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# DIRECT AND ITERATIVE SPARSE LINEAR SOLVERS APPLIED TO GROUNDWATER FLOW SIMULATIONS

J. ERHEL \*, A. BEAUDOIN , AND J.-R. DE DREUZY †

**Abstract.** Subsurface hydraulic properties are mainly governed by the heterogeneity of geological formations. Moreover, it is not possible to measure the permeability in the whole site. In order to take into account the effects of heterogeneity on groundwater flows and the uncertainty in the input data, we use a probabilistic model in our numerical simulations. In the flow equations, governed by the Darcy law and mass conservation, the permeability is thus a stochastic given function. The Darcy velocity and the hydraulic head become then unknown stochastic functions. When dealing with fractured rocks, the domain itself and the geometry are random variables. With non intrusive methods such as Monte Carlo sampling or stochastic collocation, statistics of outputs are obtained through many simulations of deterministic equations. High performance computing is required to solve the flow equations. We consider two different cases. In porous media, the domain has a simple geometry. We use then a regular mesh and a finite volume method, leading to a structured sparse matrix. In fractured media, as the domain is a complex network of fractures, we use an unstructured mesh and a mixed hybrid finite volume method, leading to a general sparse matrix. In both cases, we compare direct methods, Krylov iterative methods and multigrid iterative methods, used to solve the linear systems obtained by discretizing the flow equations. We study the impact of the random distribution function on the condition number of the matrix and on the accuracy of the result. We also make a complexity analysis and a scalability analysis on parallel computers.

**Key words.** flow equation, heterogeneity, sparse linear solvers

**AMS subject classification.**

**1. Introduction.** groundwater flow and solute transport  
heterogeneous porous media and fractured media  
stochastic numerical models  
large sparse linear systems

**2. Numerical model.** flow equations  
probability distribution of permeability variance  $\sigma$   
discretization with 2D structured grids and finite volume method ; number of grid  
points  $N = n^2$   
Monte-Carlo method and non intrusive UQ methods  
sparse linear system  
scientific platform with interfaces to libraries and to Matlab  
fully parallel for large scale computations

**3. Accuracy.** theoretical estimation of condition number ; impact of variance  
and size : cond in  $O(e^\sigma)$  and in  $O(n)$   
numerical estimation with MUMPS or conddest : cond in  $O(e^{\sigma^2})$ , larger than  
expected  
scaling and componentwise estimations with Matlab : cond in  $O(e^\sigma)$  as expected  
estimation for  $\sigma = 1$  with MUMPS and scaling with Matlab : cond in  $O(n)$  as  
expected  
conclusion : good accuracy for  $\sigma \leq 3$  and for large  $n$ , up to 8000

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\*Supported by ACI GRID'5000

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**4. Complexity.** CPU and memory requirements; fill-in for direct methods and number of iterations for iterative methods  
 impact of variance and size  
 theoretical estimation of CPU time for direct methods :  $O(n^3)$  and no sensitivity to variance  
 numerical measure with UMFPACK : as expected  
 direct methods not sensitive to variance but very expensive in CPU and memory for large size  
 numerical measure with PCG / IC(0) :  
 not too sensitive to variance but number of iterations increases with size; plateau before superlinear convergence  
 PCG suitable only with multilevel preconditioner  
 multilevel methods : subdomain decomposition or multigrid  
 theory : number of V-cycles independent of  $n$ , complexity of multigrid in  $O(n^2)$   
 theory: geometric multigrid sensitive to  $\sigma$  and algebraic multigrid robust to  $\sigma$   
 numerical measure with HYPRE : as expected  
 SMG better for small  $\sigma$  and AMG better for large  $\sigma$

**5. Large scale computations.** Clusters with distributed memory ; SPMD and MPI ; fully parallel code with data distributed from scratch  
 experiments with P=16 and  $n$  up to 4096  
 measures on clusters from Grid'5000  
 same conclusions as in sequential case: SMG better for small  $\sigma$  and AMG better for large  $\sigma$   
 Able to solve systems with 16 millions in 1 minute  
 Monte-Carlo or UQ methods feasible ; multiparametric studies feasible ; done

**6. Future.** 3D computations with subdomain decomposition  
 Grid computing

x









