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Seismoelectric signals produced by mesoscopic heterogeneities: spectroscopic analysis of fractured media

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Key Points:

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11	•	Mesoscopic compressibility contrasts cause wave-induced fluid flow
12	•	Measurable seismoelectric signals are generated by wave-induced fluid flow

• Energy-based approach to study the seismoelectric conversion in fracture networks

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14 Abstract

In fluid-saturated porous rocks, the presence of mesoscopic heterogeneities such as, for 15 example, fractures, can produce measurable seismoelectric signals. The conversion of mechan-16 ical energy into electromagnetic energy is related to wave-induced fluid flow (WIFF) between 17 the heterogeneities and the embedding background. This physical mechanism is a well-known 18 cause of seismic attenuation, which exhibits a strong frequency dependence related to rock phys-19 ical and structural properties. Consequently, seismoelectric signals arising from WIFF are also 20 expected to depend on various material properties, such as the background permeability and 21 fracture characteristics. We present analytical and numerical approaches to study the effects 22 of mesoscopic heterogeneities on seismoelectric signals. We develop an energy-based approach 23 to quantify the total energy converted to seismoelectric signals at the sample scale. In partic-24 ular, we apply our theoretical framework to synthetic models of fractured rock samples and 25 study the spectral signature of the resulting seismoelectric signals. This study highlights the 26 influence of the mechanical and hydraulic properties, as well as the geometrical characteris-27 tics, such as degree of fracture connectivity, of the probed medium on the resulting seismo-28 electric signal. 29

30 **1 Introduction**

One common assumption in seismoelectric studies is that the involved media are homo-31 geneous at the sub-wavelength scale. However, most geological environments typically con-32 tain mesoscopic heterogeneities, that is, heterogeneities larger than the pore size but smaller 33 than the dominant seismic wavelength. In presence of contrasts in elastic compliance, the stress 34 field associated with a propagating seismic wave produces a pore fluid pressure gradient and, 35 consequently, wave-induced fluid flow (WIFF), which results in energy dissipation due to vis-36 cous friction. Indeed, WIFF is currently considered to be one of the major causes of seismic 37 wave attenuation in the upper part of the Earth's crust [e.g. Müller et al., 2010]. For this rea-38 son, efforts directed towards a better understanding of WIFF involving theoretical analyses [e.g., 39 Müller and Gurevich, 2005], laboratory measurements [e.g., Batzle et al., 2006; Tisato and Madonna, 40 2012; Subramaniyan et al., 2014], and numerical simulations [e.g., Masson and Pride, 2007; 41 Rubino et al., 2009; Solazzi et al., 2016] have been increasing during the last decades. WIFF 42 is a frequency-dependent physical process that is mainly controlled by the permeability, the 43 compressibility contrasts between the heterogeneities and the embedding background, and the 44 geometrical characteristics of the heterogeneities. These properties are of significant relevance 45 for flow and transport modeling, especially in fractured media [e.g. Berkowitz, 2002] and hence 46 the analysis of the impact of WIFF on seismoelectric signals is of broad interest. 47

Despite its potential importance, only few studies have focused on the understanding of 48 seismoelectric signals due to mesoscopic heterogeneities. In their pioneering numerical work, 49 Haartsen and Pride [1997] mention a significant signal enhancement when considering a thin 50 bed between two half-spaces, but no results or corresponding detailed physical explanations 51 are presented. Similarly, Haines and Pride [2006] showed that layers that are up to 20 times 52 thinner than the seismic wavelength could be detected by the seismoelectric method. More re-53 cently, Grobbe and Slob [2016] used numerical simulations to explore the enhancement of the 54 interface response in the contact of two half-spaces when thin beds are located in between these 55 half-spaces (Cite Grobbe and Slob in this book). Although they study constructive and destruc-56 tive interference resulting from different separations and thicknesses of thin beds, they do not 57 focus on the physical phenomena taking place in the thin bed itself. 58

A likely explanation why mesoscopic effects on the seismoelectric signal have so far been largely ignored in the scientific literature is high computational cost of corresponding numerical simulations. This cost is due to the fact that the dominant scales at which WIFF takes place, as characterized by the corresponding diffusion lengths, are much smaller than the prevailing seismic wavelengths. Recently, *Jougnot et al.* [2013] presented a new approach for studying the seismoelectric response of mesoscopic heterogeneities that circumvents this limitation. In-

stead of performing numerical simulations of wave propagation, they simulated the seismo-65 electric response of oscillatory compressibility tests on synthetic samples at different frequen-66 cies. Since the size of the probed sample can be much smaller than the seismic wavelengths, 67 this approach avoids the inherent problems related to the large contrasts in spatial scale be-68 tween seismic wavelengths and diffusion lengths. The work by Jougnot et al. [2013] thus opens 69 an avenue for detailed analyses of seismoelectric responses to mesoscopic heterogeneities. Here, 70 we extend and complement this study. We first describe the underlying theoretical framework 71 used to compute the seismoelectric response of an heterogeneous sample subjected to an os-72 cillatory compressibility test. Next, we present an energy-based approach to characterize the 73 seismoelectric response at the sample scale, as a substitute to relying on a certain experimen-74 tal setup, such as, for example, a particular electrode configuration. In order to gain insights 75 into the physical processes that contribute to the seismoelectric response in the presence of 76 mesoscopic heterogeneities, we proceed to explore an analytical solution for a rock sample con-77 taining a horizontal layer centered in an otherwise homogeneous rock in an initial case and 78 containing a fracture in a second analysis. We then perform a numerical sensitivity analysis 79 of the seismoelectric signals generated in 2D fractured media. For different fracture proper-80 ties, we present the dependence on frequency and space of the electrical potential amplitude, 81 as well as the frequency-dependent total energy converted to seismoelectric signal in an os-82 cillation cycle. 83

84 2 Theory

To explore the seismoelectric signals produced by mesoscopic heterogeneities, we em-85 ploy the methodology proposed by Jougnot et al. [2013]. We consider a 2D, fluid-saturated, 86 heterogeneous porous rock sample and study its seismoelectric response to an oscillatory com-87 pression (Fig. 1). The mechanical response of the probed sample is obtained by solving Biot's 88 (1941) quasi-static equations in the space-frequency domain with adequate boundary condi-89 tions. The resulting fluid velocity field is then used to calculate the seismoelectric response 90 of the sample. That is, we decouple the seismic and electrical problems [e.g., Haines and Pride, 91 2006; Jardani et al., 2010; Zyserman et al., 2010]. In the following, we present the details of 92 the proposed methodology. It is important to mention here that the extension of this approach 93 to 3D is conceptually straightforward, but computationally cumbersome. 94

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2.1 Mechanical response

Let $\Omega = (0, L_x) \times (0, L_y)$ be a domain that represents the probed 2D sample and Γ its boundary given by

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$$\Gamma = \Gamma^L \cup \Gamma^B \cup \Gamma^R \cup \Gamma^T, \tag{1}$$

where the subscripts L, R, B, and T stand for left, right, bottom, and top boundary, respectively,

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$$\Gamma^L = \{(x, y) \in \Omega : x = 0\},$$
 (2)

$$\Gamma^R = \{ (x, y) \in \Omega : x = L_x \}, \tag{3}$$

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$$\Gamma^B = \{(x, y) \in \Omega : y = 0\},$$
 (4)

$$\Gamma^T = \{(x, y) \in \Omega : y = L_y\}.$$
(5)

We apply a time-harmonic normal compression at the top boundary of the sample. The solid is neither allowed to move on the bottom boundary nor to have horizontal displacements on the lateral boundaries. No tangential forces are applied on the lateral boundaries, and the pore fluid is not allowed to flow into or out of the sample. Thus, the following boundary conditions are imposed

