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# On the use of the wave distribution function concept to determine the directions of arrivals of radar echoes

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Conditions of application of the Storey and Lefeuvre (Geophys. J. R. Astron. Soc. 56 (1979) 255–270; Geophys. J. R. Astron. Soc. 62 (1980) 173–194) wave distribution function concept (WDF) to the determination of the directions of arrival of the NETLANDER Ground Penetrating Radar echoes are examined. The hypotheses used to make a WDF analysis from wave field components measured in a space plasma are recalled. An example of application is given. Following Lagoutte (1981 Rapport de DEA, Université d'Orléans) the WDF concept is extended to the case of an isotropic and non-ionized medium. The application to the analysis of the radar echoes within the Mars subsurface is discussed. The main point is to have a thickness of the dry layer between the surface and the first reflector much larger than the wavelengths.

## 1. Introduction

This is a complement to the Berthelier et al. (2000) NETLANDER Ground Penetrating Radar paper on the use of a fixed radar to identify icy water within the Mars subsurface. It supposes: (1) that pulses are emitted inside the ground in the 2–4 MHz frequency band; second, that after reflection on icy water and/or on other transition layers, waves are returned to the emission point before the following pulse is transmitted; and third, that magnetic and electric sensors are sensitive enough to detect the returned waves, i.e. the echoes. The present paper is essentially devoted to the determination of the directions of arrivals of the radar echoes from the waveforms associated with the measurements of the three electric and the three magnetic wave field components. The analysis is supposed to be performed at ground, which means that the waveforms have to be transmitted.

The main hypothesis made in this paper is that the observed echoes can be considered as propagating waves. This implies that the observation be done far from the zone of reflection, i.e. at distances much greater than the wavelength. It is a severe restriction, but, it will be shown in Section 3 that the hypothesis may be done for icy water on Mars. In such a case, the problem to be solved is similar in several

respects to the problem of determination of direction of arrivals for plasma waves detected on a satellite or at ground.

Numerous approaches have been proposed to determine the directions of arrival of plasma waves. Most of them assume that the electromagnetic field observed at the output of a narrow-band filter is locally and instantaneously like that of a single plane wave. When this hypothesis is valid, the electromagnetic field is fully described by a single angular frequency  $\omega_0$ , a single wave normal direction  $\mathbf{k}_0$ , and the polarization ellipses of the electric and magnetic fields. But it often happens that the waves are spread in frequency and in the direction of propagation. In that case, it has been suggested (Storey, 1971) to describe the electromagnetic field by a distribution function similar to the distribution function for particles. This function, called the wave distribution function (WDF), specifies, in each propagation mode, the distribution of wave energy density as regards to the angular frequency  $\omega_0$  and the wave-normal direction  $\mathbf{k}_0$  (Storey and Lefeuvre, 1979). It is related to the auto- and cross-power spectra of the electromagnetic field components via a set of integral equations (Storey and Lefeuvre, 1980). Solutions to the inverse problem, namely, to the determination of a WDF from a set of estimated values of the auto- and cross-power spectra of the field components, were proposed by Lefeuvre (1977). First applications on satellite data were published by Lefeuvre and Delannoy (1979) and Lefeuvre et al.

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(1981). Analyses of electromagnetic waves generated in the plasma and detected in the Earth atmosphere by ground based equipments have been published by Hayakawa et al. (1992).

The fact that the waves are generated by a radar and are propagated in the Mars sub-surface before being detected in the planet atmosphere does not change the nature of the problem. However two points have to be examined carefully: the modeling of the Mars subsurface and the derivation of the wave reflection pattern from the wave-field components measurements performed by the NETLANDER electric and magnetic antennas. Considering that an application of the Descartes Snell's law solves the problem for the horizontal electric- and magnetic-field components we will not develop on the second point. The transformation of the vertical component is deferred to a future study. The true question we have to consider is about the modeling of Mars.

Clearly, the observations available at present, do not allow to have direct information on the internal structure of the planet and on the physical properties of the constitutive materials. The only way to approach the problem is to extrapolate what has been learnt from Earth's probing. Under this hypothesis one may model the Mars subsurface by homogeneous layers separated by heterogeneous transition zones with complex structures. In the first approximation, such zones may be modeled by an infinity of perfect reflectors (i.e. of reflector with plane surfaces) randomly distributed in size, direction and space. In that case, the penetrating waves are randomly reflected by reflectors of sizes comparable to or greater than the wavelengths. The reflected signal is stochastic in nature. The wave field may be described by a wave distribution function similar to that defined for plasma waves.

