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# On the Direct Identification of Propagation Modes for HF Plasma Waves

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We examine the problem of interpreting the direct measurement of the polarization sense for HF plasma waves, in a cold and collisionless magnetoplasma when two propagation modes are simultaneously present. Two types of propagation mode estimators are considered. The first, generally more sensitive to quasi-parallel waves, consists in the determination of the signs of cross-spectra between two electric or two magnetic wave field components, both perpendicular to each other and to  $B_0$ . The second, more sensitive to quasi-perpendicular waves, is obtained by testing the ratio of the parallel to the perpendicular energy, against physical thresholds. Interpretations of the results are shown to be a function of the wave and plasma parameters. Applications performed on synthetic data constructed from two propagation mode models with different wave distribution functions (WDF) are presented. They show that the combination of the different propagation mode estimators often enables us to point out the presence of the two modes. Moreover, when the WDFs of the two modes are very different and when the wave frequency is far from the characteristic frequencies of the medium, the main propagation properties of each mode may be derived.

## 1. INTRODUCTION

The direct identification of the magnetoionic mode of high-latitude wave emissions is still controversial. As discussed by *Stix* [1962], cold-plasma theory predicts four distinct electromagnetic modes of propagation at frequencies above the ion gyrofrequency. These modes are the free-space *L-O* mode (left-hand polarized, ordinary mode), the free-space *R-X* mode (right-hand polarized, extraordinary mode), the whistler mode, and the *Z* mode. With the exception of a few regions in the Clemmow-Mullaly-Allis (CMA) diagram, two modes may coexist [*Stix*, 1962; *Allis et al.*, 1963].

At present, the controversy is particularly strong as to the mode of auroral kilometric radiation (AKR). Because the radiation escapes freely from the Earth, it must be propagated in the free-space modes. Several polarization studies, mainly based on comparisons between the wave frequencies and the characteristic frequencies of the medium, conclude that the radiation is generated in the *R-X* mode [*Gurnett and Green*, 1978; *Kaiser et al.*, 1978; *Benson and Calvert*, 1979]. However, *Oya and Morioka* [1983] have presented evidence that radiation is propagated in the *L-O* mode. One explanation proposed by *Benson* [1984] is that the *R-X* mode emanates from low-density cavities above the auroral regions and that the *L-O* mode is dominant in regions of relatively high electron density. Direct polarization measurements performed on DE 1, using orthogonal dipole electric antennas oriented such that, from time to time, they become perpendicular to the local magnetic field vector  $B_0$ , have pointed out the presence of *R-X* modes [*Shawhan and Gurnett*, 1982]. The detection of the *L-O* mode in other regions is obviously not excluded. The point is that the identification of the AKR wave mode in satellite wave observations is important to the development of a correct theory for the AKR generation mechanism; for a review on AKR, see *Grabbe* [1981] and *Oya and Morioka* [1983].

As shown by *Lefeuvre et al.* [1986], the mode that is

identified from electric or magnetic wave field component measurements is not always the one that conveys the maximum wave energy density. Suppose that the *L-O* and the *R-X* modes are simultaneously present in the wave observations presented by *Shawhan and Gurnett* [1982]. As the *R-X* mode gives the sense of polarization, one may be led to conclude that the *R-X* mode conveys the most energy. However, this may not be the case. According to *Lefeuvre et al.* [1986], the kind of estimators used by *Shawhan and Gurnett*, first, may be strongly biased in the vicinity of the critical frequencies, and so probably around the electron gyrofrequency where the AKR is supposed to be generated [*de Feraudy et al.*, 1987], and, second, may favor the mode corresponding to waves having the wave normal direction  $K$  along the Earth's magnetic field  $B_0$ . Then, before interpreting polarization measurements, it is indispensable to evaluate the possible bias of the estimators and to find a way to check whether oblique waves propagating in the *L-O* mode are simultaneously present with the *R-X* mode or not.

The aim of the present paper is to give a general approach allowing us to answer these requirements. No hypothesis is made, a priori, about the distribution of the angle  $\theta$  between  $K$  and  $B_0$ . The study is not restricted to AKR phenomena but is extended to all HF waves, i.e., to all waves with frequencies well above the proton gyrofrequency. The medium is supposed to be a cold collisionless magnetoplasma. In the absence of satellite data with more than three wave field components, we have tested our ideas on synthetic data only. But the methods described here will find full applications in future satellite missions such as Interball, where three magnetic wave field components and one electric field component will be transmitted to the ground in the HF range. Furthermore, they will be of prime importance in the design of future satellite missions when selecting which field components to measure and transmit to the ground.

The plan of the paper is as follows: Section 2 recaptures the dispersion relation and the characteristic frequencies of the medium. The wave distribution function (WDF) concept is used to evaluate the contribution to the sense of polarization, when different waves modes are simultaneously present. It is shown that two polarization estimators (for

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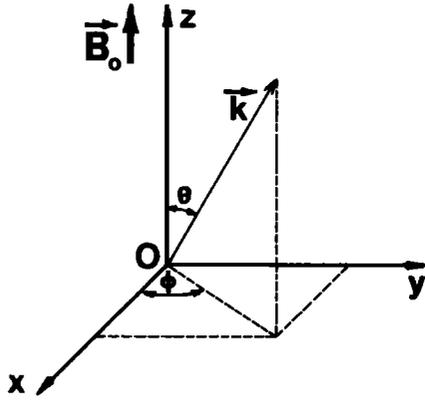


Fig. 1. Cartesian coordinate system used in the paper.

quasi-parallel and quasi-perpendicular waves) can give the polarization mode in the general case where the  $\theta$  distribution is unknown. Examples of synthetic data are given. The validity domains and thresholds needed to compare to experimental data are given in section 3. Section 4 offers some general conclusions.

## 2. ESTIMATORS OF THE POLARIZATION SENSE

### The Dispersion Relations

A Cartesian coordinate system  $Oxyz$  (Figure 1) is assumed, in which the  $z$  axis lies along  $\mathbf{B}_0$ , the  $x$  axis is in the local magnetic meridian plane and points in the direction away from the Earth, and the  $y$  axis is oriented eastward. In this system, the wave normal direction  $\mathbf{K}$  is characterized by the polar angle  $\theta$  and the azimuthal angle  $\phi$ , as measured from the local magnetic meridian plane. Considering variations of the electric and magnetic fields proportional to  $\exp i(2\pi ft - \mathbf{K} \cdot \mathbf{r})$ , and taking  $\phi = 0$ , the dispersion relation is expressed as

$$\begin{pmatrix} S - n^2 \cos^2 \theta & iD & n^2 \cos \theta \sin \theta \\ -iD & S - n^2 & 0 \\ n^2 \cos \theta \sin \theta & 0 & P - n^2 \sin^2 \theta \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0 \quad (1)$$

where  $n$  is the refractive index ( $n = kc/2\pi f$  with  $c$  the velocity of light). The quantities  $S$ ,  $D$ , and  $P$  are defined as by *Stix* [1962], but for frequencies well above the proton gyrofrequency they can be written

$$\begin{aligned} S &= \frac{1}{2}(R + L) & D &= \frac{1}{2}(R - L) \\ R &\approx 1 - \frac{f_{pe}^2}{f(f - f_{ce})} & L &\approx 1 - \frac{f_{pe}^2}{f(f + f_{ce})} \\ P &\approx 1 - \frac{f_{pe}^2}{f^2} \end{aligned}$$

where  $f_{ce}$  and  $f_{pe}$  are the electron gyrofrequency and plasma frequency.

The main types of electromagnetic plasma waves observed in high-latitude auroral regions are represented in the CMA diagram [*Stix*, 1962] in Figure 2. Since the wave frequency is high, the ion effect is negligible, and we do not consider regions above 8. The  $R$ - $X$  mode is a fast extraordi-

nary mode, whose low-frequency cutoff is the  $R = 0$  cutoff in the CMA diagram:

$$f_x = \sqrt{f_{pe}^2 + \left(\frac{f_{ce}}{2}\right)^2} + \frac{f_{ce}}{2} \quad (2)$$

The low-frequency cutoff of the  $L$ - $O$  mode is at the electron plasma frequency  $f_{pe}$ , i.e.,  $P = 0$  in the CMA diagram. The  $Z$  mode is a slow extraordinary mode bounded by the upper hybrid resonance ( $S = 0$  in the CMA diagram):

$$f_{UHR} = \sqrt{f_{pe}^2 + f_{ce}^2} \quad (3)$$

and the  $L = 0$  cutoff:

$$f_z = \sqrt{f_{pe}^2 + \left(\frac{f_{ce}}{2}\right)^2} - \frac{f_{ce}}{2} \quad (4)$$

The  $Z$  mode radiation may be a right-hand ( $f_{pe}/f < 1$ ) or a left-hand ( $f_{pe}/f > 1$ ) polarized mode. The whistler mode is limited by either  $f_{pe}$  or  $f_{ce}$ , whichever is smaller. Other representations of auroral wave emission can be seen in Figure 2 of *Gurnett et al.* [1983] and in Figures 3 and 4 of *Grabbe* [1981]. From the CMA diagrams in Figure 2, it is seen that the  $R$ - $X$  mode and the  $L$ - $O$  mode can coexist, as well as the  $L$ - $O$  mode and the  $Z$  right-handed mode, or the  $Z$  left-handed mode and the whistler mode.

