$ , large  $n$  are very excited levels in this branch, *consistently* with the fact that this branch is the dual of branch (+).

## X. THE SNYDER-YANG ALGEBRA AND QUANTUM DE SITTER SPACE-TIME

The space-time coordinates in the planckian and super-planckian domain are no longer commuting, but they obey non-zero commutation relations: The concept of space-time is replaced by a quantum algebra. The classical space-time is recovered from the quantum algebra as a particular case in which the quantum space and time coordinate operators become the classical space-time continuum coordinates (c-numbers) with all commutators vanishing and the discrete spectrum becomes the classical continuum space-time.

Here the quantum space-time description is reached *directly* from the quantum non-commuting space-time coordinates and not through the quantization procedure of the classical gravitational field. This is so because the gravity field is itself a classical concept which loose meaning at the planck scale. The space-time (the arena of events) is a classical concept which is more direct to extend to, or to replace by, *a quantum algebra* of space-time position and momenta

$$[X_i, X_j] = iM_{ij}$$

The Snyder algebra is a Lorentz covariant deformation of the Heisenberg algebra, where the position operators are non-commuting and have discrete spectra [36] soon extended by Yang [37] to include one more length parameter. It describes a non-commutative discrete space-time compatible with Lorentz-Poincare symmetry. The discrete position spectra, representations of the algebra imply a discrete space description of space.

- The Snyder algebra is precisely a description of a 4D constant curvature space of momenta, this corresponds to a de Sitter hyperboloid embedded in a 5D Minkowski momentum space. In the space of 5D momenta  $p_A$ , this includes precisely the motion of a particle of mass  $m$  and momentum on the de Sitter momentum hyperboloid  $\eta^{AB}p_Ap_B = m^2$ .

- In geometric terms, the Snyder quantized space-time is a projective geometry approach to the phase space or momentum de Sitter space in which the space-time coordinates are identified with the 4-translation generators of the  $SO(1, 4)$  de Sitter group (and are therefore non-commutative), and with other operators as the angular momentum in  $SO(1, 3)$ .
- In projective or Beltrami coordinates, the Euclid, Riemann and Lobachevsky spaces [39] corresponding to zero, positive and negative spatial curvature respectively, are upon Wick rotation the Minkowski, de Sitter and Anti de Sitter space-times with the invariance groups  $ISO(1, 3)$ ,  $SO(1, 4)$ ,  $SO(2, 3)$  respectively.

In  $D$  dimensions, the Lorentz-covariant Snyder-Yang quantum algebra follows from the Inonu-Wigner [38] group contraction of the  $SO(D - 1, 1)$  algebra with the generators:

$$\Sigma_{AB} = i(q_A \partial_{q_B} - q_B \partial_{q_A}), \quad (10.1)$$

$\Sigma_{AB}$  live on the  $(D + 2)$  parameter space  $q_A$  (hyperboloid) which satisfies

$$-q_0^2 + q_1^2 + \dots + q_{D-1}^2 + q_a^2 + q_b^2 = L^2 \quad (10.2)$$

$$A = (\mu, a, b); (\mu = 1, 2, \dots, D); (a, b) \text{ being extra space dimensions, and } q_0 \equiv q_D. \quad (10.3)$$

The  $D$ -dimensional operators  $(X_\mu, P_\mu, M_{\mu\nu})$ : space-time operator  $X_\mu$ , momentum operator  $P_\mu$ , angular momentum operators  $M_{\mu\nu}$  and the completing operator  $N_{ab}$  are all defined by the generators  $\Sigma_{\mu a}$  Eq.(10.1) as following:

$$X_\mu \equiv l_P \Sigma_{\mu a}, \quad P_\mu \equiv (\hbar/L) \Sigma_{\mu b}, \quad M_{\mu\nu} \equiv \hbar \Sigma_{\mu\nu}, \quad N_{ab} \equiv (l_P/L) \Sigma_{ab}. \quad (10.4)$$

This set of operators  $(X_\mu, P_\mu, M_{\mu\nu}, N)$  satisfy the contracted algebra of  $SO(D+1, 1)$ , namely the quantum Yang-Snyder space-time algebra:

$$[X_\mu, X_\nu] = -i(l_P^2/\hbar) M_{\mu\nu}, \quad [P_\mu, P_\nu] = -i(\hbar/L^2) M_{\mu\nu}, \quad (10.5)$$

$$[X_\mu, P_\nu] = -i\hbar N \delta_{\mu\nu}, \quad [X_\mu, N] = i(l_P^2/\hbar) P_\mu, \quad [P_\mu, N] = -i(\hbar/L^2) X_\mu \quad (10.6)$$