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$$\boldsymbol{\tau} \cdot \boldsymbol{\nu} = (0, -\Delta P), \quad (x, y) \in \Gamma^T,$$
 (6)

$$\boldsymbol{u} = \boldsymbol{0}, \quad (x, y) \in \Gamma^B, \tag{7}$$

(
$$\boldsymbol{\tau} \cdot \boldsymbol{\nu}$$
) $\cdot \boldsymbol{\chi} = 0, \quad (x, y) \in \Gamma^L \cup \Gamma^R,$ (8)

$$\mathbf{u} \cdot \boldsymbol{\nu} = 0, \quad (x, y) \in \Gamma^L \cup \Gamma^R, \tag{9}$$

$$\mathbf{w} \cdot \boldsymbol{\nu} = 0, \quad (x, y) \in \Gamma, \tag{10}$$

where ν denotes the unit outer normal on Γ and χ is a unit tangent so that $\{\nu, \chi\}$ is an orthonormal system on Γ . In addition, τ is the total stress tensor, u is the average displacement of the solid phase, and w is the relative fluid-solid displacement.

As we are interested in quantifying WIFF effects on the seismoelectric signal, the response of the sample subjected to the oscillatory compressibility test is obtained by solving Biot's (1941) quasi-static equations. This approach is valid because the physical process is controlled by fluid-pressure diffusion and, thus, inertial effects can be neglected. In the space-frequency domain, these equations can be written as

$$\nabla \cdot \boldsymbol{\tau} = 0, \tag{11}$$

(14)

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$$i\omega\frac{\eta}{k}\boldsymbol{w} = -\nabla p_f,\tag{12}$$

where $i = \sqrt{-1}$ is the imaginary number, ω the angular frequency, p_f the fluid pressure, kthe permeability, and η the fluid viscosity. Equation (11) represents the stress equilibrium within the sample, while Eq. (12) is Darcy's law. These two equations are coupled through the stressstrain relations

$$\boldsymbol{\tau} = \left(\lambda_u \nabla \cdot \boldsymbol{u} + \alpha_B M \nabla \cdot \boldsymbol{w}\right) \boldsymbol{I} + 2G^{fr} \boldsymbol{\epsilon},\tag{13}$$

 $p_f = -\alpha_B M \nabla \cdot \boldsymbol{u} - M \nabla \cdot \boldsymbol{w}.$

¹³³ In these equations, the involved coefficients are given by

$$M = \left[\frac{\alpha_B - \phi}{K^s} + \frac{\phi}{K^f}\right]^{-1},\tag{15}$$

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$$\alpha_B = 1 - \frac{K^{fr}}{K^s},\tag{16}$$

$$\lambda_u = K^{fr} + M\alpha_B^2 - \frac{2}{3}G^{fr},$$
(17)

where K^{fr} , K^s , and K^f are the bulk moduli of the solid matrix, the solid grains, and the fluid phase, respectively, λ_u is the undrained Lamé constant, ϵ is the strain tensor, ϕ is the porosity, and G^{fr} is the shear modulus of the bulk material, which is equal to that of the dry matrix.

The mechanical response of the sample subjected to the oscillatory compression is obtained by solving Eqs. (11) to (14) with the boundary conditions described by Eqs. (6) to (10). Since the methodology is based on Biot's (1941) quasi-static equations, it is limited to frequencies for which the resulting fluid flow is laminar. That is, the frequencies considered in the simulations should be smaller than Biot's (1962) critical frequency ω_c

$$\omega_c = 2\pi f_c = \frac{\phi\eta}{k\rho^f},\tag{18}$$

where ρ^f the density of the pore fluid.

In order to determine the spatial scales involved in the WIFF process in response to the applied oscillatory test, we consider a locally homogeneous medium. Without loss of generality, we explore the one-dimensional case for which the solid and relative fluid displacements, u_y and w_y , occur in the vertical direction. Combining Eqs. (11) and (13) as well as Eqs. (12) and (14) leads to

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$$\nabla^2 u_y = -\frac{\alpha_B M}{H_u} \nabla^2 w_y,\tag{19}$$

156 and

$$i\omega\frac{\eta}{k}w_y = \alpha_B M \nabla^2 u_y + M \nabla^2 w_y, \tag{20}$$

respectively. Next, substituting Eq. (19) into Eq. (20) results in

$$i\omega w_y = D\nabla^2 w_y. \tag{21}$$

Equation (21) is a diffusion equation with the diffusivity D given by

$$D = \frac{kN}{\eta},\tag{22}$$

where $N = M - \alpha_B^2 M^2 / H_u$, with H_u being the undrained plane-wave modulus. The spatial scale at which WIFF is significant is determined by the diffusion length

$$L_d \equiv \sqrt{D/\omega}.$$
(23)

165 **2.2 Electrical response**

The relative displacement between the pore fluid and the solid frame in response to the 166 applied oscillatory compression results in a drag on the electrical excess charges of the elec-167 trical double layer (EDL). This, in turn, generates a source or streaming current density $\mathbf{J}^{1,e}$. 168 Since the distributions of both the excess charge and the microscopic relative velocity of the 169 pore fluid are highly dependent on their distance to the mineral grains, not all the excess charge 170 is dragged at the same velocity. Correspondingly, a moveable charge density \hat{Q}_V^0 smaller than 171 the total charge density \underline{Q}_{V} has to be considered [e.g., Jougnot et al., 2012; \overrightarrow{Revil} and Mahardika, 172 2013; Revil et al., 2015; Jougnot et al., 2015]. Note that, in the literature, the moveable charge 173 density may also be referred to as effective excess charge and denoted by \bar{Q}_{e}^{eff} [e.g. Joug-174 not et al., 2012, 2015; Guarracino and Jougnot]. In the considered case, the source current den-175 sity takes the form [e.g., Jardani et al., 2010; Jougnot et al., 2013] 176

$$\mathbf{J}^{\mathrm{i,e}} = \underline{\hat{Q}}_{V}^{0} \mathrm{i} \omega \boldsymbol{w}, \qquad (24)$$

where $i\omega w$ is the relative fluid velocity. The moveable charge density formulation, which al-178 lows us to explicitly express the role played by the relative fluid velocity in the source cur-179 rent density generation, provides, for the same assumptions, equivalent results to the electroki-180 netic coupling coefficient formulation commonly used in the seismoelectric literature [e.g., Pride, 181 1994; Jouniaux and Zyserman, 2016]. The relationship between the moveable charge density 182 and the electrokinetic coupling coefficient can be found in many works [e.g. Revil and Leroy, 183 2004; Jougnot et al., 2012, 2015; Revil and Mahardika, 2013]. In the absence of an external 184 current density, the electrical potential φ in response to a given source current density satis-185 fies [Sill, 1983] 186

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$$\nabla \cdot (\sigma^e \nabla \varphi) = \nabla \cdot \mathbf{J}^{\mathrm{i},\mathrm{e}},\tag{25}$$

where σ^e denotes the electrical conductivity, which strongly depends on the saturating pore fluid as well as on textural properties of the medium, such as the porosity and the tortuosity [e.g. *Archie*, 1942; *Clennell*, 1997; *Revil and Linde*, 2006]. As we only consider low frequencies, that is, lower than Biot's critical frequency, we assume that displacement currents can be neglected.

In conclusion, we obtain the relative fluid-solid displacement field by solving Eqs. (11) to (14) under the boundary conditions corresponding to the applied test (Eqs. 6 to 10). Next, this field is employed to determine the source current density field through Eq. (24). Finally, the electrical potential is obtained by solving Eq. (25) under pertinent boundary conditions.

