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Rotational threshold in global numerical dynamo simulations

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ABSTRACT

Magnetic field observations of low-mass stars reveal an increase of magnetic activity with increasing rotation rate. The so-called activity–rotation relation is usually attributed to changes in the underlying dynamo processes generating the magnetic field. We examine the dependence of the field strength on rotation in global numerical dynamo models and interpret our results on the basis of energy considerations. In agreement with the scaling law proposed by Christensen and Aubert, the field strength in our simulations is set by the fraction of the available power used for the magnetic field generation. This is controlled by the dynamo efficiency calculated as the ratio of ohmic to total dissipation in our models. The dynamo efficiency grows strongly with increasing rotation rate at a Rossby number of 0.1 until it reaches its upper bound of 1 and becomes independent of rotation. This gain in efficiency is related to the strong rotational dependence of the mean electromotive force in this parameter regime. For multipolar models at Rossby numbers clearly larger than 0.1, on the other hand, we do not find a systematic dependence of the field strength on rotation. Whether the enhancement of the dynamo efficiency found in our dipolar models explains the observed activity–rotation relation needs to be further assessed.

Key words: dynamo – methods: numerical – stars: magnetic fields.

1 INTRODUCTION

There is considerable observational evidence that magnetic activity on low-mass stars increases with increasing stellar rotation rate until it saturates and reaches a constant level for very fast rotating stars (e.g. Reiners 2012). This so-called activity–rotation relation is primarily based on observations of chromospheric or coronal magnetic activity indicators, i.e. on chromospheric or coronal emission (Skumanich 1972; Noyes et al. 1984a; Delfosse et al. 1998; Pizzolato et al. 2003). Furthermore, it is supported by magnetic flux measurements in stars of spectral types G–M (Saar 2001; Reiners, Basri & Browning 2009). Noyes et al. (1984a) pointed out that chromospheric emission and thus magnetic activity depending on spectral type and rotation is well described by a single parameter, the Rossby number $\mathrm{Ro} = P_{\mathrm{rot}}/\tau_{\mathrm{c}}$, with P_{rot} being the observed rotation period of a star and τ_{c} a convective turnover time derived from mixing-length theory. The observed increase of magnetic activity with decreasing Rossby number is usually attributed to changes in the underlying dynamo processes generating the magnetic field (Donati & Landstreet 2009; Reiners 2012). However, until now, theoretical arguments explaining the activity–rotation relation are poorly developed and formulated only on a heuristic level.

The standard argument worked out in the framework of linear mean-field theory refers to the so-called dynamo number, D, a dimensionless parameter which determines the distance to the dynamo threshold of a given dynamo model. If D exceeds some critical value, $D > D_{\mathrm{c}}$, a small initial magnetic perturbation may grow exponentially, i.e. the field-free state is linearly unstable. The larger D, the larger the growth rates of the magnetic field expected in this scenario. Under some simplifying assumptions, D scales inversely proportional to the square of the Rossby number (Noyes, Weiss & Vaughan 1984b). Therefore, one might argue that dynamo action can be easily excited and thus leads to larger field strengths if the Rossby number decreases.

The principal objection against this argument is that it is entirely based on linear, kinematic theory. For stellar dynamos with $D \gg D_{\mathrm{c}}$, the magnetic field would grow rapidly until the Lorentz force changes the velocity field and as a result the magnetic field saturates. In this dynamical regime, linear theory is in general no longer applicable. Even if the dynamo number predicted the onset of dynamo action properly, its predictive power for the saturation level of the magnetic field would remain uncertain.