In some sense the physics of the problem is simpler on Mars. In contrast to what happens in space there is no relative movement between the source and observation points, which mean that the field components may be measured one after the other and not simultaneously. The only point is to make sure that the UT times are given with a sufficient accuracy to correct for the phase shifts between the cross-power spectra of the field components. As an example, the phase-shift  $\Delta\phi$  produced by a  $\Delta t$  time translation being given by  $\Delta\phi = \omega\Delta t$ , one observes that a  $\sim 10^\circ$  phase error is produced at 3 MHz by a 10 ms time uncertainty.

Now, let us examine in more detail the case of the determination of any WDF from measurements made in the atmosphere. Surprisingly, it is slightly more complex that for measurements made in a space plasma. In a space plasma, i.e. in an anisotropic ionized medium, there is a finite number of modes (the ordinary and the extraordinary modes in a cold magnetoplasma) and so a finite number of parameters are to be determined. Oppositely, in isotropic non-ionized or weakly ionized media such as the Earth and Mars atmospheres, the number of propagation modes and so of parameters to be determines may be infinite. The problem is

relatively simple to solve when, for physical reasons, one or two modes only may be present. This happens in the Earth atmosphere for waves generated in the magnetosphere (i.e. in the ordinary or the extraordinary mode) then observed at ground after very few modifications in the polarization characteristics (see e.g. Hayakawa et al., 1992). We will consider that the same type of hypothesis may be done for waves generated by the GPR radar and reflected in the Mars subsurface.

The plan of the paper is as follows. In Section 2 the Storey and Lefeuvre (1979, 1980) formulation of the WDF concept is recalled. In order to illustrate the capacity of the method, an example of WDF solution obtained from satellite data is given. Section 3 deals with a possible extension to the determination of the direction of arrival of the radar echoes on NETLANDER. Hypotheses on the existence of propagating waves are discussed. A new formulation of the WDF problem is given using calculations already published in Lagoutte (1981). Finally, provisional conclusions are presented in Section 4.

## 2. The wave distribution function concept in a magnetoplasma

### 2.1. Hypotheses

Storey and Lefeuvre make basically three hypotheses: (1) on the linearity of the wave field; (2) on the uniformity of the medium, and (3) on the stationarity in time of the observation. Let examine them.

- At the output of a narrow-band filter centered on the frequency  $\omega_0$ , the wave field may be regarded as the sum of a finite or infinite number of elementary monochromatic plane waves, each of which being a characteristic wave, i.e. a solution of the dispersion equation for the medium. The linearity implies that the monochromatic plane waves propagate without any mutual phase coherence. If a coherence effect occurs at the source, one must assume that the phase coherence between different frequencies is destroyed by dispersion along the path to the point of observation. Although Ronmark and Larsson (1988) demonstrated that the WDF may be defined on a more general way, without referring to a statistical description, the Storey and Lefeuvre model seems to be a good physical representation of what is observed on a satellite.
- The medium is supposed to be uniform on a scale much larger than the wavelength. This means that the structure of a single wave be very little different from what it could be in an infinite uniform medium. The residual large-scale non-uniformity limits the resolution in the  $\mathbf{k}$  vectors.

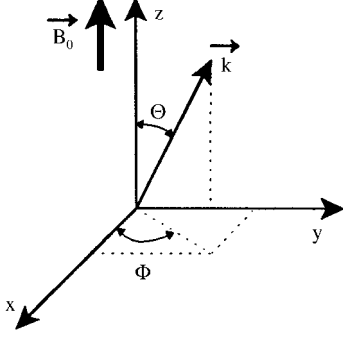


Fig. 1. Coordinate system used in the definition of the wave distribution function.

- The field is stationary in time over time intervals much longer than a wave period. This allows, first, to obtain a good resolution frequency, and second, to get accurate estimates for the auto- and cross-power spectra of the wave field components.

It is also generally assumed that the plasma is cold and collisionless. This restriction is not compulsory. Although the calculations of the dispersion equations be more difficult, Santolik and Parrot (1998) have shown the WDF concept could be extended to a warm plasma. But what seems to be the true limitation is the number of propagation modes that may coexist. For more than two modes, the number of variables is probably too important to get stable WDF solutions.