### The Use of the WDF Concept

Assuming that  $Oz$  is parallel to  $\mathbf{B}_0$  (Figure 1), the electric and magnetic wave field components are combined for the sake of convenience in a generalized electric vector  $\boldsymbol{\varepsilon}$  whose components are

$$\boldsymbol{\varepsilon}_{1,2,3} = E_{x,y,z} \quad \boldsymbol{\varepsilon}_{4,5,6} = Z_0 H_{x,y,z} \quad (5)$$

with  $Z_0$  being the wave impedance in free space. The WDF specifies how the wave energy is distributed relative to the two angles  $\theta$  and  $\phi$ , the wave frequency  $f$ , and the propagation mode  $m$  (ordinary or extraordinary). It is written  $F_m(f, \cos \theta, \phi)$  and defined as being everywhere nonnegative. [*Storey and Lefeuvre*, 1979]. At a given wave frequency  $f_0$ , it is related to the autospectra and cross-power spectra of the six wave field components by the expression

$$S_{ij}(f_0) = \frac{\pi}{2} \sum_m \iint a_{ijm}(f_0, \cos \theta, \phi) F_m(f_0, \cos \theta, \phi) d\sigma \quad (6)$$

The integral is taken over the surface of a unit sphere, where  $d\sigma = d \cos \theta d\phi$  is the solid-angle element.  $S_{ij}$  is either the autospectrum ( $i = j$ ) or the cross-spectrum ( $i \neq j$ ) between the  $\boldsymbol{\varepsilon}_i$  and  $\boldsymbol{\varepsilon}_j$  components. The coefficients  $a_{ijm}$ , which are the kernels of the integral equations, are directly related to the corresponding autospectra and cross-spectra for an elementary plane wave in the magnetoionic mode  $m$ . Their algebraic expression, derived from the Maxwell equations, is given by *Storey and Lefeuvre* [1980]. They implicitly depend on the plasma parameters  $f_{pe}$  and  $f_{ce}$ . The kernels whose expressions are needed here are listed in the appendix.

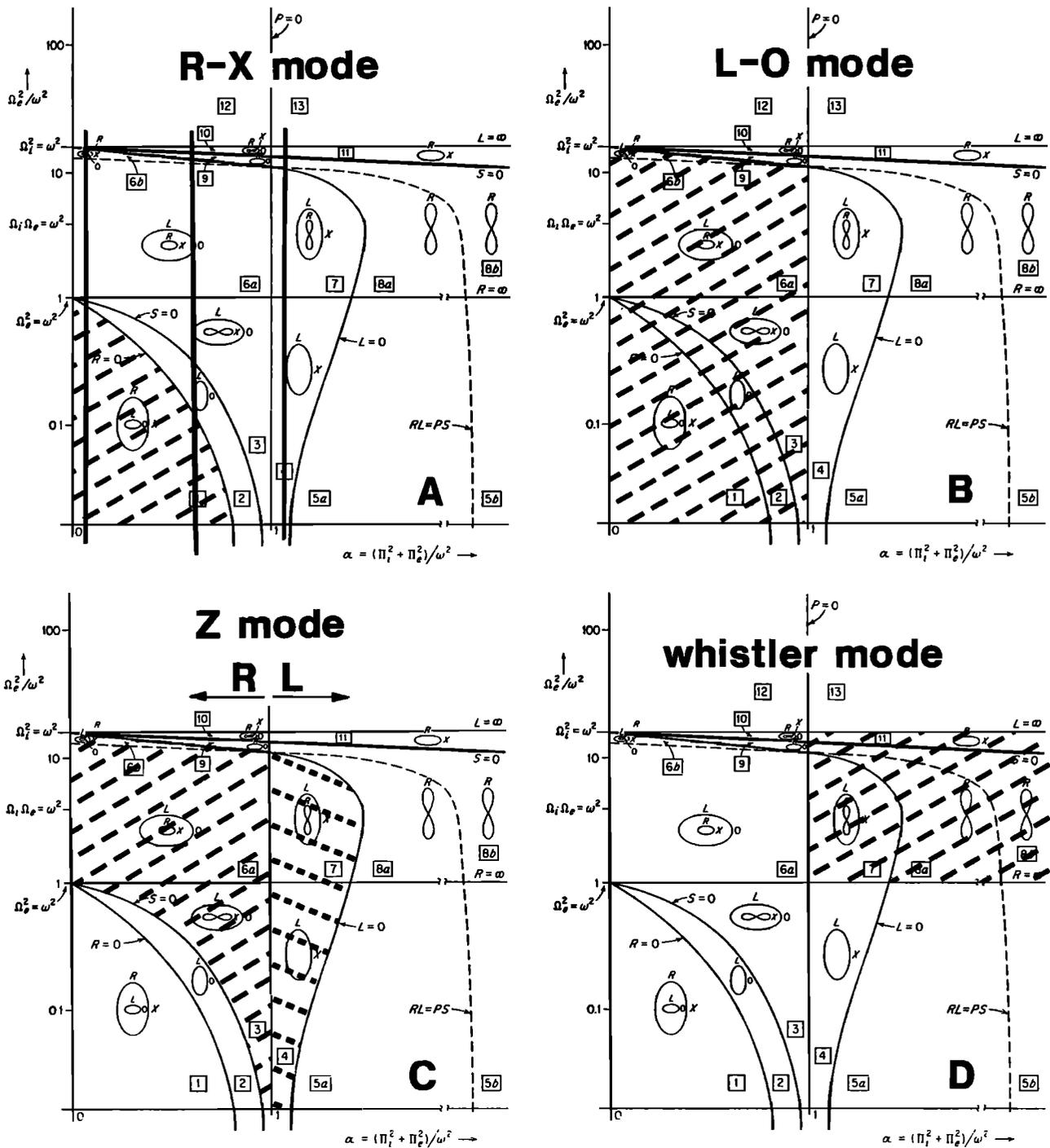


Fig. 2. CMA diagram from Stix [1962] with the main types of electromagnetic waves observed in the Earth's polar magnetosphere and with negligible ion effect. Hatched areas represent (a) the R-X mode; (b) the L-O mode; (c) the Z mode with right-hand polarization on the left of  $f = f_{pe}$  and left-hand polarization on the right of  $f = f_{pe}$ ; and (d) the whistler mode.

**Polarization Estimator for Quasi-Parallel Waves**

Lefeuvre et al. [1986] pointed out that for quasi-parallel waves, the signs of  $\text{Im}(a_{12m})$  and  $(n^2 - S)/D$  are the same in most regions of the CMA diagram: positive for right-handed and negative for left-handed waves. However, in a few regions the situation is more complicated, as shown by Allis et al. [1963]. In the regions denoted 3 and 10 by Stix [1962]

(5 and 11 by Allis et al.), the quantity  $(n^2 - S)/D$  has the same sign for the two magnetoionic modes. This means that in those regions it is impossible to identify the propagation mode of a wave field from the cross-power spectral measurements of two electric components taken in a plane perpendicular to  $\mathbf{B}_0$ . In regions denoted 6-a and 12 by Stix (13 and 7-a by Allis et al.), the situation is even more complicated. Here, the quantity  $(n^2 - S)$  takes a zero value for a critical

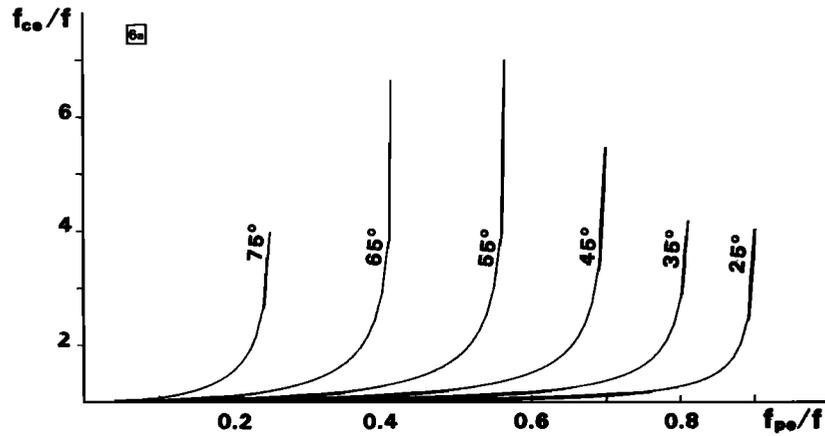


Fig. 3. Representation of  $n^2 = S$  in region 6-a of the CMA diagram (Stix) for different  $\theta_c$  values. When  $f_{ce}/f \gg 1$  the asymptotic solution is given by  $\cos \theta = f_{pe}/f$ .

angle  $\theta_c$ , given by  $\sin^2 \theta_c = P/S$ . Examples of  $\theta_c$  values in region 6-a of the Stix diagram are displayed in Figure 3. For  $n^2 < S$  to the right of the figure, it is impossible to identify the mode because the signs for the two modes are identical, while for  $n^2 > S$  to the left, the  $R$  mode as well as the  $L$  mode can be distinguished. With the exception of the regions mentioned above, we obtain the relations for the cross-power spectra of the two components of the electric field orthogonal to  $\mathbf{B}_0$ :

$$\begin{aligned} \text{Im}(S_{12}) > 0 &\leftrightarrow R \text{ waves} \\ \text{Im}(S_{12}) < 0 &\leftrightarrow L \text{ waves} \end{aligned} \quad (7)$$

A null value is associated either with a linear polarization or with an equal contribution of the  $L$  and  $R$  waves to the sign of  $\text{Im}(S_{12})$ .

The sign of  $\text{Im}(a_{45m})$  also characterizes the polarization of quasi-parallel waves. It is the same as the sign of the quantity  $(n^2 - S)P/[D(P - n^2 \sin^2 \theta)]$ . Substituting for  $\text{Im}(a_{45m})$  in (6), we get the relations for the cross-power spectra of the two components of the magnetic field orthogonal to  $\mathbf{B}_0$ :

$$\begin{aligned} \text{Im}(S_{45}) > 0 &\leftrightarrow R \text{ waves} \\ \text{Im}(S_{45}) < 0 &\leftrightarrow L \text{ waves} \end{aligned} \quad (8)$$

which are independent of  $\theta$  [Marouan, 1988] and valid in all regions of the CMA diagram without any restriction. A null value is associated either with a linear polarization or with an equal contribution of the  $L$  and  $R$  waves to the sign of  $\text{Im}(S_{45})$ .