197 **2.3 Energy-based approach**

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The sensitivity of seismoelectric signals to parameters of interest, such as the background 198 permeability or fracture properties, can be studied in different ways. Analytical expressions 199 are helpful to build conceptual understanding based on idealized situations, while for more com-200 plex and realistic scenarios it is necessary to resort to numerical simulations. Typically, a par-201 ticular experimental configuration is considered and differences in amplitude and spatial vari-202 ations are studied [e.g., Revil and Jardani, 2009; Jougnot et al., 2013; Grobbe and Slob, 2016]. 203 From a theoretical point of view, it is interesting to consider an energy-based approach and 204 study the total energy converted into seismoelectric signals. Although it would be impossible to quantify this parameter experimentally, it constitutes an attractive theoretical approach to 206 obtain a global estimate of the sensitivity of the method that is independent of the specific ex-207 perimental configuration. 208

The energy density of an electric field $\mathbf{E}(t)$ is given, in the space-time domain, by [e.g. *Feynman et al.*, 1965]

$$e(t) = \frac{1}{2}\varepsilon |\mathbf{E}(t)|^2, \tag{26}$$

where ε is the electric permittivity of the medium. Since displacement currents are negligible, the electric field at any time can be calculated as

$$\mathbf{E}(t) = \Re \left(\nabla (\varphi_0 e^{i\omega t}) \right), \tag{27}$$

where φ_0 is the complex amplitude of the electrical potential derived for each frequency as explained in the previous section. The corresponding real part is taken because we solve the equations in the space-frequency domain.

²¹⁸ Using average properties of time-harmonic complex-valued variables [*Rubino et al.*, 2006], ²¹⁹ it is straightforward to show that

$$<\Re\left(\nabla(\varphi_0 e^{i\omega t})\right)^{T_p}\cdot\Re\left(\nabla(\varphi_0 e^{i\omega t})\right)>=\frac{1}{2}\Re\left(\nabla\varphi_0^{T_p}\nabla\varphi_0^*\right),\tag{28}$$

where the operator $\langle \cdot \rangle$ denotes the average value over one oscillation cycle. Using Eqs. (27) and (28), we obtain

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$$|\mathbf{E}(t)|^2 >= \frac{1}{2} |\nabla \varphi_0|^2.$$
⁽²⁹⁾

²²⁴ Using the expression for the energy density (Eq. (26)), we finally get

$$<\frac{1}{2}\varepsilon|\mathbf{E}(t)|^2>=\frac{1}{4}\varepsilon|\nabla\varphi_0|^2=.$$
(30)

Locally, the energy density converted into seismoelectric signal in one period of oscillation T_p can therefore be computed using

$$\int_{o}^{T_{p}} e(t)dt = \frac{1}{4}\varepsilon |\nabla\varphi_{0}|^{2}T_{p}.$$
(31)

The total converted energy in the sample can then be calculated by integrating Eq. (31) over the spatial domain. Doing so for each frequency yields a spectrum of the total converted energy. This spectroscopic analysis makes it possible to determine a frequency at which this energy is maximum over the sample. In the following, we shall refer to this as the peak frequency.

233 2.4 Rock physical relationships considered in this study

Our focus is on the physics governing the generation of the seismoelectric signal in response to WIFF. For this reason, we only consider clean sandstones with different porosities of the matrix and idealized rock physical relationships to link material properties. To relate the porosity ϕ to the permeability k, we use the Kozeny-Carman equation [e.g., *Mavko et al.*, 2009]

$$k = b \frac{\phi^3}{(1-\phi)^2} d^2,$$
(32)

where *b* is a geometrical factor that depends on the tortuosity of the porous medium, and *d* the mean grain diameter. In this analysis, we take b = 0.003 [*Carcione and Picotti*, 2006] and $d = 8 \times 10^{-5}$ m [*Rubino et al.*, 2009]. These properties are characteristic of a well-sorted, fine-grained sandstone. In addition to changes in permeability, porosity variations also imply changes in the mechanical properties. To link the porosity and the solid grain properties with the elastic moduli of the dry frame, we use the empirical model of *Krief et al.* [1990]

$$K^{fr} = K^s \left(1 - \phi\right)^{3/(1-\phi)},\tag{33}$$

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$$G^m = \frac{K^{fr}G^s}{K^s},\tag{34}$$

where G^s is the shear modulus of the solid grains.

For the numerical study, we follow *Nakagawa and Schoenberg* [2007] and compute the elastic properties of the drained fracture in terms of the shear and drained normal compliances

$$\eta_T = \frac{h}{G_h^m},\tag{35}$$

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$$\eta_N = \frac{h}{K_h^{fr} + \frac{4}{2}G_h^m},$$
(36)

where h is the fracture aperture and K_h^{fr} and G_h^m are its drained-frame bulk and shear moduli, respectively.

In this work, we consider only clean sandstones in which the surface conductivity can be neglected. Also, as we consider low frequencies, that is, frequencies lower than Biot's critical frequency, we can safely neglect EDL polarization effects and assume that the electrical conductivity has no imaginary part. Under this assumption, the electrical conductivity is given by

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$$e^{2} = \sigma^{f} \phi^{m^{c}} = \frac{\sigma^{f}}{F},$$
(37)

where σ^{f} denotes the electrical conductivity of the pore water, while m^{c} and F are the cementation exponent and the formation factor as defined by *Archie* [1942], respectively. The pore water conductivity depends strongly on the amount of total dissolved salts [e.g. *Sen and Goode*, 1992].

 σ^{ϵ}

The remaining electrical parameter \hat{Q}_V^0 can be obtained by employing the empirical relationship proposed by *Jardani et al.* [2007]

$$\log\left(\underline{\hat{Q}}_{V}^{0}\right) = -9.2349 - 0.8219\log(k),\tag{38}$$

where k and \hat{Q}_V^0 are in units of m² and C/m³, respectively. Below Biot's critical frequency, 270 the effective excess charge density is similar to the one at zero frequency [e.g., Tardif et al., 271 2011; Revil and Mahardika, 2013] and, hence, boundary layer effects can be neglected in the 272 test cases considered in the following. We use idealized rock physical relationships to link σ^e 273 and \hat{Q}_{V}^{0} to porosity, but these properties can also be inferred independently by laboratory ex-274 periments [e.g., Jouniaux and Pozzi, 1995; Suski et al., 2006]. Although \hat{Q}_{V}^{0} mainly depends 275 on the permeability of the medium (Eq. 38), a recent study of Jougnot et al. [2015] highlighted 276 that the pore water salinity also has a significant effect on its amplitude (around one order-277 of-magnitude change for a salinity change of four orders-of-magnitude). 278

The dielectric permittivity of the medium is usually expressed as the product of the dielectric permittivity of the vacuum ε_0 and the relative dielectric permittivity ε_r

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 $\varepsilon = \varepsilon_r \varepsilon_0. \tag{39}$

The parameter ε_r can be determined using a volume averaging approach [*Pride*, 1994; *Linde et al.*, 2006]

$$\varepsilon_r = \frac{1}{F} \left[\varepsilon_r^f + (F-1) \varepsilon_r^s \right],\tag{40}$$

where ε_r^f and ε_r^s are the relative permittivity of the water ($\varepsilon_r^f \simeq 81$) and the solid grains ($\varepsilon_r^s \simeq 5$), respectively. This model depends on the same parameter as the electrical conductivity, that is, the formation factor (Eq. 37), and thus, is directly related to the porosity ($F = \phi^{-m^c}$).