A successful scaling law for the field strength of fast rotating stars and planets, on the other hand, was presented by Christensen, Holzwarth & Reiners (2009). They showed that a scaling originally derived from geodynamo models in the Boussinesq approximation (Christensen & Aubert 2006) also applies to certain classes of stars. The scaling law reads

$$B^2/(2\mu_0) \sim f_{\mathrm{ohm}}\varrho^{(1/3)}(q_{\mathrm{c}}\, L/H_{\mathrm{T}})^{(2/3)}, \quad (1)$$

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and is based on the available energy flux q_c, which sets the field strength apart from the density ϱ, and a convective length-scale relative to the temperature scaleheight, L/H_T. The coefficient $f_{ohm} \leq 1$ gives the ratio of ohmic to total dissipation and was set to unity in this context. Christensen et al. (2009) highlighted two remarkable findings. First, the flux-based scaling law is at first glance independent of rotation, and secondly, it is only applicable to fast rotating stars and planets.

In the following, we will argue that the flux-based scaling law (equation 1) is in fact valid for a wide range of rotation rates, but it is not independent of rotation. On the contrary, we claim that the rotational dependence is captured by f_{ohm} and simply eliminated by setting f_{ohm} to unity. This indeed seems to be justified only for stars in the rotationally saturated regime. Our analysis is based on 30 numerical dynamo models in the Boussinesq approximation. The modelling strategy and the models are briefly described in the next section. Hereafter, we present results for f_{ohm} revealing its dependence on the rotation rate and discuss the implications of our finding for the activity–rotation relation.

2 DYNAMO CALCULATIONS

Our dynamo models are solutions of the magnetohydrodynamic equations for a conducting Boussinesq fluid in a rotating spherical shell. Convection is driven by an applied temperature difference ΔT between the inner boundary at radius r_i and the outer boundary at r_o. The governing equations for the velocity $\boldsymbol{v}$, the magnetic field $\boldsymbol{B}$ and the temperature T written in a dimensionless form proposed by Olson, Christensen & Glatzmaier (1999) are

$$E\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{v} - \nabla^2\boldsymbol{v}\right) + 2\boldsymbol{z}\times\boldsymbol{v} + \nabla P = \mathrm{Ra}\frac{\boldsymbol{r}}{r_o}T + \frac{1}{\mathrm{Pm}}(\nabla\times\boldsymbol{B})\times\boldsymbol{B}\,, \tag{2}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{v}\cdot\nabla T = \frac{1}{\mathrm{Pr}}\nabla^2 T\,, \tag{3}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{B}) + \frac{1}{\mathrm{Pm}}\nabla^2\boldsymbol{B}, \tag{4}$$

$$\nabla\cdot\boldsymbol{v} = 0, \qquad \nabla\cdot\boldsymbol{B} = 0. \tag{5}$$

They are controlled by four dimensionless parameters, the Ekman number $E = \nu/\Omega L^2$, the (modified) Rayleigh number $\mathrm{Ra} = \alpha_T g_o \Delta T L/\nu\Omega$, the Prandtl number $\mathrm{Pr} = \nu/\kappa$ and the magnetic Prandtl number $\mathrm{Pm} = \nu/\eta$. In these definitions, L denotes the width of the convection zone, Ω stands for the angular velocity, α is the thermal expansion coefficient, g_o is the gravitational acceleration at the outer boundary and ν, η, κ are the kinematic viscosity, the magnetic and the thermal diffusivity. The mechanical boundary conditions are no slip and the magnetic field continues as a potential field outside the fluid shell.

In the following paragraph, we need to define some further quantities which are used throughout in this Letter. The time-averaged ratio of ohmic to total dissipation, f_{ohm}, is computed as:

$$f_{ohm} = \frac{W_J}{W_b}, \tag{6}$$

with the rate of ohmic dissipation

$$W_J = \frac{1}{\mathrm{Pm}^2\,E}\int(\nabla\times\boldsymbol{B})^2\,\mathrm{d}v, \tag{7}$$