And the operators  $M_{\mu\nu}$  satisfy angular momentum's type relations:

$$[M_\mu, M_\nu] = -i(l_P^2/\hbar) M_{\mu\nu} \quad (10.7)$$

**Classical-quantum duality in the Snyder-Yang algebra:** The Snyder-Yang algebra contains two parameters  $(a, L)$ : small scale parameter  $a$  and large scale parameter  $L$  which in our context are naturally the planck length  $l_P$  and the universe radius  $L_H$ . Our complete (classical and quantum) radius  $L_{QH}$  Eq.(5.1) *contains intrinsically the both lengths*, the classical length  $L_H$  and its quantum dual  $L_Q$  (Compton radius of the universe), and provides a basis for a framework naturally free of infrared and ultraviolet divergences:

$$a \equiv l_P, \quad L \equiv L_{QH} = L_H + L_Q = l_P \left( \frac{L_H}{l_P} + \frac{l_P}{L_H} \right) \quad (10.8)$$

We see that the Snyder-Yang algebra with the complete length  $L_{QH}(l_P, L_H)$  as a parameter provides a quantum operator realization of the complete (classical and quantum) de Sitter universe, including the quantum early and classical late de Sitter phases duals of each other. This provides further description of the pre-planckian and post-planckian de Sitter phases, within a group-theory realization of the quantum discrete de Sitter space-time and of classical-quantum gravity duality.

Finally, let us mention as an example of the different classical and quantum de Sitter phases: the cosmological vacuum energy, the most direct candidate to the dark energy today , [6],[7],[8],[4],[5],[9],[10], for which the observed value is:

$$\rho_\Lambda = \Omega_\Lambda \rho_c = 3.28 \cdot 10^{-11} (eV)^4 = (2.39 \text{ meV})^4, \quad \text{meV} = 10^{-3} eV \quad (10.9)$$

corresponding to  $h = 0.73$ ,  $\Omega_\Lambda = 0.76$ ,  $H = 1.558 \cdot 10^{-33} eV$ . The CMB data yield the values [10]:

$$H = 67.4 \pm 0.5 \text{ Km sec}^{-1} \text{ Mpc}^{-1}, \quad \Omega_\Lambda h^2 = 0.0224 \pm 10^{-4} \quad (10.10)$$

and

$$\Omega_\Lambda = 0.6847 \pm 0.0073, \quad \Omega_\Lambda h^2 = 0.3107 \pm 0.0082, \quad (10.11)$$

which implies for the cosmological vacuum **today**:

$$\Lambda = (4.24 \pm 0.11) \cdot 10^{-66} (eV)^2 = (2.846 \pm 0.076) \cdot 10^{-122} m_P^2 \quad (10.12)$$

The density  $\rho_\Lambda$  associated to  $\Lambda$  Eq.(10.9) is precisely:

$$\rho_\Lambda = \Lambda/8\pi G = \rho_P (\Lambda/\lambda_P), \quad (10.13)$$

where the planck scale values  $\rho_P, \lambda_P$  are:  $\rho_P = \lambda_P/8\pi G$ ,  $\lambda_P = 3h_P^2$ . The quantum vacuum value expected from microscopic particle physics is evaluated to be  $\Lambda_Q \approx 10^{122}$ .

**Crossing the planck scale.** The two values:  $(\Lambda, \Lambda_Q)$ , refer to the same concept of vacuum energy but they are in two huge different vacuum states and two huge different cosmological epochs: classical state and classical dilute epoch today for  $\Lambda$  observed today with the most classical levels, and quantum state and quantum very early epoch with the most excited levels for the quantum mechanical transplanckian value  $\Lambda_Q$ . The classical value today  $\Lambda = 3H^2$  corresponds to the classical universe today of classical rate  $H$  and classical cosmological radius  $L_H = c/H$ . The quantum mechanical value  $\Lambda_Q = 3H_Q^2$  corresponds to the early quantum universe of quantum rate  $H_Q$  and quantum radius  $L_Q = l_P^2/L_H = \hbar/M_H c$  which is exactly the quantum dual of the classical horizon radius  $L_H$ :  $L_Q$  is *precisely* the quantum compton length of the universe for the gravitational mass  $M_H = L_H c^2/G$ .