One may evaluate the contribution of each mode to the sign of  $\text{Im}(S_{12})$  and  $\text{Im}(S_{45})$  by plotting  $\text{Im}(a_{12m})$  and  $\text{Im}(a_{45m})$  versus  $\theta$ . One example was given in Figure 2 of Lefeuvre et al. [1986], and further examples are presented in Figure 4 for a range of plasma parameters. It can be observed that the estimators generally are more sensitive to small  $\theta$  values. In region 1 of the CMA diagram (Figure 4: panels A<sub>1</sub>, A<sub>3</sub>, B<sub>1</sub>, and B<sub>3</sub>), electric as well as magnetic components can be used to determine the sense of polarization. For the case shown in Figure 4, panels A<sub>1</sub> and B<sub>1</sub>, the contributions of the  $R$  and  $L$  modes are approximately the same with a slight tendency to enhance the  $L$  mode. Unless

a very intense wave energy density is conveyed at large  $\theta$  waves, the sign of  $\text{Im}(a_{12})$  and  $\text{Im}(a_{45})$  is controlled by small  $\theta$  waves. The reader will note that in the  $R$  mode the  $\text{Im}(a_{12m})$  coefficient is not null at  $\theta = 90^\circ$ , which means that large  $\theta$  waves may contribute to the sign of  $\text{Im}(S_{12})$ .

For the case shown in panels A<sub>3</sub> and B<sub>3</sub>, the variation with  $\theta$  is completely different for the electric and the magnetic components. When considering first the magnetic ones, it can be seen that a large scale factor exists between  $\text{Im}(a_{45m})$  in the  $L$  mode and in the  $R$  mode. The  $L$  mode is clearly dominant, with the consequence that unless the wave energy density in the  $L$  mode is negligible relative to the wave energy density in the  $R$  mode, it is the  $L$  mode only that contributes to the sign of  $\text{Im}(S_{45})$ . In other words, it is practically impossible to detect the presence of the  $R$  mode when the two modes coexist. As far as the electric measurements are concerned, the main point is the nearly constant value of  $\text{Im}(a_{12m})$  in the  $R$  modes. At small  $\theta$  values the  $L$  mode is dominant, but as soon as  $\theta > 30^\circ$ , only the  $R$  waves contribute to the sign of  $\text{Im}(S_{12})$ , and it becomes impossible to use the sign of  $\text{Im}(S_{12})$  as an estimator of the sense of polarization for quasi-parallel waves.

The variations of  $\text{Im}(a_{45m})$  in region 3 of the CMA diagram are illustrated in Figure 4, panel B<sub>4</sub>. At smaller  $\theta$  values the  $R$  mode is limited by resonance angle. Since  $\text{Im}(a_{12m})$  cannot be used as an estimator in this region, as mentioned above, it has not been shown.

Two examples are given for region 6-a of the CMA diagram in Figure 4, panels B<sub>2</sub> and B<sub>5</sub>. Once more, only the magnetic estimator can be used. For the case of a low plasma frequency, shown in panel B<sub>2</sub>, a slight tendency exists to enhance the  $L$  mode, whereas for the case of a higher plasma frequency, shown in panel B<sub>5</sub>, the  $L$  mode has clearly the more important contribution to the sign of  $\text{Im}(S_{45})$ , at least for  $\theta$  values less than  $50^\circ$ . The reader will note that the maximum contribution of the  $R$  waves is at  $\theta = 30^\circ$  and not at  $\theta = 0^\circ$ .

#### Polarization Estimators for Quasi-Perpendicular Waves

Intuitively the polarization sense for perpendicular waves may be estimated from the quantities

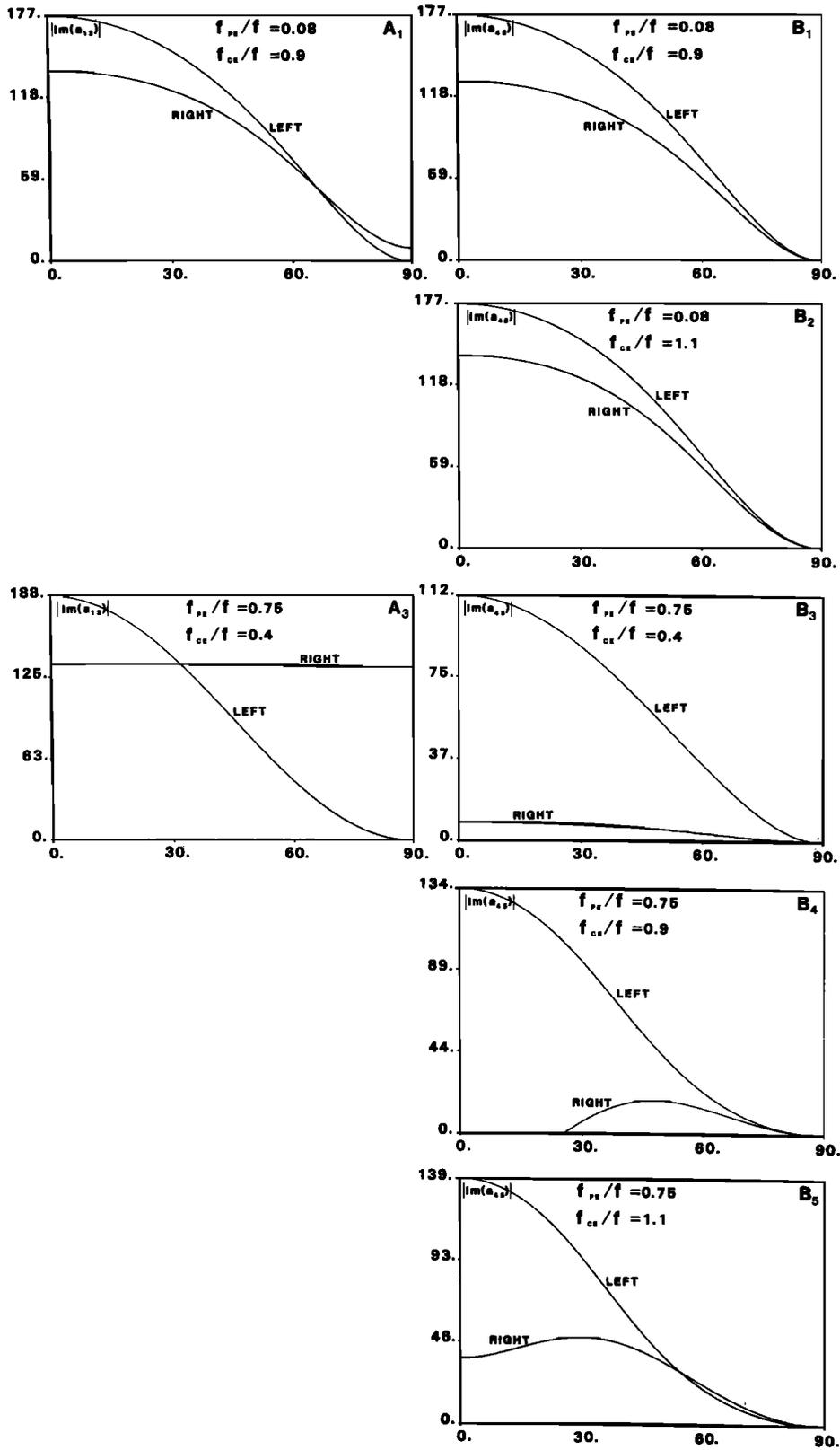


Fig. 4. Variations of the absolute values of  $Im(a_{12})$  and  $Im(a_{45})$  versus  $\theta$ . The wave frequency is  $f = 300$  kHz. We have (A<sub>1</sub> and B<sub>1</sub>)  $f_{pe}/f = 0.08$  and  $f_{ce}/f = 0.9$ , (B<sub>2</sub>)  $f_{pe}/f = 0.08$  and  $f_{ce}/f = 1.1$ , (A<sub>3</sub> and B<sub>3</sub>)  $f_{pe}/f = 0.75$  and  $f_{ce}/f = 0.4$ , (B<sub>4</sub>)  $f_{pe}/f = 0.75$  and  $f_{ce}/f = 0.9$ , and (B<sub>5</sub>)  $f_{pe}/f = 0.75$  and  $f_{ce}/f = 1.1$ . In each panel, right-hand polarization and left-hand polarization are considered. The units are arbitrary.

TABLE 1. Expressions of the Kernels of the Appendix in Special Cases

$\theta$	$n^2$	$\text{Im}(a_{12})$	$\text{Im}(a_{45})$	$a_{33}$	$a_{11} + a_{22}$	$a_{66}$	$a_{44} + a_{55}$
$0^\circ$	$R$	$\alpha R^2 \nu_R$	$\alpha R \nu_R$	0	$2\alpha \nu_R R^2$	0	$2\alpha \nu_R R$
$0^\circ$	$L$	$-\alpha L^2 \nu_L$	$\alpha L \nu_L$	0	$2\alpha \nu_L L^2$	0	$2\alpha \nu_L L$
$90^\circ$	$P$	0	0	$2\alpha \nu_O P$	0	0	$2\alpha \nu_O P$
$90^\circ$	$RL/S$	$\frac{\alpha}{2\nu_X} \left( \frac{1}{R^2} - \frac{1}{L^2} \right)$	0	0	$\frac{\alpha}{\nu_X} \left( \frac{1}{R^2} + \frac{1}{L^2} \right)$	$2 \frac{\alpha}{\nu_X} \frac{S}{RL}$	0

Here  $\alpha = 1/\epsilon_0$  and  $\nu_i = 1/2 \omega^3 \{\partial/\partial\omega [-(\omega n)^{-2}]\}_i$ ;  $i = R, L, O$  or  $X$ .

$$P_E = \frac{S_{33}}{S_{11} + S_{22}} \quad (9)$$

$$P_H = \frac{S_{66}}{S_{44} + S_{55}}$$

which represent the ratios of the parallel to the perpendicular energy for the electric and magnetic wave fields. For  $\theta \rightarrow \pi/2$ ,  $P_E \rightarrow \infty$  and  $P_H \rightarrow 0$  for  $O$  polarization ( $n^2 \rightarrow P$ ), whereas  $P_E \rightarrow 0$  and  $P_H \rightarrow \infty$  for  $X$  polarization ( $n^2 \rightarrow RL/S$ ). In the following we will show that there are thresholds in  $P_E$  and  $P_H$  in certain cases allowing the identification of both the  $O$  and the  $X$  modes.