and the power W_b generated by buoyancy forces,

$$W_b = \frac{\mathrm{Ra}}{E}\int\frac{r}{r_o}\,v_r\,T\,\mathrm{d}v. \tag{8}$$

Moreover, we define a Rossby number for our models similar to the observational one. It is given by the ratio of the Rossby radius to a typical convective length-scale ℓ_c, $\mathrm{Ro}_\ell = v_{rms}/(\Omega\ell_c)$, where v_{rms} stands for the rms velocity of the flow and ℓ_c is derived from the kinetic energy spectrum (Christensen & Aubert 2006; Schrinner, Petitdemange & Dormy 2012). A non-dimensional measure for the convective energy flux is the Nusselt number, Nu, defined as the ratio of the total heat flow to the conducted heat flow. Finally, we note that the temperature scaleheight H_T for our models is given by $H_T = c_P/(\alpha g_o)$ with the heat capacity c_P.

We aim at comparing models which differ only in their rotation rates. Therefore, we keep the thermodynamically available energy flux and the diffusivities for a sequence of models constant and vary only the angular velocity. Translated in non-dimensional quantities, this means that we keep (in a first approximation) the Rayleigh number over some critical Rayleigh number Ra_c, the Prandtl number and the magnetic Prandtl number constant and change successively the Ekman number. More precisely, we try to keep the Nusselt number Nu for a sequence of models constant. Because Nu is an output parameter in our simulations and only roughly determined by Ra normalized by its critical value, $\mathrm{Ra}/\mathrm{Ra}_c$ has to be adjusted accordingly. Some more details are given in Appendix A.

Due to computational limitations all current numerical dynamo simulations run in a parameter regime which is not appropriate for stellar interiors. Moreover, Boussinesq models do not account for the strong density variation in stars and thus certainly do not reproduce stellar dynamo processes in realistic detail. However, our models are adequate to study the flux-based scaling law which was originally derived from Boussinesq models, too.

3 RESULTS

We considered sequences of models with $\mathrm{Nu} \approx 2.2$, 3.5 and 7, and Pm varying between 3 and 7. The Prandtl number Pr was always set to unity in our simulations. For given Pm and Nu, we obtained a *sequence of models* by varying the Ekman number between $E = 10^{-3}$ and $E = 10^{-5}$; some models with $E = 3\times10^{-3}$ and $E = 3\times10^{-6}$ could also be added to our sample. On the other hand, simulations with $\mathrm{Nu} \approx 7$ and $E \leq 3\times10^{-5}$ were numerically not feasible. The magnetic Reynolds number of our models, $\mathrm{Rm} = v_{rms}\,L/\eta$, is always larger than 100 and thus far above the minimum value of $\mathrm{Rm} = 40$ needed to obtain dynamo action in this setting (Olson & Christensen 2006). We show in Appendix A that there is a slight but systematic increase of Rm with rotation rate for a sequence of models at constant Nusselt and Prandtl numbers.

Following Christensen & Aubert (2006), a non-dimensional form of the flux-based scaling law is obtained by dividing relation (1) by $\varrho\,\Omega^2L^2$,

$$B^2/(2\mu_0\varrho\Omega^2L^2) \sim f_{ohm}(q_c/(\varrho\Omega^3L^3)\,L/H_T)^{(2/3)}. \tag{9}$$

With the Lorentz number $\mathrm{Lo} = B/(\mu\varrho\Omega^2L^2)^{1/2}$, the flux-based Rayleigh number $\mathrm{Ra}_Q = q_c\alpha g_o/(4\pi\varrho c_P\Omega^3L^2)$, and $L/H_T = L\,\alpha g_:/c_P$, relation (9) may simply be written as

$$\mathrm{Lo}/f_{ohm}^{1/2} = c\,\mathrm{Ra}_Q^{1/3}. \tag{10}$$

Christensen & Aubert (2006) found the prefactor c to be 0.92 for models with a predominantly dipolar field geometry, and a slightly