**Two extremely different physical conditions and gravity regimes.** This is a realistic, clear and precise illustration of the *physical classical-quantum duality* between the two extreme Universe scales and gravity regimes or phases *through the planck scale*: the dilute state and horizon size of the universe today on the one largest known side, and the transplanckian scales and highest density state on the smallest side: size, mass, and their associated time (Hubble rate) and vacuum energy density  $(\Lambda, \rho_\Lambda)$  of the universe *today* are truly *classical*, while its extreme past at  $10^{-61} t_P = 10^{-105}$  sec deep inside the transplanckian domain of extremely small size and high vacuum density value  $(\Lambda_Q, \rho_Q)$  are truly *quantum and trans-planckian*. This manifests the *classical-quantum or wave-particle duality* between the classical macroscopic (cosmological) gravity physical phase and the quantum microscopic particle physics and transplanckian phase through the *crossing* of the planck scale, planck scale duality in short.

**An unifying picture:** Starting from the earliest past quantum era from  $10^{-61} t_P$  to  $t_P$ , with the quantum excited level  $n = 10^{122}$ , the entropy  $S_{Qn}$  increases in discrete levels  $s_P/(2n + 1)$  from its extreme small value  $S_Q = 10^{-122} s_P$  at the earliest time  $10^{-61} t_P$  till for instance its quantum inflation value  $10^{-12} s_P$ , ( $n_{Qinfl} = 10^{12}$ ), at time  $10^{-6} t_P$ , to its planck value ( $n = 0$ ):  $S_Q = s_P = \pi \kappa_B$  at the planck time  $t_P$ , the *crossing scale*, after which it goes to its semi-classical and classical levels  $(2n + 1)s_P$ , e.g. inflationary value  $S_{\Lambda inflation} = 10^{12} s_P$ , ( $n = 10^{12}$ ) at the classical inflationary stage at  $10^6 t_P$  and it follows *increasing and classicalizes* till the most classical level today  $n = 10^{122}$ :  $S_\Lambda = 10^{122} s_P$  at the present time  $10^{61} t_P$ . And as far as the universe will continue expanding its horizon as  $l_P \sqrt{(2n + 1)}$ ,  $S_{\Lambda n}$  will continue increasing as  $(2n + 1)$ .



The *total*  $Q\Lambda$  gravitational entropy (for the whole history) is the sum of the three values above discussed corresponding to the three regimes: classical  $\Lambda$ , quantum  $Q$  and planck values (subscript  $P$ ). In the past remote and more quantum (Q) eras:  $10^{-61} t_P \leq t \leq t_P$ , the planck entropy value ( $n = 0$ ):  $s_P = \pi\kappa_B$  dominates  $S_Q$ . In the classical eras:  $t_P \leq t \leq 10^{61}t_P$ , the today entropy value ( $n = 10^{+122}$ ):  $S_\Lambda = 10^{+122}s_P$  dominates.