3 Insights from 1D analytical solutions

In some simple cases, the set of equations that govern the generation of seismoelectric signals due to WIFF can be solved analytically. Equations that explicitly relate the dependence of the resulting electrical potential on rock properties can be useful to understand the underlying physical processes. Recently, *Monachesi et al.* [2015] solved the governing equations presented in the previous section for a 1D case. Here, we present their main analytical solutions and results, based on which we then study the seismoelectric signal dependence on the background permeability and on the pore water salinity.

3.1 General solution for a thin layer

Monachesi et al. [2015] consider a thin layer of thickness $2L_h$ located at the center of 297 an otherwise homogeneous rock sample (Fig. 1). In the following, properties related to the thin 298 layer are identified by the subscript "h" for heterogeneity and the ones corresponding to the 299 rest of sample by the subscript "b" for background. The thicknesses of the two embedding re-300 gions constituting the background are L_b and, thus, the total thickness of the sample is $2(L_h +$ 301 L_b = L. Assuming a set of boundary conditions analogous to Eqs. (6) to (10), the bound-302 ary value problem given by Eqs. (11) to (14) can be solved in terms of the relative fluid-solid 303 displacement $w(y,\omega)$. Then, the current density $\mathbf{J}^{\mathbf{i},\mathbf{e}}(y,\omega)$ can be computed from $w(y,\omega)$ us-304 ing Eq. (24). Finally, the electrical potential is obtained by solving Eq. (25) with the adequate 305 boundary conditions. The resulting electrical potential as a function of the vertical position y306 and frequency ω is given by [Monachesi et al., 2015] 307

 $\varphi(y,\omega) = \begin{cases} -\frac{i\omega\hat{Q}_{V}^{0,h}}{\sigma_{h}^{e}}\frac{A_{h}}{\kappa_{h}}\left(e^{-\kappa_{h}|y|} + e^{\kappa_{h}|y|}\right) + S_{h}, & 0 \le |y| \le L_{h}, \\ -\frac{i\omega\hat{Q}_{V}^{0,b}}{\sigma_{b}^{e}}\frac{A_{b}}{\kappa_{b}}\left(e^{-\kappa_{b}|y|} + e^{-\kappa_{b}(L-|y|)}\right) + S_{b}, & L_{h} \le |y| \le L/2, \end{cases}$

where S_h , S_b , A_h , and A_b are given by

$$S_{h} = \frac{i\omega\hat{Q}_{V}^{0,h}}{\sigma_{h}^{e}}\frac{A_{h}}{\kappa_{h}}\left(e^{-\kappa_{h}L_{h}} + e^{\kappa_{h}L_{h}}\right) - \frac{i\omega\hat{Q}_{V}^{0,b}}{\sigma_{b}^{e}}\frac{A_{b}}{\kappa_{b}}\left(-\kappa_{b}L_{h} + e^{-\kappa_{b}(L-L_{h})} - 2e^{-\kappa_{b}L/2}\right), \quad (42)$$

$$S_b = \frac{2i\omega \hat{Q}_V^{0,b}}{\sigma_b^e} \frac{A_b}{\kappa_b} e^{-\kappa_b L/2},\tag{43}$$

(41)

$$A_{h} = \left(e^{-\kappa_{h}L_{h}} - e^{\kappa_{h}L_{h}}\right)^{-1} \frac{\Delta P\left(\beta_{h} - \beta_{b}\right)}{\sum_{i=h,b} N_{j}\kappa_{i} \ coth(\kappa_{i}L_{j})},\tag{44}$$

$$A_{b} = \left(e^{-\kappa_{b}L_{h}} - e^{-\kappa_{b}(L-L_{h})}\right)^{-1} \frac{\Delta P\left(\beta_{h} - \beta_{b}\right)}{\sum_{j=h,b} N_{j}\kappa_{j} \ coth(\kappa_{j}L_{j})}.$$
(45)

Note that the seismoelectric signal depends on the parameter k, which is related to the dif-

fusion length, and, thus, among other parameters, to the permeability (see Eqs. 22 and 23) by

$$\kappa = \frac{\sqrt{i}}{L_d} = \sqrt{\frac{i\omega\eta}{kN}},\tag{46}$$

and to the 1D Skempton coefficient β defined by 320

$$\beta \equiv \frac{\alpha_B M}{H_u}.\tag{47}$$

Equation (41), together with Eqs. (42) to (45), constitute the analytical solution of the seis-322 moelectric response of a rock sample containing a central horizontal layer subjected to an os-323 cillatory compressibility test as shown in Fig. 1. It is interesting to note that the seismoelec-324 tric response is highly dependent on the medium permeability through $\hat{Q}_V^{0,b}$ and κ . It also de-325 pends on the Skempton coefficient difference between the heterogeneity and the embedding 326 background $\beta_h - \beta_b$, and thus on the compressibility contrast between heterogeneity and back-327 ground. This finding is consistent with the literature on WIFF [e.g. Müller et al., 2010]. 328

To explore the dependence of the analytical solution on the various rock physical and 329 structural parameters, we first consider a sample with a vertical side length of 20 cm composed 330 of a stiff, low-permeability background with a porosity of 0.05 (Material 1 in Table 1), per-331 meated at its center by a compliant, high-permeability horizontal layer with a thickness of 6 332 cm and a porosity of 0.4 (Material 2 in Table 1). The sample is fully saturated in water and 333 subjected to a harmonic compression of amplitude $\Delta P=1$ kPa at frequencies of 10¹, 10², and 334 10^{3} Hz. 335

Table 1. Material properties employed in this work. Materials 1 and 2 are the same as the Materials 1 and 3 336 used by Monachesi et al. [2015], respectively. 337

Quartz grain bulk modulus K^s [GPa]	37		
Quartz grain shear modulus G^s [GPa]	44		
Water bulk modulus K^f [GPa]	2.25		
Water viscosity η [Pa \times s]	0.001		
Water electrical conductivity σ^f [S m ⁻¹]	0.01		
Water density $\rho^f [\text{Kg m}^{-3}]$	10^{3}		
	Material 1	Material 2	Material 3
Porosity ϕ	0.05	0.4	0.5
Dry rock bulk modulus K^{fr} [GPa]	31.47	2.88	0.017 / 0.04 *
Dry rock shear modulus G^{fr} [GPa]	37.42	3.42	0.01 / 0.02 *
Permeability k [mD]	2.66	3410	9600
Electrical conductivity σ^e [S m ⁻¹]	2.5×10^{-5}	$1.6 imes 10^{-3}$	$2.5 imes 10^{-3}$
Moveable charge density \hat{Q}_{V}^{0} [C m ⁻³]	526.8	1.49	0.637
Biot's critical frequency f_c [Hz]	2.99×10^6	1.8×10^4	8.29×10^3

* Calculated using Eqs. (35) and (36) for apertures of 0.03 / 0.06 cm, respectively.

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Figure 2a shows the amplitude profile of the resulting relative fluid velocity dw/dt =338 $i\omega w$ along the y-axis $(y \in [-L/2, L/2])$ for the three frequencies considered. Due to the strong contrast between the Skempton coefficients of the two materials, significant relative fluid ve-340 locities arise in both the background and the layer. The relative fluid velocity is higher near 341 the contact between the layer and the background and vanishes at the center and at both edges 342 of the sample. Under compression, the compliant layers undergoes more deformation than the 343 material on either side of it with the result that water is forced out of the layer. The ampli-344 tude of dw/dt reaches larger values for higher frequencies. A significant current density $\mathbf{J}^{1,e}$ 345 prevails in the background (Fig. 2b) due to the relative fluid velocity field (Fig. 2a) produced 346 by the compression and the relatively large excess charge (Table 1). The maximum current 347 densities occur at the contacts between the two materials, where the relative fluid velocity is 348

also highest. Inside the layer, even though significant fluid flow also takes place, the result ing source current density is small since the effective excess charge is much smaller in this
 material characterized by a larger permeability (Table 1, Eq. 38).