## 2.2. The direct problem

Let us consider the Cartesian coordinate system  $O_{xyz}$  (Fig. 1) in which the  $O_z$ -axis is parallel to the earth's magnetic field  $\mathbf{B}_0$ , the  $O_x$ -axis is in the local magnetic meridian plane and points away from the earth, and the  $O_y$ -axis is oriented eastward. In that system, the  $\mathbf{k}$  vector is characterized by the polar angle  $\theta$  made by  $\mathbf{k}$  with  $\mathbf{B}_0$  and by the azimuthal angle  $\phi$ , the origin of which is in the local meridian plane. The WDF is a function of these two angles and of the frequency  $\omega_0$  at which the analysis is done. It is written  $F(\omega_0, \cos \theta, \phi)$ .

Let us gather the three electric wave field components  $E_x, E_y, E_z$  and the three magnetic components  $H_x, H_y, H_z$  in a generalized electric vector  $\xi$  whose components are

$$\xi_{1,2,3} = E_{x,y,z}, \quad \xi_{4,5,6} = Z_0 H_{x,y,z}, \quad (1)$$

with  $Z_0$  the wave impedance of free space. The WDF is related to the spectral matrix element  $S_{ij}$  of the six wave field components ( $i, j = 1, \dots, 6$ ) by the set of integral equations

$$S_{ij}(\omega_0) = \sum_m \int \int a_{ijm}(\omega_0, \cos \theta, \phi) \times F_m(\omega_0, \cos \theta, \phi) |d \cos \theta d \phi|, \quad (2)$$

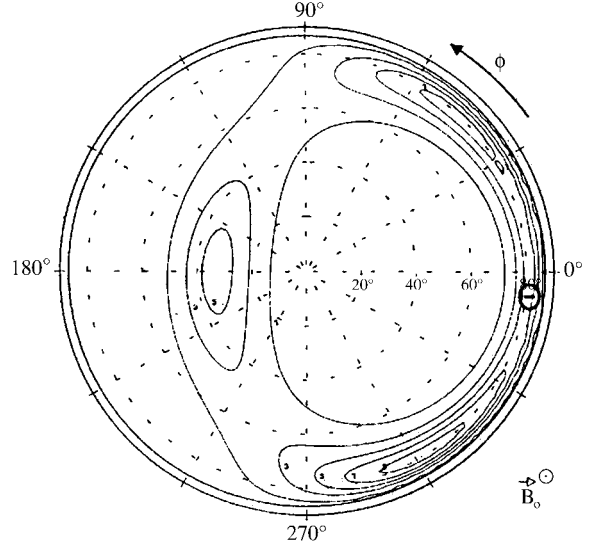


Fig. 2. WDF solution for a hiss event observed by ISEE1 satellite at 416 Hz (from Storey et al., 1991).

where the index  $m$  indicates the magneto-ionic mode, the functions  $a_{ijm}(\omega_0, \cos \theta, \phi)$ , which are the kernels of the set of integral equations, are defined below, the function  $F_m(\omega_0, \cos \theta, \phi)$  is the WDF in the  $m$  mode,  $|d \cos \theta d \phi|$ , is an element of solid angle centered on  $\cos \theta$  and  $\phi$ ,  $\cos \theta$  varies between  $-1$  and  $+1$  and  $\phi$  varies between  $0$  and  $2\pi$ .

The basic formula for the kernels is given in Eq. (18) of the Storey and Lefeuvre (1979) paper, and is written as

$$a_{ijm}(\omega_0, \cos \theta, \phi) = \xi_i \xi_j^* / \rho, \quad (3)$$

where  $\xi_i$  and  $\xi_j$  are the complex amplitudes of the  $i$ th and  $j$ th components of the generalized electric field of an elementary plane wave in the mode  $m$  and characterized by the variables  $\omega_0, \theta$  and  $\phi$ , and where  $\rho$  is the wave energy density. Details of their calculations and exact analytical expressions are given in Lefeuvre (1977). The algebraic expressions are given in Storey and Lefeuvre (1980). However, a correction must be introduced on one term (see Lefeuvre et al., 1986).