**The whole picture is depicted in figure (1)**, where:  $\Lambda$  refers to the cosmological constant (or associated Hubble-Lemaitre constant  $H$ ) in the classical gravity phase.  $Q$  means quantum,  $P$  means planck scale, planck's units, natural to the system, greatly simplify the history. (The complete history is a theory of pure numbers). Each stage is characterized by the set of main physical gravitational quantities: ( $\Lambda$ , density  $\rho_\Lambda$ , size  $L_\Lambda$ , and gravitational entropy  $S_\Lambda$ ). In the quantum trans-planckian phase, levels are labeled with the subscript  $Q$ . Total means the whole history including the two phases or regimes. The present age of the universe  $10^{61}$ , (with  $\Lambda = \rho_\Lambda = 10^{-122} = 1/S_\Lambda$ ) is a *lower bound* to the future universe age and similarly for the present entropy level  $S_\Lambda$ . The past  $10^{-61}$ , (with  $\Lambda_Q = 10^{122} = \rho_Q = 1/S_Q$ ) is an *upper bound* to the extreme past (origin) of the universe and quantum initial entropy, (arrow of time). [Similarly, the values given in Fig.1 (in planck units) for the CMB are the classical CMB age ( $3.8 \cdot 10^5 yr = 10^{57}t_P$ ) and the set of gravitational properties of the universe at this age, and their corresponding precursors in the quantum preceding era at  $10^{-57}t_P$ .  $S_\Lambda$  constitute also an upper bound to the entropy of the CMB photon radiation.] **Figure caption:** The quantum discrete levels of the universe from its early trans-planckian era to today classical vacuum energy (dark energy), namely, the standard model of the universe completed back in time with quantum physics in terms of its vacuum history. The universe is composed of two main phases: after and before the planck scale (planck time  $t_P$  and planck units). The complete history goes from  $10^{-61}t_P$  to  $10^{61}t_P$ : In the pre-planckian (trans-planckian) phase  $10^{-61}t_P \leq t \leq t_P \equiv t_{planck}$  the quantum levels are:  $H_{Qn} = \sqrt{(2n+1)}$ ,  $\Lambda_{Qn} = (2n+1)$ ,  $S_{Qn} = 1/(2n+1)$ ,  $n = 0, 1, 2, \dots$ ,  $Q$  denoting quantum. The  $n$ -levels cover *all* scales starting from the past highest excited trans-planckian level  $n = 10^{122}$  with *finite curvature*  $R_Q = 10^{122}$ ,  $\Lambda_Q = 10^{122}$  and minimum entropy  $S_Q = 10^{-122}$ , as  $n$  decreases:  $S_{Qn}$  increases,  $(H_{Qn}, \Lambda_{Qn})$  decrease passing the planck level ( $n = 0$ ):  $H_{planck} = 1 = \Lambda_{planck} = S_{planck}$  and entering the post-planckian phase e.g.  $n = 1, 2, \dots, n_{inflation} = 10^{12}$ ,  $\dots, n_{cmb} = 10^{114}$ ,  $\dots, n_{reoin} = 10^{118}$ ,  $\dots, n_{today} = 10^{122}$ . In the post-planckian universe  $t_P \leq t \leq 10^{61}t_P$  the levels are:  $H_n = 1/\sqrt{(2n+1)}$ ,  $\Lambda_n = 1/(2n+1)$ ,  $S_n =$