Significant electrical potential amplitudes (Fig. 2c), well above the $\simeq 0.01$ mV detectability threshold of laboratory experiments (e.g. *Zhu and Toksöz* [2005]; *Schakel et al.* [2012]), arise in response to the oscillatory compression. These results are consistent with those by *Jougnot et al.* [2013] for fractured rocks and point to the importance of WIFF effects on seismoelectric signals in the presence of porosity variations. Inside the layer, the amplitude of the electrical potential is constant. This is due to the negligible source current density in this highpermeability material. Because the electrical potential is continuous, this corresponds to the value of the electrical potential at the contact between the two materials.

The resulting electrical potential is not only characterized by its amplitude but also by 360 its phase θ . In the background, θ shows rapid spatial changes when the frequency is high (Fig. 361 2d). Inside the layer, θ remains constant, which is in agreement with the behavior observed 362 for the amplitude of the electrical potential in this region (Fig. 2c). In general, the phase val-363 ues vary strongly within the medium and cover a much larger range than could be expected 364 from a frequency-dependent electrical conductivity. For example, Kruschwitz et al. [2010] re-365 port a typical induced polarization phase of less than 0.6° for a large frequency range ($f \in [10^{-3}]$; 366 10^4] Hz), while our calculations show a distribution -180 to +180° (Fig. 2d). This confirms 367 that our assumption concerning the negligible effect of complex conductivity at low frequen-368 cies is valid (see section 2.4). 369

The behavior of the electrical potential curves as a function of normalized time is shown 370 in Fig. 3 for the three frequencies considered. The curves correspond to the electrical poten-371 tial differences $\Delta \varphi$ recorded by an electrode located at the center (y = 0) and a reference 372 electrode located at one edge of the sample (y = L/2 or y = -L/2). Note that the integer 373 values of t/T_p correspond to the moment of maximum applied stress. This representation al-374 lows us to interpret the physical mechanisms in a simple manner: during the compression cy-375 cle of the applied normal stress, the fluid inside the compliant layer experiences a pressure in-376 crease and thus water flows from the layer into the background, generating a significant seis-377 moelectric signal. Conversely, during the extension cycle, water flows from the background 378 into the layer, generating a seismoelectric signal with an opposite sign. Note that the ampli-379 tude and phase of the electrical potential at 10^2 and 10^3 Hz are similar with a negligible phase 380 lag with respect to the applied pressure. In contrast, the 10 Hz signal depicts different ampli-381 tude and phase values. These differences in amplitude and phase are also evident in Figs. 2c 382 and d. 383

In order to explore in detail the dependence of the electrical potential on the frequency 384 of the oscillatory compression, we show in Fig. 4a the amplitude of the electrical potential along 385 the y-axis of the sample for frequencies between 1 Hz and 10^4 Hz [see Monachesi et al., 2015, 386 for the spatial-frequency dependence of the phase]. Between ~ 10 and ~ 100 Hz, the spatial 387 extent and amplitude of the electrical potential in both the background and the layer are larger 388 than for other frequencies. It is not straightforward to assign a frequency of maximum spatial extent since different amplitude iso-values have different corresponding frequencies of max-390 imum spatial extent. At low frequencies (1-10 Hz), the electrical potential tends to become 391 negligible. At higher frequencies (100-10000 Hz), WIFF is comprised in the immediate vicin-392 ity of the boundaries of the layer and the magnitude of the electrical potential is non-zero only 393 inside the layer. In agreement with Figs. 2c and d, the amplitude of the electrical potential re-394 mains constant inside the layer at each frequency. 395

In Figure 4b we show the distribution of the electrical potential amplitude obtained when the material properties of the background and the layer are interchanged. Due to the imposed boundary conditions, when the layer is stiffer and less permeable than the background, the electrical potential has a significant amplitude only inside the layer. The electrical potential am⁴⁰⁰ plitude is also frequency-dependent, with a maximum at the center of the layer and for a fre-⁴⁰¹ quency that is higher compared to the previous situation [*Monachesi et al.*, 2015].

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3.2 Particular solution for a single fracture

Monachesi et al. [2015] also studied the seismoelectric signal of an homogeneous rock sample that is permeated by a single horizontal fracture. This was done by adapting their analytical solution to an infinitely thin layer at the center of the sample. This yields a simpler expression of the seismoelectric response

$$\varphi(y,\omega) = -\frac{i\omega\hat{Q}_V^{0,b}}{\sigma_b^e} \frac{\bar{A}_b}{\kappa_b} \left(e^{-\kappa_b|y|} + e^{-\kappa_b(L-|y|)} - 2e^{-\kappa_b(L/2)} \right),\tag{48}$$

408 where

$$\bar{A}_{b} = \lim_{L_{h} \to 0} A_{b} = \frac{\Delta P \left(1 - \beta_{b}\right)}{\frac{2}{Z_{N}} \left(1 - e^{-\kappa_{b}L}\right) + N_{b}\kappa_{b} \left(1 + e^{-\kappa_{b}L}\right)},$$
(49)

and Z_N is the drained normal compliance of the fracture. Note that Z_N is the only fracture parameter in these equations, while the only structural parameter is the total thickness of the sample *L*. It is also interesting that the seismoelectric signal mainly depends on the background permeability k_b through κ_b (Eq. 46) and $\hat{Q}_V^{0,b}$ and on the background Skempton coefficient β_b . Figure 5a shows the spectroscopic analysis for a sample with the same size and background material as in Fig. 4a (Material 1 in Table 1) permeated by a fracture. Note the high amplitudes reached in this case due to the strong compressibility of the fracture.

3.3 Sensitivity to the background permeability

From the presented analytical solutions, it is clear that the background permeability has 418 a predominant role in the generation of seismoelectric signals. Figure 5b shows the resulting 419 seismoelectric signal when the background permeability is one order-of-magnitude larger than 420 in Material 1, that is, $k_b = 26.6$ mD. As opposed to what was presented by *Monachesi et al.* 421 [2015], we let the permeability vary independently of the porosity, which does not change, so 422 that the changes observed are uniquely related to permeability changes. We can observe two 423 different effects: as the permeability increases, the maximum amplitude of the signal increases 424 and the frequency of maximum extent of the signal is shifted towards higher frequencies. This 425 result is consistent with the ones discussed by Jougnot et al. [2013]. The spatial scale at which 426 WIFF occurs depends on the diffusion length and, therefore, on the background permeabil-427 ity (Eq. 46). Therefore, the frequency of maximum extent of the signal is mainly controlled 428 by the background hydraulic properties, which is consistent with the asymptotic analysis by 429 Monachesi et al. [2015]. The shift of maximum WIFF to higher frequencies related to a larger 430 permeability also implies a higher fluid velocity and thus a higher amplitude of the electrical 431 potential (Fig. 2). The amplitude of the seismoelectric signal is also affected by the imposed 432 relationship between the moveable charge density and the background permeability (Eq. 38). A larger permeability implies a smaller moveable charge density and thus a decrease of the 434 amplitude. The significant increase in amplitude shown in Fig. 5b suggests that the effect of 435 a larger fluid velocity due to the shift to higher frequencies dominates over the amplitude de-436 crease due to the smaller moveable charge density. 437