Consider first the following quantities:

$$P'_E(\theta)_m = \frac{a_{33m}(\theta)}{a_{11m}(\theta) + a_{22m}(\theta)} \quad m = O, X \quad (10)$$

$$P'_H(\theta)_m = \frac{a_{66m}(\theta)}{a_{44m}(\theta) + a_{55m}(\theta)} \quad m = O, X$$

Examples of the functions  $P'(\theta)_m$  are shown in Figure 5 (solid lines) for the wave and plasma parameters corresponding to Figure 4. Their characteristics can be understood with the aid of the expressions for  $a_{ij}$  listed in Table 1, from which it follows that  $P'_E(0)_X = P'_E(\pi/2)_X = 0$ . With the exception of regions 3, 10, 6-a, and 12, discussed in the previous section,  $P'_E(\theta)_X$  is bounded by a maximum value  $Q_E$  at  $\theta = \theta_E$ . Similarly for the magnetic components,  $P'_H(0)_O = P'_H(\pi/2)_O = 0$ . In all regions of the CMA diagram,  $P'_H(\theta)_O$  is bounded by a maximum value  $Q_H$  at  $\theta = \theta_H$ . Formally,  $Q_E$  and  $Q_H$  can be expressed as

$$Q_E = \max [P'_E(\theta)_X] \quad 0 \leq \theta \leq \pi/2 \quad (11)$$

$$Q_H = \max [P'_H(\theta)_O] \quad 0 \leq \theta \leq \pi/2$$

For the electric component of the  $O$  mode we find that  $P'_E(0)_O = 0$  and  $P'_E(\pi/2)_O = \infty$ . Since the wave normal surfaces never cross, it follows that  $P'_E(\theta)_O > P'_E(\theta)_X$  for  $0 \leq \theta \leq \pi/2$ , again in all regions except the ones mentioned above. Similar arguments for the magnetic component of the  $X$  mode lead to  $P'_H(\theta)_X > P'_H(\theta)_O$ , which is valid in all regions.

A wave field may consist of both  $O$  mode and  $X$  mode. An example of the  $P$  values for a mixed wave field is shown with dotted lines in Figure 5. The field is composed of two WDFs identical to Dirac distributions and centered on the same value of  $\theta$ . One WDF is the  $O$  mode, and the other the  $X$  mode.

It can be observed that the  $P'$  functions are generally more sensitive to large  $\theta$  values. Region 1 of the CMA diagram is again the only one where electric as well as magnetic estimators can be used. For the case shown in Figure 5, panels A<sub>1</sub> and B<sub>1</sub>, the electric and magnetic curves are quite similar, and  $P'_{EX}$  and  $P'_{HO}$  reach their maximum ( $\approx 0.2$ ) at  $\theta_E \approx \theta_H \approx 50^\circ$ , whereas  $P'_{EO}$  and  $P'_{HX}$  are monotonic functions that run from zero at  $\theta = 0^\circ$  to  $\infty$  at  $\theta = 90^\circ$ .  $P'_{E(O+X)}$  and  $P'_{H(O+X)}$  are mean curves varying from 0 to 1. Referring to the relations (11), we see that  $P_E > 0.2$  indicates the presence of an  $O$  mode whereas  $P_H > 0.2$  identifies the presence of an  $X$  mode. It is likely that the hypothesis of a single propagating mode can be made for  $P_E$  or  $P_H \gg 0.2$ . The main contribution to the final value of the  $P_E$  and  $P_H$  estimators is due to large  $\theta$  waves, one possible definition of the large  $\theta$  waves being  $\theta > \theta_E, \theta_H$ . For the case shown in Figure 5, panels A<sub>3</sub> and B<sub>3</sub>, the situation is almost the same for  $P_H, P'_{HO}$  having a maximum  $\approx 0.2$  at  $\theta_H = 52^\circ$ . Due to the logarithmic scale, the  $P'_{EX}$  curve is not represented, but a threshold  $< 0.01$  may be adopted to point out the presence of the  $O$  mode (see Figure 11).

The situation in region 3 is illustrated in Figure 5, panel B<sub>4</sub>. There is a resonance angle for the  $X$  mode at  $\theta = 25^\circ$ . The maximum of the  $P'_{HO}$  curve is equal to  $\approx 0.1$  for  $\theta_H = 40^\circ$ . The two examples of region 6-a (panels B<sub>2</sub> and B<sub>5</sub>) have slightly different maxima, but do not present any particularity.

Returning to (11), it can be concluded that  $P_E$  values greater than  $Q_E$  indicate the presence of the  $O$  mode. The exact  $P_E$  value depends on  $\theta$  and on the wave energy density in each mode. In the same way,  $P_H$  values greater than  $Q_H$  correspond to the presence of the  $X$  mode. On the other hand, a value of  $P_E$  less than  $Q_E$ , or of  $P_H$  less than  $Q_H$ , cannot be readily interpreted since it may be due to parallel waves in any mode as well as to nonparallel waves in the  $X$  mode for  $P_E$  and in the  $O$  mode for  $P_H$ .

### The Mode Identification

Combining the results obtained for the quasi-parallel and quasi-perpendicular estimators, one generally obtains good information on the propagation mode present in the medium and on their propagation characteristics, without any hypothesis on the  $\theta$  distribution. This is illustrated in Figure 6 for two different values of the wave and plasma parameters. The symbols ( $\parallel$ ) and ( $\perp$ ) represent quasi-parallel and quasi-perpendicular propagation and are absent for propagation that is both parallel and perpendicular;  $\theta_r$  is the resonance angle of the  $R$ - $X$  mode at  $f_{pe}/f = 0.75$  and  $f_{ce}/f = 0.9$ . We assume that in regions where they can be compared, there is no inconsistency between the signs of  $\text{Im}(S_{12})$  and  $\text{Im}(S_{45})$ . Obviously, simultaneous values of  $P_E$  and  $P_H$ , such as

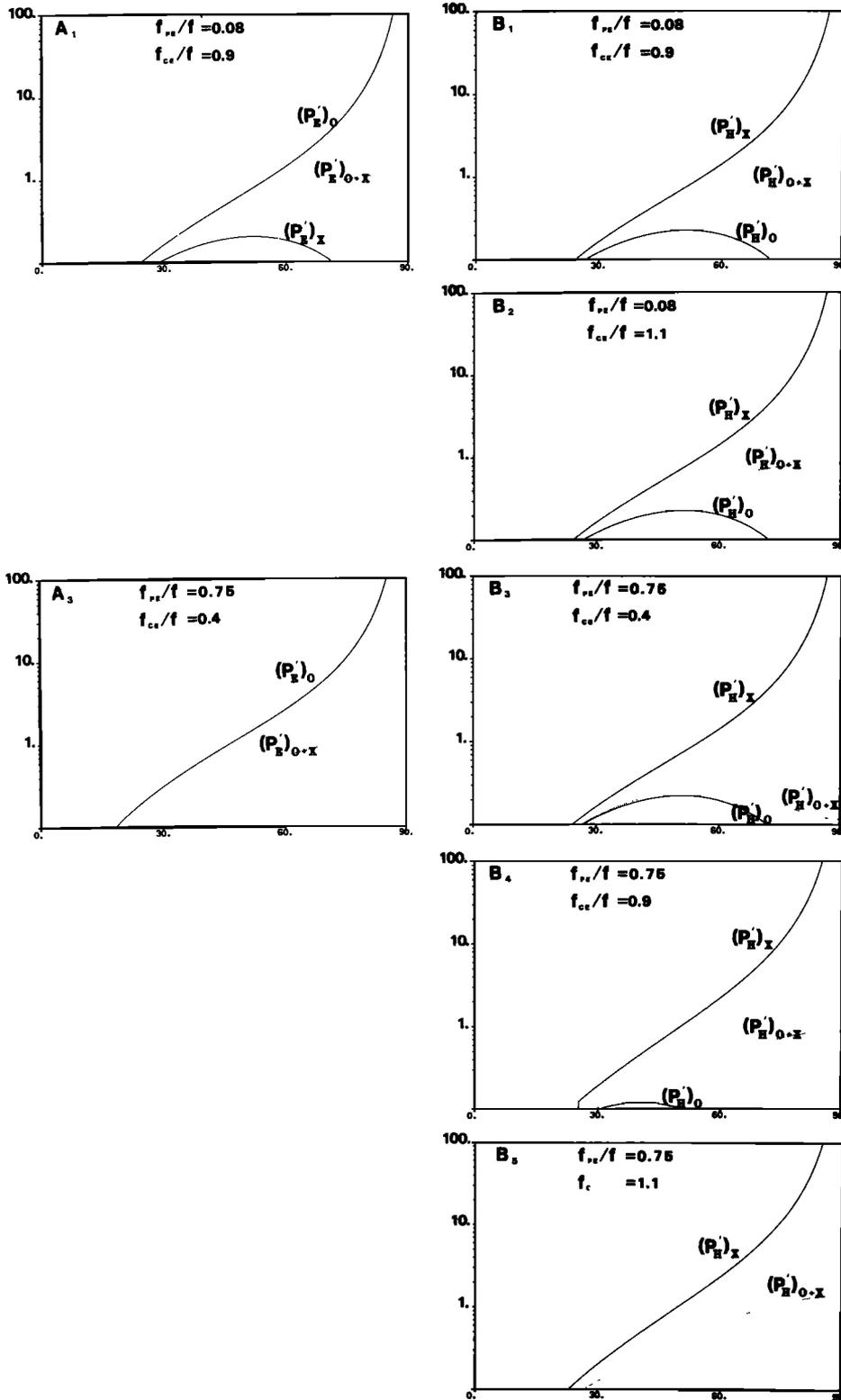


Fig. 5. Variations of  $(P'_E)_X$ ,  $(P'_E)_O$ ,  $(P'_H)_X$ , and  $(P'_H)_O$  versus  $\theta$  (solid line);  $(P'_E)_{X+O}$  and  $(P'_H)_{X+O}$  (dotted line). The plasma parameters are the same as in Figure 4.