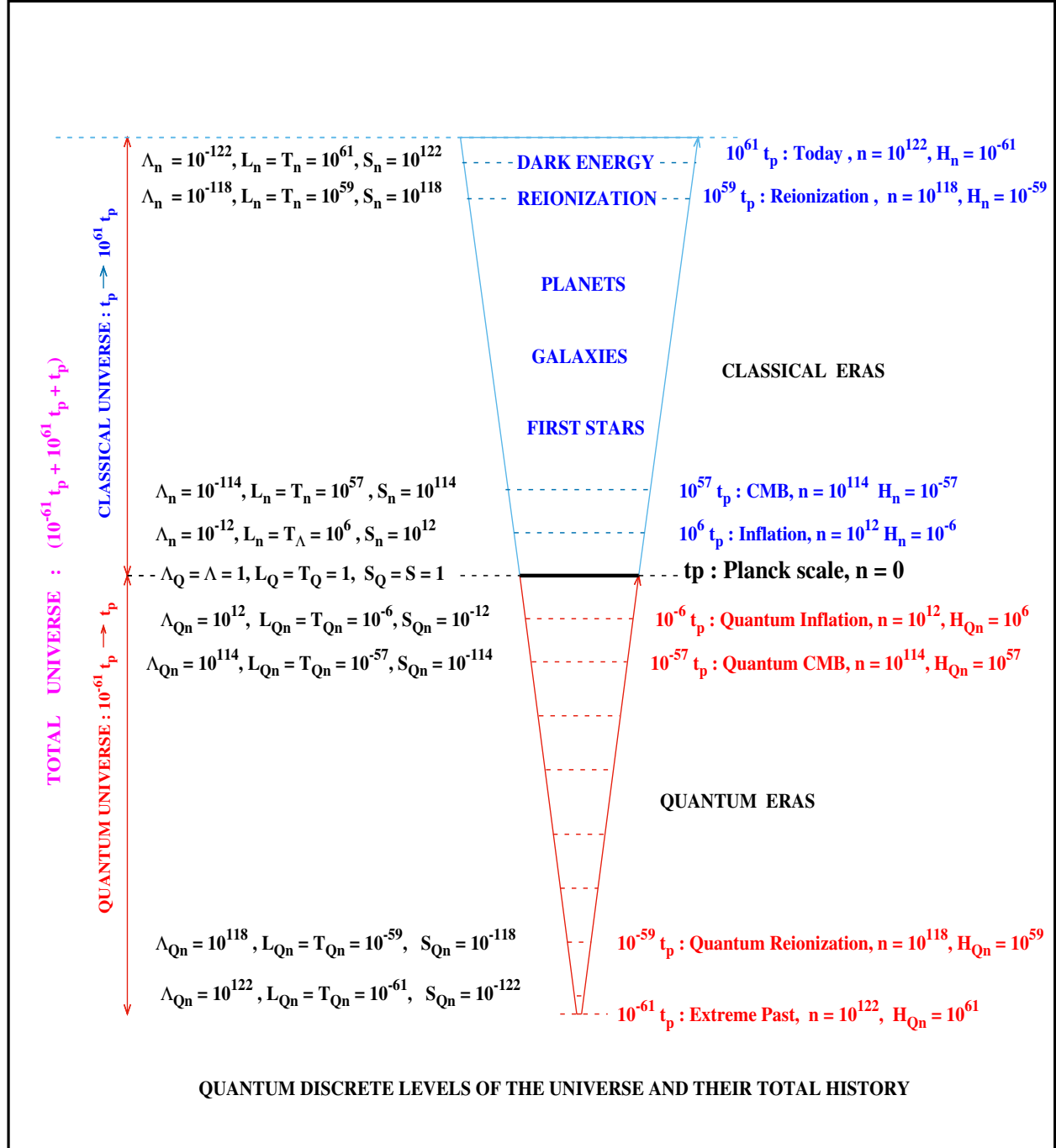


FIG. 1. The quantum discrete levels of the universe from its early trans-planckian era to today dark energy. In the pre-planckian (trans-planckian) phase  $10^{-61}t_P \leq t \leq t_P \equiv t_{\text{planck}}$  the quantum levels are (in planck units):  $H_{Qn} = \sqrt{(2n+1)}$ ,  $\Lambda_{Qn} = (2n+1)$ ,  $S_{Qn} = 1/(2n+1)$ ,  $n = 0, 1, 2, \dots, Q$  denoting quantum. The  $n$ -levels cover *all* scales from the past highest excited trans-planckian level  $n = 10^{122}$ , passing the planck level ( $n = 0$ ) and entering the post-planckian phase e.g.  $n = 1, 2, \dots, n_{\text{inflation}} = 10^{12}, \dots, n_{\text{cmb}} = 10^{114}, \dots, n_{\text{reoin}} = 10^{118}, \dots, n_{\text{today}} = 10^{122}$ . In the post-planckian universe  $t_P \leq t \leq 10^{61}t_P$  the levels are:  $H_n = 1/\sqrt{(2n+1)}$ ,  $\Lambda_n = 1/(2n+1)$ ,  $S_n = (2n+1)$ : as  $n$  increases, radius, mass and  $S_n$  increase and *consistently* the universe *classicalizes*. See the complete figure caption in the text at the end of Section X.

$(2n + 1)$ : As  $n$  increases, radius, mass and  $S_n$  increase,  $(H_n, \Lambda_n)$  decrease and *consistently* the universe *classicalizes*. The present age of the universe  $10^{61}t_P$  with its most classical value  $H_{today} = 10^{-61}$ ,  $\Lambda_{today} = 10^{-122} = 1/S_{today}$  is a *lower bound* to the future universe age and similarly for the present entropy level  $S_n$ . The far past  $10^{-61}t_P$ , (with  $\Lambda_Q = 10^{122} = 1/S_Q$ ) is an *upper bound* to the extreme known past ("origin") of the universe and quantum initial entropy, (arrow of time).

## XI. CONCLUSIONS

We have accounted in the introduction and along the paper the main results and will not include all of them here. We synthesize below some conclusions and remarks.