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3.4 Sensitivity to the pore water conductivity

The moveable charge density is not only influenced by the permeability. As discussed by *Jougnot et al.* [2015], the pore water salinity, and thus the pore water electrical conductivity σ^f , also affects their moveable charge density through its influence on the thickness of the EDL and the associated changes of the Zeta potential [e.g. *Revil et al.*, 1999]. In addition, the pore water conductivity also strongly affects the bulk electrical conductivity (Eq. 37). To study the effect of salinity changes on the seismoelectric signal, we complement our spectroscopy analysis for a pore water conductivity that is one order-of-magnitude smaller ($\sigma^f = 0.001$ S

m⁻¹, Fig. 6a) and one order-of-magnitude larger ($\sigma^f = 0.1$ S m⁻¹, Fig. 6b) than in the pre-446 vious cases ($\sigma^f = 0.01 \text{ Sm}^{-1}$, Fig. 5a). We calculate the corresponding moveable charge den-447 sity as deviations from the value given by Eq. (38) with the model proposed by *Jougnot et al.* [2015]. This results in values of $\hat{Q}_V^{0,b}$ = 790.12 C m⁻³ and 351.31 C m⁻³, for σ^f = 0.001 S m⁻¹, and σ^f = 0.1 S m⁻¹, respectively. Not surprisingly, the impact of salinity/pore water con-448 449 450 ductivity upon the seismoelectric signal is significant: the lower the pore water conductivity, 451 the higher the amplitude of the signal. The frequency of maximum extent of the signal is not 452 affected by a change in pore water conductivity. Note here that the influence of fluid conduc-453 tivity on the seismoelectric response is more important due to its effect on the bulk electrical conductivity (Eq. 37) than to its impact on $\hat{Q}_V^{0,b}$, which can be considered a secondary ef-454 455 fect [Jougnot et al., 2015]. 456

457 **4** Numerical study of fractured rock samples

In this section, we numerically solve the governing equations described in the theory sec-458 tion in order to consider 2D fracture geometries. We employ the numerical strategy presented 459 by Jougnot et al. [2013] for exploring the generation of seismoelectric signals due to WIFF 460 in the presence of fractures. That is, we consider a 2D synthetic rock sample containing meso-461 scopic heterogeneities. Equations (11) to (14) are solved, with the boundary conditions described by Eqs. (6) to (10), using a finite element procedure [Rubino et al., 2009]. From the result-463 ing 2D velocity fields, we compute the electrical current density (Eq. 24) and then numeri-464 cally solve Eq. (25) assuming perfect electrical insulation along the boundaries using a finite 465 volume approach. To do so, we adapted an open source finite volume numerical code that was originally conceived to solve subsurface fluid flow problems [Künze et al., 2014] to the con-467 sidered electrical problem. In an initial analysis, we consider a synthetic homogeneous rock 468 sample containing a simple 2D fracture. We then study the effects of different fracture lengths, 469 different fracture orientations, and different numbers of fractures in the sample. Finally, frac-470 ture networks with varying degrees of connectivity are explored. 471

472 **4.1 Analysis for a single fracture**

We first consider a simple case corresponding to a homogeneous rock containing a hor-473 izontal fracture at its center (Fig. 7a). For the background material, we use for all cases the 474 same sandstone as in the analytical study (Material 1 in Table 1). The fracture is modeled as 475 a very compliant poroelastic rectangle that is characterized by large values of porosity and per-476 meability (Material 3 in Table 1), with the elastic properties being calculated using Eqs. (35) 477 and (36). Given that the fracture does not permeate the entire sample, the analytical solution 478 presented in the previous section cannot be used and, instead, the numerical approach is em-479 ployed. This initial case will be the basic geometry for which we will perform the 2D sen-480 sitivity analysis. 481

We consider a sample of $6 \times 6 \text{ cm}^2$ with a horizontal fracture of 3 cm length and 0.03 cm aperture located at its center (Fig. 7a). We use 600×600 elements to discretize the entire domain. The numerical simulations using this mesh were compared to simulations using finer meshes to ensure the accuracy of the calculations. We compute the seismoelectric response of oscillatory compressions with $\Delta P = 1$ kPa at 40 different frequencies equally spaced on a logarithmic scale between 1 and 10000 Hz.

Figure 7b shows the resulting seismoelectric signal amplitude for a frequency of 142 Hz. Note that the electrical problem has been solved using a reference electrode at the origin (xe 0 cm, y = 0 cm). The electrical potential generated by this small heterogeneity is maximal in the immediate vicinity of the fracture and easily measurable with typical experimental setup in the laboratory. This large signal is due to the high compressibility contrast between the fracture and the background, which in turn results in significant WIFF.

In Fig. 7, we display the detailed spatial distribution of the electrical potential for a sin-494 gle frequency in order to stress the 2D nature of the signal. However, the main interest of our 495 approach is to study the spectral dependence of the signal generated by the oscillatory compression through a spectroscopic analysis. In order to best represent these results, Figure 8a 497 shows vertical cuts of the seismoelectric signal amplitude through the center of the sample shown 498 in Fig. 7a (x = 3 cm) as a function of frequency. These vertical cuts, which pass through the 499 center of the fracture, are similar to those presented in the analytical section, but it should be 500 noted that in this case the sample is 2D. To complete our study, we use the results of the energy-501 based analysis that we developed in Section 2.3 to provide a global measure of the frequency 502 dependence of the seismoelectric signal in the sample. Figure 8b shows the total energy con-503 verted to the seismoelectric signal in one compression cycle as a function of frequency. To calculate this value, we numerically computed for each frequency the gradient of the electrical 505 potential amplitude (Eq. 31) and summed the squared contribution of each pixel weighted by 506 its electrical permittivity (Eqs. (39) and (40)) multiplied by one fourth of the corresponding 507 period. The resulting spectrum shows a strong dependence of the converted electric energy on 508 frequency and a clearly defined peak frequency for which the converted electric energy is max-509 imum. In this case, the peak frequency corresponds to 142 Hz, which is the frequency used 510 for the 2D representation in Fig. 7b. 511

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4.2 Sensitivity to the fracture length

In this subsection, we investigate the effect of the fracture extent along the x-axis in the 513 sample on the amplitude of the seismoelectric signal. We consider two cases where the frac-514 ture length is smaller than in the previous section, with fracture lengths of 0.6 and 1.8 cm (Figs. 515 9a and d, respectively), and two cases where the extent is larger, that is, 4.2 and 6 cm (Figs. 516 9g and j, respectively). The latter corresponds to the extreme case of a fracture that permeates the whole sample. As the fracture length increases, so does the spectral range at which 518 the fracture can be detected, the amplitude of the signal, and the vertical extent of the mea-519 surable electrical potential (Figs. 9b, e, h, and k). From the converted energy (Figs. 9c, f, i, 520 and 1) we can also see that the fracture length changes the peak frequency at which the con-521 verted energy is higher; larger fractures imply a lower peak frequency and a higher amount 522 of converted energy. Note that a one order-of-magnitude change in the fracture length from 523 the sample in Fig. 9a to the one in Fig. 9j implies a shift of almost two orders-of-magnitude 524 in the peak frequency and an increase of more than two orders-of-magnitude in the converted 525 energy. 526

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4.3 Sensitivity to the fracture orientation