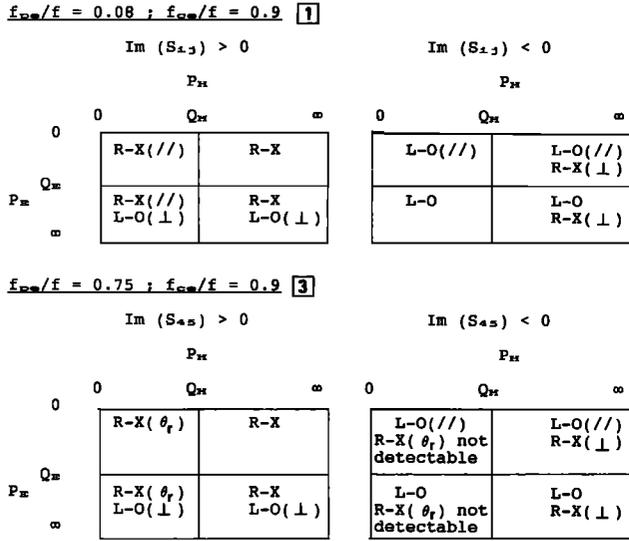


Fig. 6. Identification of the propagation modes and the main propagation characteristics when combining the quasi-parallel and quasi-perpendicular estimations of polarization. In the top panel,  $\text{Im}(S_{ij})$  means either  $\text{Im}(S_{12})$  or  $\text{Im}(S_{45})$ .

$P_E \gg Q_E$  and  $P_H \gg Q_H$ , correspond to cases with two modes.

All these considerations being taken into account, it is observed that, provided the WDFs in each mode have different  $\theta$  distributions, one can make the distinction between propagations in a single mode or in two modes, with indications on the general propagation characteristics: quasi-parallel or quasi-perpendicular. There are exceptions in cases with a dominant mode, i.e., when there is a large difference in the value of the  $\text{Im}(a_{12m})$  and  $\text{Im}(a_{45m})$  coefficients. In such cases there will always exist combinations of the sign of  $\text{Im}(S_{12})$  and  $\text{Im}(S_{45})$ , and of the value of  $P_E$  and  $P_H$ , for which it is impossible to see whether the nondominant mode is present or not. The case of a dominant mode is discussed further in section 3.

The technique can be applied to experimental data. Knowing the plasma parameters  $f_{ce}$  and  $f_{pe}$  and the wave frequency, one needs only to look to the sign of  $\text{Im}(S_{12})$  and/or  $\text{Im}(S_{45})$  and to calculate the ratios  $P_E$  and/or  $P_H$ . These ratios must be compared to thresholds evaluated as in (11). An example of the identification of the modes with synthetic data is given below.

*Examples of Applications*

As an example of application, consider five different models of WDF (see Figure 7) where waves in the  $R-X$  and in the  $L-O$  modes are simultaneously present. For the sake of simplicity, it is assumed that the WDF in each mode is constant over a given  $\theta$  interval, the central  $\theta$  values of the WDFs being different for the two modes. No information on the  $\Phi$  distribution is needed since  $\text{Im}(a_{12m})$  and  $\text{Im}(a_{45m})$  are not  $\Phi$  dependent. For each model, taking the plasma parameters of Figure 4, one obtains with (6) the values of  $\text{Im}[S_{12}(f)]$  and  $\text{Im}[S_{45}(f)]$ . Then, applying (7) and (8), one estimates a propagation mode. The set of results is summarized in Table 2a. In order to avoid too specific discussions, we have not considered the plasma parameters of Figure 4,

panel B<sub>4</sub>, for which a resonance angle has to be taken into account.

The following two points come out:

1. For wave frequencies very close to a critical frequency ( $f = f_X$  in panel B<sub>3</sub> of Figure 4) it is always the  $L-O$  mode which is identified, unless it has very little energy compared to the  $R-X$  mode, which is consistent with panel B<sub>3</sub> of Figure 4, where the  $\text{Im}(a_{45})$  values in the  $R-X$  mode are always negligible in relation to the  $\text{Im}(a_{45})$  values in the  $L-O$  mode.

2. There is no simple rule to determine which waves have the most important contribution to the signs of  $\text{Im}(S_{12})$  and  $\text{Im}(S_{45})$ .

The only way to improve our mode identification is to compute the values of the  $P_E$  and  $P_H$  estimators and to compare them to the  $Q_E$  and  $Q_H$  thresholds. This has been done for the same conditions as above, substituting the five WDF models into (6). The  $Q_E$  and  $Q_H$  values are directly derived from Figure 5. The set of all relevant values is given in Table 2b. The results of the tests are summarized in Table 2c. The dashes mean that  $P_E < Q_E$  or  $P_H < Q_H$ . We note that (1) in most cases,  $P_E$  and  $P_H$  values of the order of 0.2 are high enough to detect the presence of a given mode, and (2) as forecast,  $P_E$  values are used to point out the presence of one mode (here  $L-O$ ) whereas  $P_H$  values are used to point out the presence of the other mode (here  $R-X$ ), the two measurements being sometimes complementary (model 5).

Now, examining Tables 2a and 2c, we see that in most cases where electric and magnetic measurements can be compared ( $A_1$  and  $B_1$ ,  $A_3$  and  $B_3$ ) one identifies the simul-

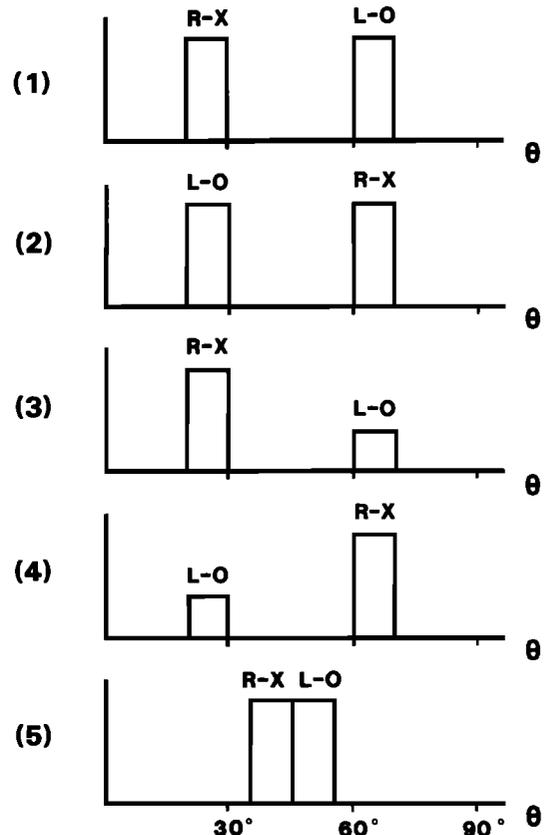


Fig. 7. Models of WDF used for a test on synthetic data. The units are arbitrary.

TABLE 2a. Results of the Test With Synthetic Data Obtained From Figure 7: Polarization Estimator for Quasi-Parallel Waves

Model	Region 1, Case A1	Region 1, Case A3	Region 1, Case B1	Region 6a, Case B2	Region 1, Case B3	Region 6a, Case B5
1	R-X	R-X	R-X	R-X	L-O	R-X
2	L-O	L-O	L-O	L-O	L-O	L-O
3	R-X	R-X	R-X	R-X	L-O	R-X
4	R-X	R-X	L-O	R-X	L-O	L-O
5	L-O	R-X	L-O	L-O	L-O	L-O

The first two columns refer to panels A<sub>1</sub> and A<sub>3</sub> of Figure 4 obtained for electric measurements, whereas the last four refer to panels B<sub>1</sub> to B<sub>5</sub> of Figure 4, obtained for magnetic measurements.

taneous presence of the two modes. In all the other cases, where we rely on the magnetic data only, the full identification depends on the sign of Im (S<sub>45</sub>). If it is negative, i.e., if it shows up the L-O mode, the P<sub>H</sub> estimator may point out the presence of the R-X mode. If it is positive, i.e., if it corresponds to the R-X mode, it is not necessary to estimate the P<sub>H</sub> value any more: it does not contain any supplementary information.

Returning to the ideal cases where magnetic plus electric data may be compared, most of the time one can obtain first approximations on the propagation characteristics in each mode. As an example, consider plasma parameters of panels A<sub>1</sub> and B<sub>1</sub> of Figures 4 and 5. Applying the strategy defined in Figure 6 and using the signs of Im (S<sub>ij</sub>) and the P<sub>E</sub>, P<sub>H</sub> values of Table 2, one observes that for model 1, the R-X mode is conveyed by rather parallel waves and the L-O mode by rather perpendicular waves; model 2 is the opposite of model 1; the propagation characteristics of model 3 are similar to those of model 1; the L-O mode of model 4 is not detected; and finally for model 5 we find L-O mode waves plus R-X mode waves propagating with K vectors nearly perpendicular to B<sub>0</sub>.

An experimental illustration of the interest of a good definition of the P<sub>E</sub> threshold can be found in the Viking data recorded by the V4H experiment [Bahnsen et al., 1987]. In this experiment, the measured electric field component is alternatively almost parallel, then perpendicular to the geomagnetic field B<sub>0</sub>. Then, assuming the wave field is time stationary, after half the spin rotation one may estimate P<sub>E</sub> (see equation (9)). This has been done for the event of Figure 1 of de Feraudy et al. [1987] just at the time (≈2033 UT) where the satellite seems to pass through the source region of AKR emissions. Assuming that the plasma frequency is given by the upper cutoff frequency of the VLF hiss [de

Feraudy et al., 1987] we obtain f<sub>pe</sub> = 20. kHz. As f<sub>ce</sub> = 208 kHz, when working at f = 233 kHz, we have f<sub>pe</sub>/f = 0.085 and f<sub>ce</sub>/f = 0.89, which is typically the case of our Figure 5, panel A<sub>1</sub>. The computation of P<sub>E</sub> gives P<sub>E</sub> = 1.12. Referring to Figure 5, panel A<sub>1</sub>, and equation (12), P<sub>E</sub> is compared against Q<sub>E</sub> = 0.20. Our conclusion is that, as suggested by de Feraudy et al. [1987], L-O mode waves are definitely present. Moreover, if we take the lower cutoff frequency of the event as well as the spin modulation pattern of the band as indicators of the presence of R-X mode waves, we find, with de Feraudy et al., that AKR emissions in the source regions are the sum of L-O and R-X mode waves. However, to be affirmative about the presence of R-X mode waves, one would need to estimate the quantities (7), (8), and (9).