- The standard model of the universe is extended back in time with planckian and trans-planckian physics before inflation in agreement with observations, classical-quantum gravity duality and quantum space-time. The quantum vacuum energy bends the space-time and produces a constant curvature de Sitter background. We find the quantum discrete cosmological levels: size, time, vacuum energy, Hubble constant and gravitational (Gibbons-Hawking) entropy and temperature from the very early trans-planckian vacuum to the classical today vacuum energy. The  $n$ -levels cover *all* scales from the far past highest excited trans-planckian level  $n = 10^{122}$  with finite curvature,  $\Lambda_Q = 10^{122}$  and minimum entropy  $S_Q = 10^{-122}$ ,  $n$  decreases till the planck level ( $n = 0$ ) with  $H_{planck} = 1 = \Lambda_{planck} = S_{planck}$  and enters the post-planckian phase e.g.  $n = 1, 2, \dots, n_{inflation} = 10^{12}, \dots, n_{cmb} = 10^{114}, \dots, n_{reoin} = 10^{118}, \dots, n_{today} = 10^{122}$  with the most classical value  $H_{today} = 10^{-61}$ ,  $\Lambda_{today} = 10^{-122}$ ,  $S_{today} = 10^{122}$ . We implement the Snyder-Yang algebra in this context yielding a consistent group-theory realization of quantum discrete de Sitter space-time, classical-quantum gravity duality symmetry and a clarifying unifying picture.
- A clear picture for the de Sitter background and the whole universe epochs emerges, both for its classical (post-planckian) and quantum (pre-planckian) regimes, depicted in Fig (1). This is achieved by considering classical-quantum gravity duality, trans-planckian physics, quantum space-time and quantum algebra to describe it. Concepts as the Hawking temperature and the usual (mass) temperature are precisely the

same concept in the different: classical gravity (post-planckian) and quantum gravity regimes respectively. Similarly, it holds for the Bekenstein-Gibbons and Hawking entropy. An unifying clarifying picture has been provided in terms of the main physical gravitational intrinsic magnitudes of the universe: age, size, mass, vacuum energy, temperature, entropy, covering the relevant gravity regimes and cosmological stages: classical, semiclassical and quantum planckian and trans-planckian eras. The total or global mass levels are  $M_n = m_P \sqrt{2n+1}$  for all  $n = 0, 1, 2, \dots$ . *Two* dual branches  $m_{n\pm} = m_P [\sqrt{2n+1} \pm \sqrt{2n}]$  do appear for the usual mass variables, covering the *whole mass range*: from the planck mass ( $n = 0$ ) until the largest cosmological ones in the post-planckian branch (+), and from the smallest masses till near the planck mass in the pre-planckian branch (-).

- The quantum space-time structure arises from the relevant non-zero space-time commutator  $[X, T]$ , or non-zero quantum uncertainty  $\Delta X \Delta T$ . The *quantum light cone* is generated by the quantum planck hyperbolae  $X^2 - T^2 = \pm[X, T]$  due to the quantum uncertainty  $[X, T] = 1$ . Inside the planck hyperbolae there is a entirely *new quantum region* which is purely quantum vacuum or zero-point planckian and trans-planckian energy and constant curvature. The quantum non-commuting coordinates  $(X, T)$  and the transverse commuting spatial coordinates  $X_{\perp j}$  generate the quantum two-sheet hyperboloid  $X^2 - T^2 + X_{\perp j}^2 = \pm 1$ . The quantum de Sitter space-time is described through the relevant quantum non-commutative coordinates and the quantum hyperbolic "light cone" hyperbolae. They generalize the classical de Sitter space-time and reduce to it in the classical zero quantum commutator coordinates. Interestingly enough, de Sitter space-time turns out to be *discretized* in quantum hyperbolic levels  $X^2 - T^2 + X_j X_j + Z^2 = (2n + 2), n = 0, 1, 2, \dots$
- In the post-planckian domain, the quantum de Sitter space-time extends in discrete levels from the planck scale hyperbolae ( $n = 0$ ) and the quantum (low  $n$ ) levels to the quasi-classical and classical levels (intermediate and large  $n$ ), tending asymptotically for the very large  $n$  to a classical continuum space-time. Consistently, these levels have larger gravitational (Gibbons-Hawking) entropy  $S_n$ , lower vacuum density  $\Lambda_n$  and lower Hubble rate  $H_n$ . In the pre-planckian trans-planckian domain, quantum de Sitter extends from the planck scale hyperbolae ( $n = 0$ ) to the lengths and time smaller

than the planck scale, the quasi-quantum transplanckian levels (small and medium  $n$ ), until the deep extreme highly excited transplanckian levels (very large  $n$ ) which are those of smaller entropy  $S_{Qn}$ , higher vacuum density  $\Lambda_{Qn}$  and higher  $H_{Qn}$ .