## 2.3. Example of WDF solution

The determination of  $F_m(\omega_0, \cos \theta, \phi)$  from the estimated values of the real and imaginary parts of the  $S_{ij}$ 's is an inverse problem that admits of infinitely many solutions. One way to get a physical solution is to select the function which satisfies the estimated  $S_{ij}$ 's and has the maximum entropy (Lefeuvre, 1977). A summary of the technique may be found in Lefeuvre and Delannoy (1979).

As an example, the result of an analysis performed on satellite data is given in Fig. 2. The wave field has been measured by the ISEE-1 satellite, within the

magnetosphere. Contours of the WDF solutions are represented on a polar plot. They represent the wave energy density conveyed by waves whose  $\mathbf{k}$  vectors are in the  $\mathbf{B}_0$  direction. The  $\theta$  and  $\phi$  values vary, respectively, from 0 to 90° and from 0 to 360°. The inner circle is due to the presence of a non-propagating zone in the vicinity of  $\theta = 90^\circ$ . The amplitude levels of the WDF contours are given in a linear scale. One observes that the wave energy density is conveyed by three wavepackets: the main, with  $\mathbf{k}$  vectors having  $\theta$  values around 75° and  $\phi$  values around 180°. The method does not provide absolute energy densities, but, we see that the first secondary wavepacket conveyed 70% of the energy density of the main, and the other 50%. In that particular case, it is very likely that the ISEE satellite observes different sources of emissions.

### 3. Direction of arrival of the radar echoes

#### 3.1. Hypotheses

The hypotheses of Section 2 must obviously be modified to take into account of the nature of the problem sets on NETLANDER. According to the assumptions made in the introduction about the time stationarity and the number of polarization modes, the only point to examine is the hypothesis on the field linearity.

The waves emitted by the radar are coherent waves with well defined polarization characteristics and  $\mathbf{k}$  vectors spread in all directions. If the reflection zone is extended and/or if there are several zones of reflections, the returned waves must reach the point of observation with different wave normal directions. Provided the paths to the points of reflection then to the point of observation are long enough, one may expect that the coherencies are destroyed and so that the observed field is a linear field which can be described by the Storey and Lefeuvre model. Then, as for the hypothesis on the uniformity of the medium, which is the second hypothesis, the crucial point is to know if the distance between the measurement devices and the reflection zones are large enough as regards to the wavelengths in the medium.

An answer to that question may be obtained by studying the condition of propagation and reflection of the electromagnetic field in geological materials. Consider the case of a harmonic electromagnetic plane wave of pulsation  $\omega$  propagating in a homogeneous medium of conductivity  $\sigma$ , permittivity  $\varepsilon$ , and magnetic permeability  $\mu$ , assumed, hereafter, to be equal to that of vacuum  $\mu_0$ , in agreement with what is observed for most of geological materials on the Earth.

When both displacement and conduction currents should be taken into account, that is situations for which  $\omega^2\mu\varepsilon$  is not negligible compared to  $\omega\mu\sigma$ , the wave is a propagating wave with a celerity  $v$ . The electric field and the magnetic field are orthogonal to each other, and to their direction

of propagation. They are attenuated along the direction of propagation with a damping factor  $\gamma$ . The parameters  $v$  and  $\gamma$  are equal to

$$v = \{0.5\mu\varepsilon[1 + \sigma^2/\varepsilon^2\omega^2]^{1/2} + 1\}^{1/2}, \quad (4)$$

$$\gamma = \{0.5\mu\varepsilon[1 + \sigma^2/\varepsilon^2\omega^2]^{1/2} - 1\}^{1/2}. \quad (5)$$

Now consider the two limiting cases where  $\omega^2\mu\varepsilon \ll \omega\mu\sigma$  and  $\omega^2\mu\varepsilon \gg \omega\mu\sigma$ . The first one corresponds to situations where the conduction current is very much greater than the displacement current. In this case, there is no displacement of an initial wave form: the wave phenomenon has degenerated into diffusion. The interaction between a time varying electromagnetic wave and a conductive medium then results in induced electric currents, which tend to oppose its penetration in the conductive medium. When penetrating in a homogeneous medium, the intensity of an electromagnetic wave, thus, decreases as  $\exp(-z/\delta)$  with a logarithmic scale factor  $\delta$  (so-called skin depth) equal to

$$\gamma = 1/\delta = [\omega\mu\varepsilon/2]^{1/2}. \quad (6)$$

$\delta$  is very small compared to the wavelength of the electromagnetic field, and it is a scaling geometrical factor of the electromagnetic field.