### 3. THE VALIDITY DOMAINS

#### The Dominant Modes

To determine the wave and plasma parameters for which the quasi-parallel polarization estimators are strongly biased toward one of the two modes (see Figure 4, panel B<sub>3</sub>), we define the quantities

$$R_{12} = \frac{\text{Im} [a_{12}(\theta)]_L}{\text{Im} [a_{12}(\theta)]_R} \tag{12}$$

$$R_{45} = \frac{\text{Im} [a_{45}(\theta)]_L}{\text{Im} [a_{45}(\theta)]_R}$$

For R<sub>12</sub> and R<sub>45</sub> greater than 1 the L mode is called the dominant mode, while for R<sub>12</sub> and R<sub>45</sub> less than 1 the R mode is dominant. When one mode is dominant, the polarization

TABLE 2b. Results of the Test With Synthetic Data Obtained From Figure 7: Polarization Estimator for Quasi-Perpendicular Waves

Model	Case A1, Q ≈ 0.22	Case A3, Q ≈ 10 <sup>-3</sup>	Case B1, Q ≈ 0.22	Case B2, Q ≈ 0.12	Case B3, Q ≈ 0.21	Case B5, Q ≈ 0.10
	P <sub>E</sub>	P <sub>E</sub>	P <sub>H</sub>	P <sub>H</sub>	P <sub>H</sub>	P <sub>H</sub>
1	0.70	0.81	0.14	0.12	0.16	0.06
2	0.13	0.11	0.54	0.59	0.14	0.64
3	0.38	0.37	0.12	0.11	0.15	0.08
4	0.13	0.06	0.94	1.05	0.23	1.23
5	0.33	0.39	0.32	0.32	0.23	0.30

The column heads refer to panels of Figure 5.

TABLE 2c. Results of the Test With Synthetic Data Obtained from Figure 7: Polarization Estimator for Quasi-Perpendicular Waves

Model	Case A1	Case A3	Case B1	Case B2	Case B3	Case B5
1	<i>L-O</i>	<i>L-O</i>	...	...	...	...
2	...	<i>L-O</i>	<i>R-X</i>	<i>R-X</i>	...	<i>R-X</i>
3	<i>L-O</i>	<i>L-O</i>	...	...	...	...
4	...	<i>L-O</i>	<i>R-X</i>	<i>R-X</i>	<i>R-X</i>	<i>R-X</i>
5	<i>L-O</i>	<i>L-O</i>	<i>R-X</i>	<i>R-X</i>	<i>R-X</i>	<i>R-X</i>

The column heads refer to panels of Figure 5.

estimators tend to favor this mode although the other mode may be of larger energy density, thereby rendering the mode identification difficult.

In order to study the behavior of  $R_{12}$  and  $R_{45}$  for different regions of the CMA diagram, the  $f_{pe}/f$  ratio was fixed, and the  $f_{ce}/f$  ratio was taken as variable. Three  $f_{pe}/f$  values are considered: 0.08 (the left vertical line in Figure 2a), 0.75 (the middle vertical line), and 1.2 (the right vertical line).

To have an idea of the  $\theta$  dependence, the computations of  $R_{12}$  and  $R_{45}$  are done for one large  $\theta$  value (solid curve) and for one small  $\theta$  value (dashed line). Generally one takes  $\theta = 80^\circ$  and  $\theta = 5^\circ$ , respectively, but those values may be modified in regions with either a resonance angle  $\theta_r$  or an angle  $\theta_c$  for which  $n^2 = S$ . The  $R_{12}$  quantity was estimated even within regions of the CMA diagram where the identification of the propagation modes is impossible from the electric measurements.

Figure 8a shows the variation of  $R_{12}$  as a function of  $f_{ce}/f$  for  $f_{pe}/f = 0.08$ . For  $R_{12} > 1$ , there is a tendency to observe the left-handed polarized mode, even if the right-handed polarized mode has the same wave energy density; for  $R_{12} < 1$  the tendency is to observe the right-handed mode. As  $f_{ce}/f$  increases, one passes from region 1 in the CMA diagram to region 6 (regions 2 and 3 are not seen in the graph due to the  $f_{pe}/f$  value and the frequency bin). In region 1 there is an equal contribution of the two modes for  $f_{ce}/f < 0.3$ . When  $f_{ce}/f$  approaches 1, large  $\theta$  waves have a higher contribution in the *R-X* mode than in the *L-O* mode, whereas low  $\theta$  waves show a higher contribution in the *L-O* mode close to  $f_{ce}/f = 1$  only. When  $f < f_{ce}$ , the *L-O* mode is greater than the *Z* right-handed mode, particularly for the large  $\theta$  waves. Note that for  $f_{ce}/f$  values between 1.0 and 1.123, the quantity  $n^2 - S$  is negative at  $\theta = 80^\circ$ , which means that no mode identification is possible, at least from the electric measurements. Figure 8b concerns the magnetic measurements ( $R_{45}$ ). The  $\theta = 5^\circ$  and  $\theta = 80^\circ$  solutions are practically identical, except in a very narrow domain around  $f = f_{ce}$  where the *L-O* mode is dominant above  $f_{ce}$  as well as below.

More important effects are seen for  $f_{pe}/f = 0.75$  (Figure 9). When the electric measurements are considered (Figure 9a), no mode identification is possible between the  $R = 0$  cutoff ( $f_X$  frequency) and the electron gyrofrequency with our method. This poses no problem for the interval between  $f_X$  and  $f_{UHR}$  (region 2 of the CMA diagram) since only one single mode propagates here. However, the interval between  $f_{UHR}$  and  $f_{ce}$  corresponds to region 3, for which the interpretation of the sign of  $\text{Im}(S_{12})$  is ambiguous. For  $f_{ce}/f > 1$ , in region 6-a, the mode identification is again not possible up to a  $f_{ce}/f$  value given by  $\theta$  (see Figure 3). The latter constraint explains why there is no  $\theta = 80^\circ$  curve below  $f_{ce}$  on the present graph. In any case, the interpretation of the sign of

$\text{Im}(S_{12})$  in this region of the CMA diagram is generally impossible since one does not know beforehand the position of the distribution in  $\theta$ , with respect to the  $\theta_c$  angle. For  $f_{ce}/f < 1$ , it is the mode with the highest wave energy that gives the sign of  $\text{Im}(S_{12})$  if the waves propagate with small  $\theta$  values. But as soon as large  $\theta$  waves are present in the *R-X* mode (for instance,  $\theta > 30^\circ$  at  $f_{ce}/f = 0.4$  in Figure 4, panel B<sub>3</sub>), they control the sign of  $\text{Im}(S_{12})$ .

The mode identification is easier with magnetic measurements (Figure 9b). The *L-O* mode is dominant below  $f_{ce}/f_X$ , the small  $\theta$  values being more sensitive to this mode. Above  $f_{ce}/f_{UHR}$  the frequency domain for which the  $R_{45}$  curves may be drawn is a function of the  $\theta$  value under consideration. It starts from the vicinity of  $f_{ce}/f_{UHR}$  for a large  $\theta$  value, but from  $f_{ce}/f = 1$  for small  $\theta$  values. Although the interpretation is difficult, it is clear that at  $f_{ce}/f$  values below  $\approx 1.5$  the *L-O* mode is dominant, whereas above this value it is the *R-X*

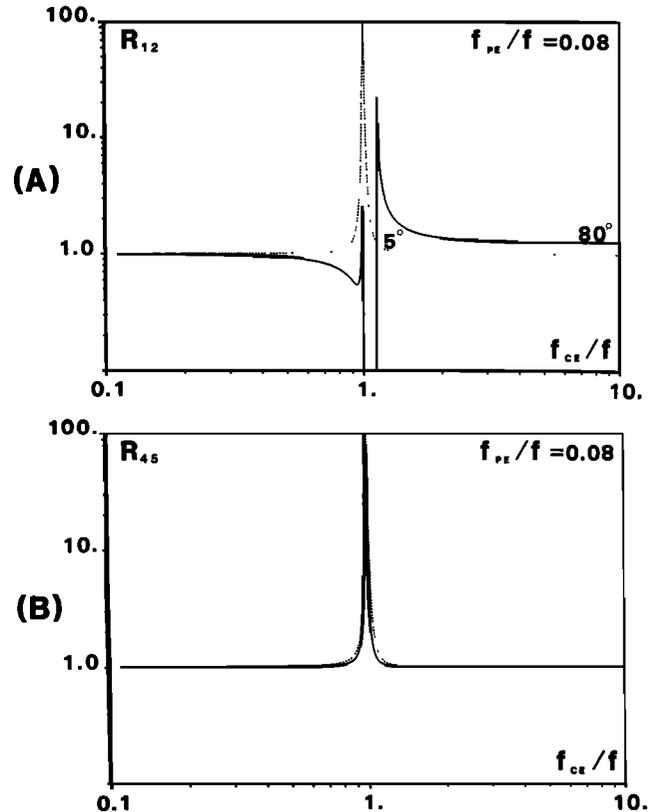


Fig. 8. (a) Ratio between  $\text{Im}(a_{12})$  in the *L* mode and  $\text{Im}(a_{12})$  in the *R* mode as a function of  $f_{ce}/f$ , with  $f_{pe}/f = 0.08$ ,  $f = 300$  kHz. The solid line corresponds to  $\theta = 80^\circ$ , and the dashed line to  $\theta = 5^\circ$ . (b) Same as Figure 8a, but for  $\text{Im}(a_{45})$ .

mode. For  $f_{ce}/f$  values in between  $f_{ce}/f_{UHR}$  and 1.0, there is no propagation of small  $\theta$  waves due to the resonance angle.