- Cosmological evolution goes from the pre-planckian or trans-planckian quantum phase to the planck scale and then to the post-planckian universe: semiclassical accelerated de Sitter era (field theory inflation), then to the classical phase until the present *diluted* de Sitter era. This evolution between the different gravity regimes could be view as a mapping between asymptotic (in and out) states characterized by the sets  $U_\Lambda$  and  $U_Q$ , and thus as a Scattering-matrix description: The most early quantum trans-planckian state in the remote past being the "in-state", and the very late classical dilute state being the far future or today "out-state".
- Inflation is part of the standard cosmological model and is supported by the CMB data of temperature and temperature-E polarisation anisotropies. This points to  $10^{-6}m_P$ , (or  $10^{-5}M_P$  for the reduced mass  $M_P = m_P/\sqrt{8\pi}$ ) as the energy scale of Inflation [19],[20], safely below the planck energy scale  $m_P$  of the onset of quantum gravity. This implies that Inflation is consistently in the *semiclassical gravity regime*. This in turn implies that the preceding phase of Inflation corresponds to a planckian and pre-planckian quantum phase. Inflation being a de Sitter, (or quasi de Sitter) stage, it has a smooth space-time curvature *without any physical space-time singularity*.
- Integrating the above results, and because the earliest stages of the universe are de Sitter (or quasi de Sitter) eras, it appears that there is **no singularity** at the universe's origin. First: the so called  $t = 0$  Friedman-Robertson Walker mathematical singularity is **not** physical: it is the result of extrapolation of the purely classical (non quantum) General Relativity theory, *out of its domain of physical validity*. The planck scale is not merely a useful system of units but a physically meaningful scale: the onset of quantum gravity, this scale precludes the extrapolation until zero time or length. This is precisely what is expected from quantum trans-planckian physics in gravity: the smoothness of the classical gravitational singularities. Second: Inflation (classical or quantum) in the very past ( $10^6t_P$  or  $10^{-6}t_P$ ) is mainly a de Sitter or quasi de Sitter smooth constant curvature era **without any curvature singularity**. Third: the

extreme past (at  $10^{-61}t_P$ ) is a trans-planckian de Sitter state of high *bounded* trans-planckian constant curvature and therefore *without singularity*. This paper is not devoted to the singularity issue but our results here and the whole picture emerging from this paper and [2] indicate the trend and insight into the problem.

- Further couplings, interactions and background fields can be added. The conceptual results here will not change by adding further couplings or interactions, or further background fields to the background here. Of course, this is just a first input in the construction of a complete physical theory and understanding *in agreement with observations*. Besides its conceptual and fundamental physics interest, this framework reveals deep and useful clarification for relevant cosmological eras and its quantum precursors and for the cosmological vacuum. This could provide realist insights and science directions where to place the theoretical effort for cosmological missions and future surveys such as Euclid, DESI, WFIRST, LSST-Vera C. Rubin Observatory and Simons Observatory for instance,[27], [28], [29], [42], [43] and for the searching of cosmological quantum gravitational signals for e-LISA [11] for instance, after the success of LIGO [12],[13]
- The exhibit of  $(c, G, h)$  helps in recognizing the different relevant scales and physical regimes. Even if a hypothetical underlying "theory of everything" could only require pure numbers (option three in [40]), physical touch at some level asks for the use of fundamental constants [41],[30]. Here we used three fundamental constants, (tension being  $c^2/G$ ). It appears from our study here and in ref [1], that a complete quantum theory of gravity would be a theory of pure numbers.
- We can similarly think in quantum string coordinates (collection of point oscillators) to describe the quantum space-time structure, (which is different from strings propagating on a fixed space-time background). This yields similar results for the string expectation values  $X^2 - T^2$  and other related operators and yields too a quantum *hyperbolic space-time width bending* for the characteristic lines and light cone generators, or for the space-time horizons [1]. Moreover, the  $\sqrt{n}$  quantization is like the string mass quantization  $M_n = m_s\sqrt{n}$ ,  $n = 0, 1, \dots$  with the planck mass  $m_P$  instead of the string mass  $m_s$ , that is to say, with the gravitational constant  $G/c^2$  instead of the string constant  $\alpha'$ .

## ACKNOWLEDGEMENTS

The author acknowledges the French National Center of Scientific Research (CNRS) for Emeritus Director of Research contract and F. Sevre for help with the figure. This work was performed in LERMA-CNRS-Observatoire de Paris- PSL University-Sorbonne University.

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