The second one corresponds to situations where the displacement current is very much greater than the conduction current. The electromagnetic field, then, is a propagating plane wave, with an attenuation factor that approaches asymptotically a maximum as  $\omega$  is increased. This maximum does not depend on  $\omega$ , and it is equal to

$$\gamma_{\max} = [\sigma\mu\varepsilon/2]^{1/2}. \quad (7)$$

If the medium is very resistive,  $\gamma_{\max}$  becomes very small ( $\gamma_{\max} = 188.3\sigma\sqrt{\varepsilon_r}$ , where  $\varepsilon_r = \varepsilon/\varepsilon_0$ ,  $\varepsilon_0$  being the permittivity of vacuum).  $1/\gamma_{\max}$  is then very large compared to the wavelength of the electromagnetic field,  $\lambda = 2\pi v/\omega = 2\pi/\omega\sqrt{\varepsilon\mu}$  which is, therefore, the scaling geometrical factor of the electromagnetic field. In a non-conducting medium,  $v = c/\sqrt{\varepsilon\mu} = 1/\sqrt{\varepsilon_r}$  where  $c$  is the velocity of the wave in free space ( $\mu = \mu_0$ ). At a given frequency, the velocity  $v$ , and the wavelength  $\lambda$  of the wave are then determined by the permittivity of the medium. The situation is in fact similar to geometrical optics, the index of refraction  $n = c/v$  being equal to  $\sqrt{\varepsilon_r} = \sqrt{\varepsilon/\varepsilon_0}$ . The behavior of an electromagnetic plane wave at the boundary between two different homogeneous mediums can, then, be described using the Snell Descartes law, and it only depends on the contrast in permittivity between the two mediums.

The wavelength  $\lambda$  and the skin depth  $\delta$  are then the geometrical scaling factors of the diffusive and propagating part of the electromagnetic field, the two limiting cases of an almost purely diffusive field or propagating wave corresponding to situations where:

$$\omega^2\mu\varepsilon = (2\pi/\lambda)^2 \ll 2/\delta^2 = \omega\mu\sigma, \quad (8)$$

or

$$\omega^2 \mu \varepsilon = (2\pi/\lambda)^2 \gg 2/\delta^2 = \omega \mu \sigma, \quad (9)$$

respectively.

Consider now a 1-D medium made up with homogeneous layers of thickness  $e_i$ , permittivity  $\varepsilon_i$ , and resistivity  $\rho_i$ . It is assumed here that plane interfaces lead to deterministic transition/reflexion processes for the radar electromagnetic wave and so that a  $\mathbf{k}$  vector depends only on the  $\mathbf{k}$  vector of the emitted wave and the direction of the normal to the plane interface. Let the pulsation  $\omega$  be such that the electromagnetic field behaves as propagating plane waves in the uppermost layers ( $\omega \varepsilon_i \gg \sigma_i$ ;  $i = 1, \dots, N$ ). The time lag necessary for the first reflected wave, that is the wave reflected at the bottom of the uppermost layer to reach the surface is  $\tau_1 = 2e_1/\sqrt{\varepsilon_1\mu}$ , and fulfilling of the second hypothesis requires the period  $2\pi/\omega$  to be small compared to  $\tau_1$ , that is ensured if the wavelength  $\lambda$  is small compared to  $e_1$ . In practice, separating the echoes corresponding to the successive reflections requires  $\lambda$  to be small compared to  $e_i$  for  $i = 1, \dots, N$ .

The objective of the NetLander Ground Penetrating Radar experiment is to probe the Mars subsurface and search for signatures of ice reservoirs. Due to sublimation processes and the porous nature of the megaregolith, the ground ice table is expected to be covered with an almost non-conductive (say  $\sigma < 10^{-8} \text{ S m}^{-1}$ ) dry layer. The top of the ground ice layer corresponds to a reflector for the electromagnetic waves. It is found at depth exceeding 300 m at equatorial latitudes and 150 m for high latitudes, with a minimum value of 30 m in Acidalia Planitia and Utopia Planitia (Costard and Kargel, 1995). Nothing is known about the actual conductivity of the Mars permafrost. Results from laboratory experiments on sandstones without clays, however, show that it is likely to be on the order of  $10^{-6} \text{ S m}^{-1}$  at temperatures below a limit  $\theta_L$  found in the range  $-10$ – $20^\circ\text{C}$ , depending on ions present in the ice on the order of  $10^{-3} \text{ S m}^{-1}$  between  $\theta_L$  and  $0^\circ\text{C}$ , and larger than  $10^{-3} \text{ S m}^{-1}$  in presence of liquid water (Guichet, 1998). The mean value of  $\varepsilon_r$  is expected to be in the order of 3.5 (Barbin et al., 1991; Herique, 1995).