To understand the sensitivity to the fracture orientation, we consider four cases where 528 a 3 cm long fracture is oriented from sub-horizontal to vertical with respect to the x-axis (see 529 Figs. 10a, d, g, j and the figure caption for the fracture angles). As the orientation of the fracture becomes more vertical, the amplitude of the electrical potential decreases. WIFF takes 531 place from the more compliant fracture to the stiff background and vice versa, and is max-532 imum when the fracture is perpendicular to the direction of applied stress. In the horizontal 533 case, the applied stress strongly deforms the fracture, increases its fluid pressure and produce 534 significant WIFF and seismoelectric conversion. Conversely, in the extreme case of a verti-535 cal fracture, the fluid mainly flows inside the fracture, which has a low $\hat{Q}_V^{0,h}$ and therefore it 536 does not produce a significant electrical source current density. The intermediate states (Figs. 537 10a, d, and g) show the smooth transition between a horizontal to a vertical fracture. The ori-538 entation does not affect the peak frequency, although the total converted energy is, as expected, 539 significantly smaller for more vertically oriented fractures (Figs. 10 c, f, i and l). 540

4.4 Sensitivity to the number of fractures

To understand the aggregate effect of multiple fractures, we consider an increasing amount of fractures in a sample of the same size as in the previous cases. The starting point is a sin-gle 541 fracture as shown in the fracture length subsection (Fig. 9j). We then consider cases with 2, 3, 4 and 5 equally spaced fractures throughout the sample (Figs. 11 a, d, g, and j). The cor-542 responding fracture spacings are 2.97, 1.97, 1.47, and 1.17 cm, respectively. Regardless of the 543 number of fractures in the sample, the maximum amplitude of electrical potential does not sig-544 nificantly change. As the number of fractures in the sample increases, the vertical extent of 545 the seismoelectric signal generated by each fracture decreases and the spectral range where 546 the signal could be detected is shifted towards higher frequencies. Correspondingly, the en-ergy 547 plots in Fig. 11 show that the peak frequency for which the maximum of energy is con-verted 548 also shifts to higher values as the number of fracture increases. This shift in frequency corresponds to the dependence of the diffusion length (Eq. 23) on the frequency; by decreas-ing 550 the space between fractures, we decrease the spatial scale at which WIFF between the fractures 551 can take place, thus the frequency corresponding to the maximum extent of fluid flow 552 is higher. It is interesting to note that although the peak frequency is affected by the number 553 of fractures in the sample, this parameter does not seem to influence the total converted energy 554 at the corresponding peak frequency (Fig. 11). This suggests that the larger number of fractures 555 compensates for the smaller spatial extent of the region in which significant electri-cal potential 556 amplitude are produced by each fracture in the sample.

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4.5 Analysis of a fracture network

In this subsection, we study the effects of fracture connectivity on the seismoelectric signal. We consider a similar setup as the one used by *Rubino et al.* [2014] to explore the dependence of the seismic attenuation on fracture connectivity. We consider a sample of 20×20 cm², discretized by 1000×1000 elements, and examine four different fracture scenarios. In the first scenario, horizontal fractures are randomly distributed in the sample (Fig. 12a). In the second scenario, the horizontal fractures are retained and vertical fractures are added randomly under the constraint that none of the fractures is connected to another one (Fig. 12c). The third case corresponds to the same number of horizontal and vertical fractures, but with some of the vertical fractures being connected to the horizontal ones (Fig. 12e). Finally, in the fourth scenario we consider the case when all the vertical fractures are connected to some of the hor-izontal ones (Fig. 12g). In all the examples, the fractures have an aperture of 0.06 cm. The fracture properties are given in Table 1 (Material 3). The maximum pressure applied is the same as in all other examples, that is, $\Delta P = 1$ kPa, and all the boundary conditions remain the same as in previous cases.

⁵⁷² Figure 12 shows the amplitude of the electrical potential of the four geometries consid-ered
⁵⁷³ for a frequency of 0.73 Hz. We observe that the presence of vertical fractures that are
⁵⁷⁴ not connected to the horizontal ones does not significantly change the amplitude of the seis⁵⁷⁵ moelectric response (Figs. 12b and d). However, when the vertical fractures are connected to the
⁵⁷⁶ horizontal ones, the spatial distribution and amplitude of the electrical potential does change
⁵⁷⁷ (Figs. 12f and h). Indeed, the maximum amplitude in the sample is lower for a higher frac-ture
⁵⁷⁸ connectivity and larger parts of the sample are "illuminated" with a measurable electrical in this case.

To study the dependence of the seismoelectric signal on fracture connectivity at the sam-ple scale, we present in Fig. 13 plots of the total converted electrical energy as a function of frequency. A clear dependence on fracture connectivity can be observed. Adding the unconnected vertical fractures results in a higher seismoelectric energy, but as the fracture connec-tivity increases, there is a decrease in the total energy of the electric field. The peak frequency is also affected by the degree of connectivity. When the vertical unconnected fractures are added, the peak frequency does not change and corresponds to 0.73 Hz, which was the frequency used in Fig. 12. Increasing the fracture connectivity shifts the peak frequency to higher values.

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592 **5 Discussion and conclusions**

Following Jougnot et al. [2013] and Monachesi et al. [2015], we performed a theoret-593 ical (analytical and numerical) study of the seismoelectric signals generated when a rock sam-594 ple containing mesoscopic heterogeneities is submitted to an oscillatory compressibility test. 595 Heterogeneities are considered mesoscopic when their size is smaller than the typical wavelength but larger than the pore-scale. In the present contribution we focused on mesoscopic-597 scale fractured media and developed a quantitative approach to characterize the dependence 598 of the seimoelectric signal with fracture connectivity. The predicted signal is highly frequency-599 dependent and hence we illustrated our results in terms of the space-frequency distribution of the seismoelectric response, which corresponds to a spectroscopic analysis. The source of this 601 frequency-dependent signal is linked to WIFF from the more compliant heterogeneities to the 602 background during the compression cycle, and in the opposite direction during the dilatation 603 cycle. Our results show that this phenomenon yields measurable seismoelectric signals under 604 typical laboratory set-ups in terms of applied pressure, frequency range, and instrument res-605 olution [e.g. Batzle et al., 2006; Subramaniyan et al., 2014; Pimienta et al., 2015]. 606

We introduced for the fist time an energy-based approach (Section 2.3) to characterize the seismoelectric conversion at the sample scale. This approach provides complementary information to our spectroscopic analysis at the sample scale by allowing for the definition of a peak frequency for which the total converted seismoelectric energy is maximum. The total converted seismoelectric energy could be compared to the elastic strain energy and energy dissipation as derived by *Solazzi et al.* [2016]. This is, however, outside the scope of this contribution and we leave this comparison for future publications.

We studied different kinds of mesoscopic heterogeneities: thin layers, single fractures 614 and fracture networks. Our results show a strong dependence of the seismoelectric signal on 615 mechanical, hydraulic, and structural properties of the background and the mesoscopic het-616 erogeneities. In particular, the background permeability via the diffusion length, fracture sep-617 aration and fracture length, control the frequency at which maximum WIFF occurs and, there-618 fore, also influences the peak f requency. The amplitude of the electrical potential is mainly 619 controlled by the background permeability, the pore water conductivity, compressibility con-620 trast between heterogeneity and background, and fracture orientation. These parameters affect 621 the bulk conductivity, moveable charge density, and source current density, which define the 622 electrical potential distribution in the sample. Similar to what was observed by Rubino et al. 623 [2013, 2014] for the seismic case, fracture orientation, extent, density, and connectivity influ-624 ence the spectroscopic signature of the seismoelectric signal. This is particularly interesting 625 for the characterization of fractured media, which is of primary importance in hydrological ap-626 plications yet extremely difficult to achieve in practice [e.g. Berkowitz, 2002]. 627