At  $f_{pe}/f = 1.2$  (Figure 10), a mode identification is required for  $f_{ce}/f > 1$  only (region 7). Due to the existence of a resonance angle the large  $\theta$  value has been fixed at  $20^\circ$ . The solutions obtained for the electric measurements are displayed in Figure 10a. The left-handed polarized mode ( $Z$  mode here) is dominant over a large frequency domain. As far as the solutions obtained for the magnetic measurements are concerned (Figure 10b), they favor the  $Z$  mode just below  $f_{ce}$ . At smaller frequencies, the  $Z$  mode is very slightly dominant for the largest  $\theta$  waves (which here are very small), whereas it is the whistler mode for the smallest  $\theta$  waves.

#### The $P_E$ and $P_H$ Thresholds

The variations of the  $P_{EX}^i$  and  $P_{HO}^i$  maximum values denoted  $Q_E$  and  $Q_H$ , respectively, as regards  $f_{ce}/f$ , are shown in Figure 11 for  $f_{pe}/f = 0.08$  ( $A_1, B_1$ ),  $0.75$  ( $A_2, B_2$ ), and  $1.2$  ( $A_3, B_3$ ). Note that the  $Q_E$  values have not been plotted in regions where the tests on the parallel to perpendicular energy of the electric field are not relevant. So one can consider that in the present field of study, electric and magnetic thresholds are always below  $\approx 0.6$ . At fixed  $f_{pe}/f$  values, they both decrease when  $f_{ce}/f$  increases, and, conversely, at fixed  $f_{ce}/f$  value they both decrease when  $f_{pe}/f$  increases.

In any case, one is very far from the constraints when assuming plane waves at  $\theta = 90^\circ$  ( $P_E \rightarrow \infty$  for the  $O$  mode and  $P_H \rightarrow \infty$  for the  $X$  mode). Values of  $P_E$  and  $P_H$  greater than

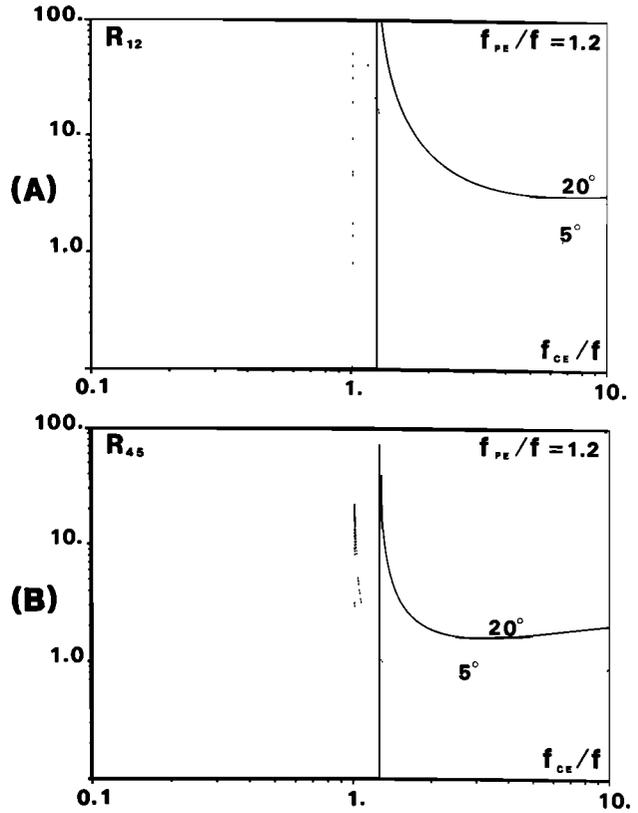


Fig. 10. (a) Same as Figure 8a but for  $f_{pe}/f = 1.2$ . The solid line corresponds to  $\theta = 20^\circ$ , and the dashed line to  $\theta = 5^\circ$ . (b) Same as Figure 10a but for  $\text{Im}(a_{45})$ .

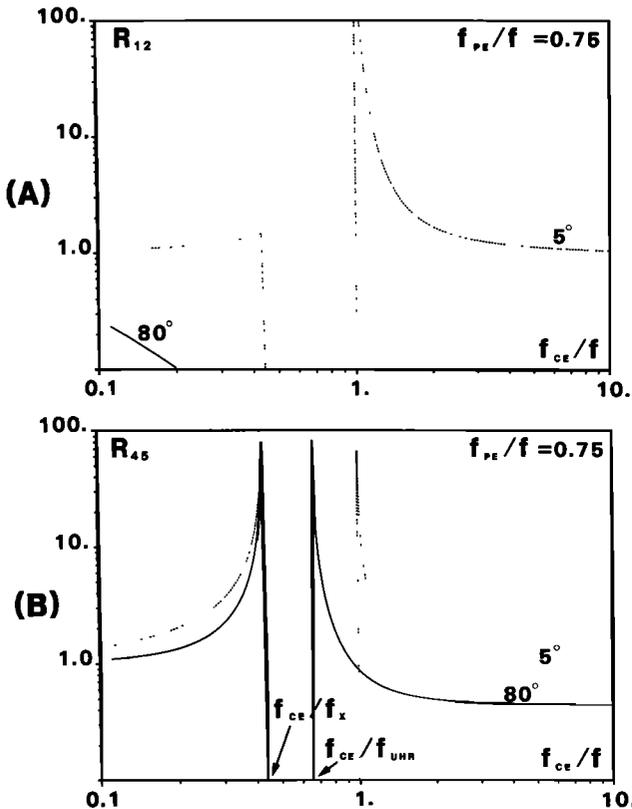


Fig. 9. (a) Same as Figure 8b but for  $f_{pe}/f = 0.75$ . The solid line corresponds to  $\theta = 80^\circ$ , and the dashed line to  $\theta = 5^\circ$ . (b) Same as Figure 8b but for  $f_{pe}/f = 0.75$ .

a fraction of unity are sufficient to point out the presence of waves propagating in  $O$  or  $X$  modes.

As an indication, Figure 12 shows the variations of the  $\theta_E$  and  $\theta_H$  angles;  $\theta_E$  values have not been plotted in regions where the polarization of the electric field is not readily interpretable. We see that the waves that contribute the most to the  $P_E$  and  $P_H$  values always have  $\theta$  values above  $70^\circ$  for the magnetic measurements but they can also include waves propagating at  $\theta \approx 20^\circ$  for the electric measurements ( $f_{pe}/f = 1.2$ ). The electric measurements, which were already found to provide a poor estimator of the polarization of quasi-parallel waves (see previous section), may also give rise to a poor estimator of the polarization of quasi-perpendicular waves.

#### 4. CONCLUSION

We have studied the problem of interpreting the direct measurements of polarization sense for HF plasma waves in a cold and collisionless magnetoplasma, in the case where two magnetoionic modes may simultaneously be present. It is shown that two types of propagation estimators could be considered.

The first estimator, generally more sensitive to the quasi-parallel waves, consists in the determination of the signs of  $\text{Im}(S_{12})$  and  $\text{Im}(S_{45})$ : the imaginary parts of measured cross-spectra between two electric or two magnetic wave field components, both perpendicular to each other and to the Earth magnetic field. According to the wave and plasma parameters at the point of measurement the estimator may be more sensitive to one mode than to the other. The