The hypothesis that the echoes can be considered as propagating waves in the uppermost layers (dry soil and upper resistive part of the permafrost) requires first the electromagnetic field not to be a diffusive one in these mediums, that corresponds to  $\delta^2/2$  on the order of, or greater than  $(\lambda/2\pi)^2$ ; for  $\sigma = 10^{-5} \text{ S m}^{-1}$  and  $\varepsilon_r$  in the range 1–10, it corresponds to frequencies  $f = \omega/2\pi$  on the order of, or greater than a few hundreds of kHz. It also requires the observation to be done far from the reflection zone. Table 1 gives the values of the damping factor  $\gamma$  and wavelength  $\lambda$ , calculated from formulas (4) and (5) for  $\varepsilon_r$  values between 1 and 10. It shows that frequencies in the range 2–4 MHz correspond to the propagating waves with wavelength in the range 24–150 m for  $\sigma = 10^{-8} \text{ S m}^{-1}$  and  $\sigma = 10^{-5} \text{ S m}^{-1}$ . As expected, they would correspond to a diffusive field cancelled out in a few

tens of meters in the deeper more conductive layers of the permafrost, with conductivity on the order of  $10^{-3} \text{ S m}^{-1}$ . The requirement for the observation to be done far from the reflection zone is, therefore, valid if the thickness  $e_1$  of the dry layer between the surface and the first reflector is large compared to 80 m, assuming an  $\varepsilon_r$  value on the order of 3.5 in this layer. This is expected to be the case at equatorial latitudes.

As indicated in the introduction, by extrapolating what has been learnt from Earth's probing, the real situation is expected to be more complex. Homogeneous layers are separated by heterogeneous transition zones we can model by an infinity of perfect reflectors (i.e. of reflector with plane surfaces) randomly distributed in size, direction and space. As a result, even if the penetrating wave is deterministic, the reflected waves are of stochastic nature.

### 3.2. The direct problem

The main difference between the WDF analysis of a wave field measured in a magnetoplasma and a wave field measured in the atmosphere or within the Mars subsurface, appears in the  $a_{ijm}$ 's kernels. According to the nature of the problem, one cannot expect to keep the same algebraic expression than for the plasma and find some limiting values. The only way to derive the new kernels is to define a new coordinate system, express the  $\zeta_i$  and  $\zeta_j$  complex amplitudes in terms of a new polarization parameter, calculate the wave energy density  $\rho$ , then establish a new set of kernels for each mode  $m$ .

Let us first consider the coordinate system. Considering that the  $O_z$ -axis is the vertical axis and the  $O_x$ - and  $O_y$ -axis are orthogonal horizontal axis defined as regards to the north/south direction or, more simply, to the NETLANDER axes, one may keep the same notations as in Fig. 1. The  $\mathbf{k}$  vector is always characterized by a polar angle  $\theta$  and azimuthal angle  $\phi$ . The plane defined by  $\mathbf{k}$  and  $O_z$  is now the incidence plane. For the sake of convenience, we note  $E_{||}$  and  $E_{\perp}$  the components of the wave electric field parallel and perpendicular to that incidence plane. Similar notations ( $H_{||}$  and  $H_{\perp}$ ) are adopted for the wave magnetic field.

According to the previous definition, the polarization characteristics may be described by three parameters:

- the modulus  $|p|$  and the phase  $\beta$  of a complex polarization parameter given by

$$p = E_{\perp}/E_{||} = |p|e^{i\beta}. \quad (10)$$

- the modulus of one of the field component:  $E_{\perp}$  or  $E_{||}$ .