Connected fractures reduce the total energy converted to the seismoelectric signal and 628 change the spatial distribution of electrical potential amplitude. For an equal number of hor-629 izontal and vertical fractures, the total converted electrical energy decreases by $\sim 50\%$ for the 630 corresponding peak frequency (Fig. 13) when these fractures are connected. The reason for 631 this is that the connection to vertical fractures enables part of the fluid pressure increase in response to the applied stress to be released from the horizontal fractures into these highly per-633 meable regions. This reduces the fluid pressure gradient and, thus, the fluid flow between frac-634 tures and background, which in turn results in a decrease of the generated electrical source 635 current density and the measurable electrical potential outside the fractures. Given that the de-636 gree of fracture connectivity controls the effective hydraulic properties of fractured rocks, this 637 connectivity effects are potentially important as they may help to extract this kind of infor-638 mation from corresponding seismoelectric measurements. 639

The present contribution describes analytical and numerical experiments and aims at understanding how mesoscopic heterogeneities can produce measurable seismoelectric signals under laboratory conditions. To the best of the author's knowledge, these prediction have not yet been tested in practice. Such experimental studies would be of significant interest for both the rock physics and the seismoelectric community as they may provide a new rock physical characterization tool: seismoelectric spectroscopy.

Besides thin layers or fractures, other types of mesoscopic heterogeneities are known to generate significant WIFF [e.g. *Batzle et al.*, 2006; *Adam et al.*, 2009; *Müller et al.*, 2010; *Pimienta et al.*, 2015] but remain unexplored in terms of their seismoelectric response. Similar effects also exist in patch-type partially saturated conditions [e.g. *Caspari et al.*, 2011; *Masson and Pride*, 2011; *Rubino and Holliger*, 2012]. Such saturation effects and the resulting seismoelectric signals could explain some discrepancies between experimental data and current models, such as those shown by *Bordes et al.* [2015].

The results of this study could also help to better understand seismoelectric conversions 653 at the field s cale. Indeed, all geological formations contain a certain degree of mesoscopic het-654 erogeneity and, therefore, seismic waves are expected to produce seismoelectric signals asso-655 ciated with such heterogeneities as they propagate. These phenomena could be one of the causes 656 for the difficulties encountered in seismoelectric field applications. For example, high noise 657 levels encountered in field applications [e.g. Strahser et al., 2011] could be related to hetero-658 geneities of different nature and size that generate multiple seismoelectric source currents when 659 traversed by the seismic waves. Further studies accounting for effects such as geometrical di-660 vergence and the co-seismic field will be carried out in the near future. These are necessary 661 to quantify the relative contribution of Biot's slow waves to the total seismoelectric signal that 662 would be measured in the field. Our results clearly illustrate that a better understanding of the 663 role played by mesoscopic heterogeneities is essential for the development of the seismoelec-664 tric method. 665

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Figure 1. Schematic representation of the oscillatory compressibility test proposed by *Jougnot et al.* [2013] applied to a sample containing a single layer.



Figure 2. (a) Amplitude of relative fluid velocity dw/dt, (b) amplitude of electrical source current density J^{i,e}, and (c) amplitude $|\varphi|$ and (d) phase θ of the electrical potential corresponding to a rectangular, stiff, lowpermeability background containing a compliant, high-permeability horizontal layer at its center. The porosity of the layer is 0.4 (Material 2 in Table 1), whereas that of the background is 0.05 (Material 1 in Table 1). In all cases, the panels show the parameters as functions of y. For visualization purposes, we denote the boundaries of the layer by dashed lines.



Figure 3. Electrical potential differences $\Delta \varphi$ between an electrode located at the center of the sample and a reference electrode located at an edge of the sample as functions of the normalized time t/T_p for frequencies of 10^1 , 10^2 , and 10^3 Hz.



Figure 4. Amplitude of the electrical potential along the *y*-axis as a function of frequency corresponding to (a) a stiff, low-permeability background containing a compliant, high-permeability horizontal layer at its center, and (b) a compliant, high-permeability background containing a stiff, low-permeability horizontal layer at its center. Adapted from *Monachesi et al.* [2015].



Figure 5. Seismoelectric response of a sample containing a fracture at its center (Eqs. 48 and 49) and its dependence on the background permeability. Amplitude of the electrical potential along the *y*-axis as a function of frequency (a) for the background properties corresponding to Material 1 (Table 1, $k_b = 2.66$ mD) and (b) for a background permeability that is one order-of-magnitude larger ($k_b = 26.6$ mD). Adapted from *Monachesi et al.* [2015].



Figure 6. Seismoelectric dependence on pore water electrical conductivity. Amplitude of the electrical potential along the *y*-axis as a function of frequency for the same sample as shown in Fig. 5a but considering a pore water salinity that is (a) one order-of-magnitude smaller ($\sigma^f = 0.001 \text{ S m}^{-1}$) and (b) one order-ofmagnitude larger ($\sigma^f = 0.1 \text{ S m}^{-1}$). Given the large amplitude difference, we use separate color scales for each subplot.



Figure 7. (a) Synthetic rock sample used as the basis to study the seismoelectric dependence on the properties of fractured media. A single horizontal fracture is embedded in an otherwise homogeneous medium. The sensitivity analysis presented in the following is based on simple variations of this initial model. (b) Amplitude of the electrical potential generated for a frequency of 142 Hz with a virtual reference electrode located at the left bottom edge of the sample. The amplitude of the stress applied on the top boundary of the sample ΔP is 1 kPa.



Figure 8. Vertical cuts of the electrical potential at the center (x = 3 cm) of the sample shown in Fig. 7a as a function of frequency. (a) Amplitude and (b) total energy converted to seismoelectric signal in one cycle as a function of frequency.

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Figure 9. Dependence of seismoelectric signals on fracture length. (a), (d), (g) and (j) Samples with the same properties as Fig.7a but with different fracture lengths. (b), (e), (h) and (k) Vertical cuts of the amplitude of the electrical potential at the center (x = 3 cm) of the corresponding samples as functions of frequency. (c),

(f), (i) and (l) Total converted electrical energy as a function of frequency.



Figure 10. Dependence of seismoelectric signal on fracture orientation. (a), (d), (g) and (j) Samples with the same properties as Fig. 7a but with different fracture orientation, ranging from sub-horizontal to vertical. The angles with respect to the *x*-axis are: (a) 27° , (d) 45° , (g) 67.5° and (j) 90° . (b), (e), (h) and (k) Verti-

cal cuts of the amplitude of the electrical potential at the center (x = 3 cm) of the corresponding samples as

functions of frequency. (c), (f), (i) and (l) Total converted electrical energy as a function of frequency.



Figure 11. Dependence of seismoelectric signal on fracture density. (a), (d), (g) and (j) Samples with the same properties as Fig.7a but with different number of fractures. (b), (e), (h) and (k) Vertical cuts of the amplitude of the electrical potential at the center (x = 3 cm) of the corresponding samples as functions of

frequency. (c), (f), (i) and (l) Total converted electrical energy as a function of frequency.



Figure 12. Left column: Rock samples used to test the effect of fracture connectivity on the seismoelectric signal. (a) Sample containing horizontal fractures that are not connected between each other. (c) Sample containing the same horizontal fractures as (a), plus vertical fractures, which are not connected to the horizontal ones. (e) Sample containing the same amount of horizontal and vertical fractures as in (c) but with some of the fractures connected. (g) Same as (b) but with most of the fractures connected. Right column: Amplitudes of the electrical potential in the samples shown in the left column for a frequency of 0.73 Hz.



Figure 13. Total energy converted to seismoelectric signal in one period as a function of frequency for the
samples shown in Fig. 12a, c, e, and g.