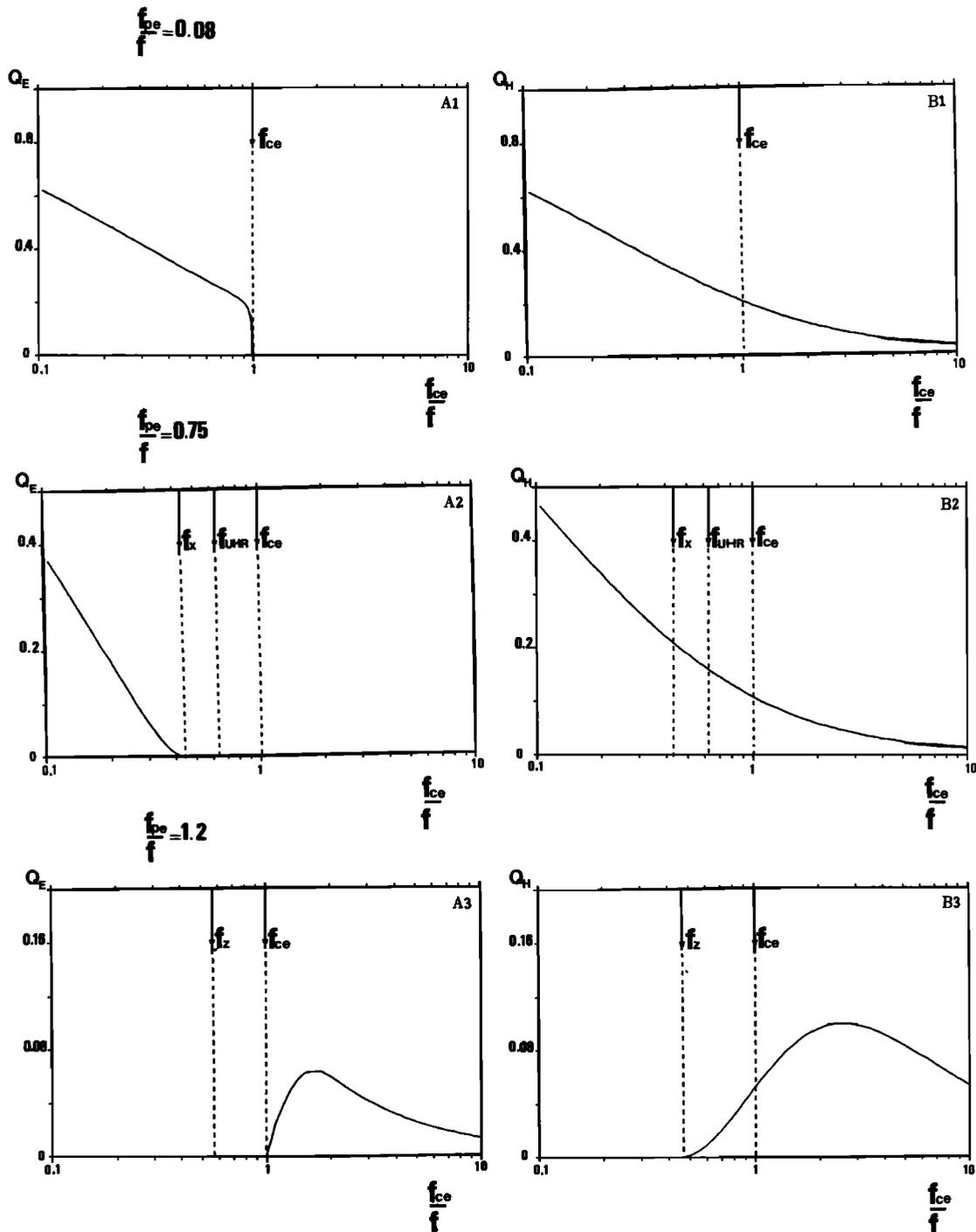


Fig. 11. Variations of  $Q_E$  and  $Q_H$  versus  $f_{ce}/f$  for  $f_{pe}/f = 0.08$  (A<sub>1</sub>, B<sub>1</sub>),  $f_{pe}/f = 0.75$  (A<sub>2</sub>, B<sub>2</sub>), and  $f_{pe}/f = 1.2$  (A<sub>3</sub>, B<sub>3</sub>). The wave frequency is  $f = 300$  kHz.

phenomenon may be so important at the vicinity of the characteristic frequencies of the medium (electron gyrofrequency, upper hybrid frequency,  $R = 0$  cutoff frequency, etc.) that the identification of a given mode may be impossible, even when it conveys the largest wave energy density. The estimator used for the electric measurements ( $\text{Im}(S_{12})$ ) presents two serious drawbacks: first, it does not apply in specific regions of the CMA diagram, and second, it may be sensitive to quasi-perpendicular waves. There is no restriction for the estimator used for the magnetic measurements ( $\text{Im}(S_{45})$ ).

The second estimator, more sensitive to the quasi-perpendicular waves, is obtained by testing the values  $P_E$  and  $P_H$  of the ratio of the parallel to the perpendicular energy, in the electric and magnetic wave field components, against thresholds. A major result of the study is the definition of thresholds for electric as well as for magnetic measurements. Their values, which depend on the wave and plasma parameters, are always much lower than 1. For instance, with the parameters used for Figures 4 and 5,  $P_E$  and  $P_H$  values greater than 0.2 are sufficient to establish the existence of  $O$  and  $X$  polarization, respectively. Infinite (or large) values of  $P_E$  and

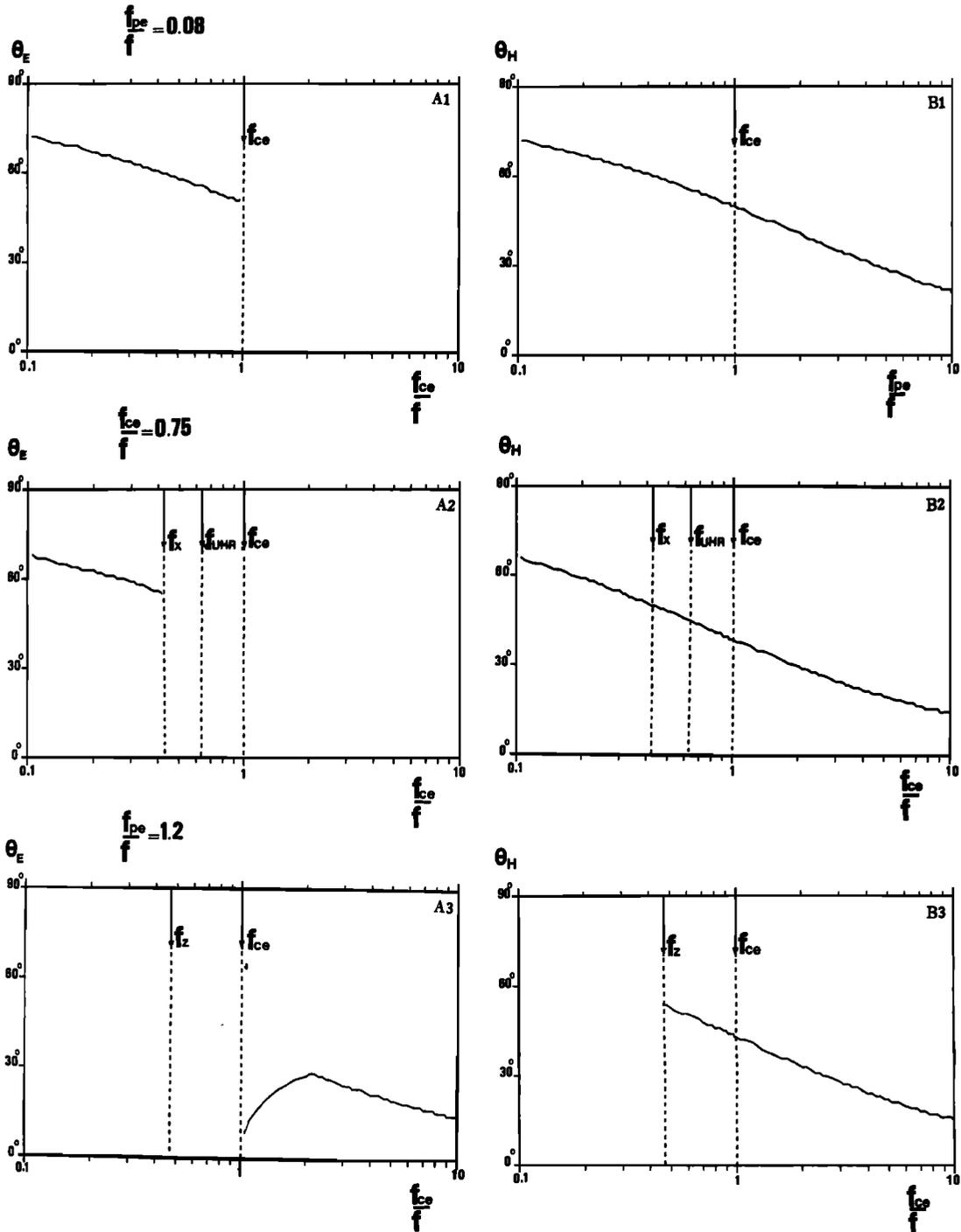


Fig. 12. Variations of  $\theta_E$  and  $\theta_H$  versus  $f_{c0}/f$  for  $f_{pe}/f = 0.08$  (A<sub>1</sub>, B<sub>1</sub>), 0.75 (A<sub>2</sub>, B<sub>2</sub>), and 1.2 (A<sub>3</sub>, B<sub>3</sub>). The wave frequency is  $f = 300$  kHz.

$P_H$  are not required. An example of application to Viking data has enabled us to check that in the source region, AKR emissions could be the sum of *L-O* and *R-X* waves.

Applications have been performed on synthetic data constructed from two-propagation mode models with different wave distribution functions. They show that the combination of the four estimators often enables us to point out the simultaneous presence of the two modes. Moreover, when the WDFs of the two modes are very different and when the wave frequency is far from the characteristic frequency of the medium, the main propagation characteristics in each

mode (propagation quasi-parallel or quasi-perpendicular) may be derived. But obviously there are cases where the use of the propagation mode estimators does not lead to reliable identifications. The only way to proceed is to perform a full determination of the WDFs in each mode, which has never been undertaken so far. As a consequence, if one wants to avoid any misinterpretations, one must carefully study the properties of the propagation mode estimators—as we did—with the wave and plasma parameters in hand, before starting the analysis of real data.

As long as the waveforms of the electric and magnetic

wave field components of HF waves are not simultaneously recorded before being transmitted to the ground, it will not be possible to fully apply the techniques proposed here to real data. However, this will become possible in the near future with the Soviet Interball project.

#### APPENDIX

Here we give expressions of the kernels  $a_{ij}$  used in this paper, as a function of the plasma parameters.

$$\begin{aligned} \text{Im}(a_{12}) &= \frac{(n^2 - P)^2}{n^4(n^2 - L)^2} (n^2 - S) D \chi \\ \text{Im}(a_{45}) &= \frac{(n^2 - P)^2 \cos^2 \theta}{n^2(n^2 - L)^2} (n^2 - S) D \frac{P}{P - n^2 \sin^2 \theta} \chi \\ a_{33} &= \frac{(n^2 - R)^2 \cot^2 \theta}{n^4} \chi \\ a_{11} + a_{22} &= \frac{(n^2 - P)^2 (n^2 - R)^2}{2n^4} \left\{ \frac{1}{(n^2 - R)^2} + \frac{1}{(n^2 - L)^2} \right\} \chi \\ a_{66} &= \frac{(n^2 - P)^2 (R - L)^2 \sin^2 \theta}{4n^2(n^2 - L)^2} \chi \\ a_{44} + a_{55} &= \frac{\cos^2 \theta}{2n^2(n^2 - L)^2} \\ &\quad \cdot \{(P - L)^2 (n^2 - R)^2 + (P - R)^2 (n^2 - L)^2\} \chi \end{aligned}$$

with

$$\begin{aligned} \chi &= 8[\varepsilon_0 \nu_g (\lambda' + \mu')]^{-1} \\ \lambda' &= 2(P - R)^2 \left\{ 1 + \frac{(P - L)^2 (n^2 - R)^2}{(P - R)^2 (n^2 - L)^2} \right\} \cos^2 \theta \\ \mu' &= \frac{(R - L)^2 (n^2 - P)^2}{(n^2 - L)^2} \sin^2 \theta \\ \nu_g &= \frac{1}{2} \omega^3 \frac{\delta}{\delta \omega} [-(\omega n)^{-2}] \end{aligned}$$

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