Three parameters are also required in all the other ways of characterizing the polarization (amplitude of the axes

Table 1  
Wavelength and damping factor of the electromagnetic wave for different frequency and permittivity values, in three different situations

f (MHz)	$\lambda$ (m)			$Y$ ( $\times 10^{-3}$ )			$1/Y$ (m)		
(a) <i>Conductive permafrost layer</i>									
	$\epsilon_r = 1$	$\epsilon_r = 3.5$	$\epsilon_r = 10$	$\epsilon_r = 1$	$\epsilon_r = 3.5$	$\epsilon_r = 10$	$\epsilon_r = 1$	$\epsilon_r = 3.5$	$\epsilon_r = 10$
2	66.9	58.5	43.8	84	73	55	11.9	13.6	18.2
3	53.1	43.8	30.4	100	82	57	10.0	12.1	17.5
4	44.8	35.0	23.2	113	88	58	8.9	11.4	17.2
$\sigma = 10^{-3}$									
(b) <i>Resistive permafrost layer</i>									
2	149.8	80.2	47.4	188	101	60	531	993	1678
3	100.0	53.5	31.6	188	101	60	531	993	1678
4	75	40.1	23.7	188	101	60	531	993	1678
$\sigma = 10^{-5}$									
(c) <i>Dry almost non conductive medium</i>									
2	150	80.2	47.4	189	101	60	530.5	992.5	1677.6
3	100	53.5	31.6	189	101	60	530.5	992.5	1677.6
4	75	40.1	23.7	189	101	60	530.5	992.5	1677.6
$\sigma = 10^{-8}$									

of the polarization ellipses + inclination angle, Stokes parameters).

Being given these definitions, the  $E_x$ ,  $E_y$  and  $E_z$  field components are calculated by mapping  $E_{\perp}$  and  $E_{\parallel}$  in the  $O_{xyz}$  coordinate system. One obtains

$$E_x = E_{\parallel}(|p|e^{i\beta} \sin \phi + \cos \theta \cos \phi),$$

$$E_y = E_{\parallel}(-|p|e^{i\beta} \cos \phi + \cos \theta \sin \phi),$$

$$E_z = E_{\parallel}(-\sin \theta). \quad (11)$$

Now, considering: (i) that the minor axis of the polarization ellipse of the  $\mathbf{E}$  vector is the major axis of the polarization ellipse of the  $\mathbf{H}$  vector and vice versa; and (ii) that the amplitudes of the  $\mathbf{E}$  and  $\mathbf{H}$  vectors are related by the equation

$$Z_0 = E/H = E_{\parallel}/H_{\perp} = E_{\perp}/H_{\parallel}, \quad (12)$$

one has for the components of the magnetic field

$$H_x = E_{\parallel}(\sin \phi - |p|e^{i\beta} \cos \theta \cos \phi)/Z_0,$$

$$H_y = -E_{\parallel}(\cos \phi + |p|e^{i\beta} \cos \theta \sin \phi)/Z_0,$$

$$H_z = E_{\parallel}(|p|e^{i\beta} \sin \theta)/Z_0. \quad (13)$$

Then, by applying (1) one obtains the six components  $\xi_i$  of the generalized electric field  $\xi$ .

The wave energy density  $\rho$  is a function that does not depend on the polarization and propagation parameters. It is given by

$$\rho = 0.5 (\epsilon_0 E^2 + \mu_0 H^2). \quad (14)$$

By applying Eqs. (1) and (3) for each mode  $m$  one obtains a set of 36 complex  $a_{ij}$  kernels ( $a_{ji}^* = a_{ij}$ ). As an example, one get for the four first kernels:

$$a_{11} = \psi[|p|^2 \sin^2 \phi + \cos^2 \theta \cos^2 \phi + |p| \cos \theta \sin^2 \phi \cos \beta],$$

$$a_{12} = \psi[-|p|^2 \sin \phi \cos \phi + \cos^2 \theta \cos \phi \sin \phi + |p|e^{i\beta} \cos \theta \sin^2 \phi - |p|e^{-i\beta} \cos \theta \cos^2 \phi],$$

$$a_{13} = \psi[-|p|e^{i\beta} \sin \theta \sin \phi - \cos \theta \sin \phi \cos \phi],$$

$$a_{14} = \psi[|p|e^{i\beta} \sin^2 \phi - |p|e^{-i\beta} \cos^2 \theta \cos^2 \phi$$

$$+ \cos \theta \cos \phi \sin \phi - |p|^2 \cos \theta \sin \phi \cos \phi], \quad (15)$$

with  $\psi = E_{\parallel}^2/\rho$ .

Provided there are a finite number of propagation modes, the direct problem associated to the NETLANDER experiment can be summarized by the set of integral equations (2) in which the  $a_{ijm}$ 's are the new kernels in each mode  $m$ .

### 3.3. On the resolution of the Inverse problem

The determination of  $F_m(\omega_0, \cos \theta, \phi)$  from a set of  $S_{ij}$ 's is an inverse problem that has infinitely many solutions. It is relatively easy to solve in the case where a single polarization mode is present. In such a case,  $E_{\parallel}$  is a constant which enters in an unknown normalization factor (the wave energy density  $\rho$  is not known) which does not affect the determination of the directions of arrival. The parameters  $|p|$  and  $\beta$  are variables on which one may iterate to find the solution  $F(\omega_0, \cos \theta, \phi)$  that provides with the best fit to the data. For each couple of  $|p|$  and  $\beta$  values a WDF solution may be obtained e.g. via the Lefeuvre (1977) maximum entropy method. For doing, so provided the  $a_{ij}$  kernels are modified, one may even use the Delannoy and Lefeuvre (1979) software. The output parameter which measures the fit to the data depends on the variance errors in the data. Obviously, the inverse problem is still simpler if first estimates of the  $|p|$  and  $\beta$  values are a priori known.

In the hypothesis of two propagation modes, the same procedure may be used. However, the level of complexity of the problem is strongly increased since one estimates six polarization parameters (three for each mode). For a greater number of modes one may find other procedures.

Now, whatever the degree of complexity of the inverse problem, the resolution in  $\theta$  and  $\phi$  depends on the number of independent data, i.e. of the independent kernels. In a magnetoplasma, when the six electromagnetic field components are available, there is a maximum of 33 independent kernels. The number is reduced to 31 in the whistler mode approximation. It is decreased to 15 when, as it is often the case, six wave field components are available.

Let us compare these numbers with the ones from our new kernels. By carefully examining the real (Re) and imaginary (Im) parts of each of them, one points out the following linear relationships:

$$\text{Re}(a_{14}) + \text{Re}(a_{25}) + \text{Re}(a_{36}) = 0, \quad (16)$$

$$a_{11} + a_{22} + a_{33} = a_{44} + a_{55} + a_{66}, \quad (17)$$

$$\text{Im}(a_{12}) = \text{Im}(a_{45}),$$

$$\text{Im}(a_{13}) = \text{Im}(a_{46}),$$

$$\text{Im}(a_{15}) = \text{Im}(a_{24}),$$

$$\text{Im}(a_{16}) = \text{Im}(a_{34}),$$

$$\text{Im}(a_{23}) = \text{Im}(a_{56}),$$

$$\text{Im}(a_{26}) = \text{Im}(a_{35}). \quad (18)$$

As a result, for six electromagnetic field components one may rely on 27 independent data, and for four electromagnetic field components (e.g. the four horizontal ones) one has 15 independent data. In other words, one is about in the same situation as in space plasmas.

#### 4. Conclusion

In the present paper, we have recalled the Storey and Lefeuvre (1979, 1980) wave distribution function (WDF) concept, derived all the required algebraic expressions to apply it to the determination of the directions of arrival of the radar echoes from the measurement of electric and magnetic wave field components, and discussed validity conditions.

The main condition to be fulfilled is that the radar echoes received on the electric and magnetic antennas be considered as propagating waves. This means that the reflection zones be at distances much larger than the observed wavelengths. According to the frequencies emitted by the radar, and to the hypothesis one can presently make about the geophysical materials, we have shown that this supposes a thickness of

the dry layer between the surface and the first reflector much greater than  $\sim 80$  m.

A difficult point we partially solved is the number of co-existing polarization modes. Assuming the reflection does not modify the polarization characteristics one can consider that the returned waves have the polarization of the emitted waves. However, this has not been demonstrated so far. Now, one may expect that emitting pulses with different polarizations, and analyzing the echoes in several narrow-band frequencies between 2 and 4 MHz we may expect to obtain cases where a dominant polarization is present.

Other conditions to be fulfilled are: (i) to have electric and magnetic antennas sensitive enough to detect the radar echoes, and (ii) to have observation times long enough to get accurate estimations of the auto- and cross- power spectra of the field components. One may probably get good results from the measurements of the horizontal field components only. But, better accuracies in the direction of arrival may be obtained if one can use the vertical components also.

Obviously, further studies are needed, particularly to model the Mars sub-surface, including the rock (sediment) –permafrost interface.

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