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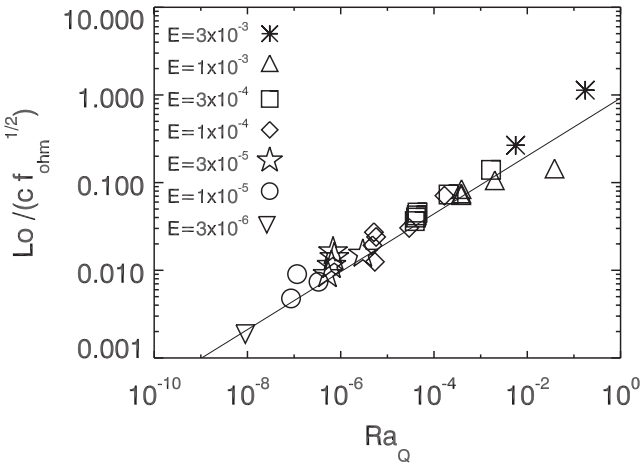

Figure 1. $\mathrm{Lo}/(c\, f_{\mathrm{ohm}}^{1/2})$ versus the flux based Rayleigh number. The constant c was chosen to be 0.92 for dipolar and 0.48 for multipolar models (Christensen & Aubert 2006; Christensen 2010). The solid line does not represent a fit but the prediction of equation (10).

lower value of $c = 0.48$ was given by Christensen (2010) for multipolar models. In Fig. 1, we plotted $\mathrm{Lo}/(c\, f_{\mathrm{ohm}}^{1/2})$ against Ra_Q in logarithmic scales for our sample of models. Independent of their Ekman number, all models are in agreement with the scaling (10) indicated in Fig. 1 by the solid line. Apparently, the flux-based scaling law holds for all models independently of their rotation rates.

However, the ratio of ohmic to total dissipation, f_{ohm}, may increase drastically with rotation rate, as demonstrated in Fig. 2. Shown is f_{ohm} versus the Rossby number for sequences of models with $\mathrm{Nu} = 2.2$ and various Pm. At $\mathrm{Ro}_\ell \approx 0.12$, the rate of ohmic diffusion increases rapidly until the steep slope flattens and f_{ohm} saturates for lower Rossby numbers. On the other hand, a strong dependence of f_{ohm} on Pm cannot be inferred from Fig. 2.

The strong increase of f_{ohm} at $\mathrm{Ro}_\ell \lesssim 0.12$ coincides with a transition from multipolar dynamo models at higher Rossby number to models with a dipole-dominated magnetic field. In fact, the dependence of f_{ohm} on the rotation rate changes crucially at the regime boundary as shown in Fig. 3. For a sequence of predominantly dipolar models with a slightly higher Nusselt number, $\mathrm{Nu} = 3.5$, the increase of f_{ohm} is qualitatively reproduced. However, the

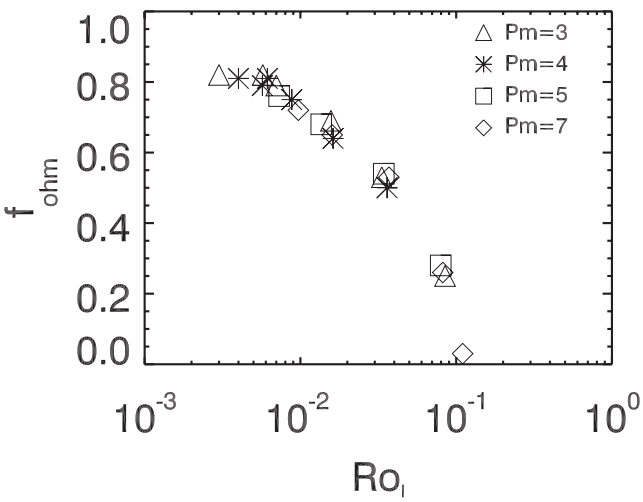

Figure 2. The ratio of ohmic to total dissipation, f_{ohm}, versus the Rossby number for sequences of models with $\mathrm{Nu} = 2.2$ and various Pm.

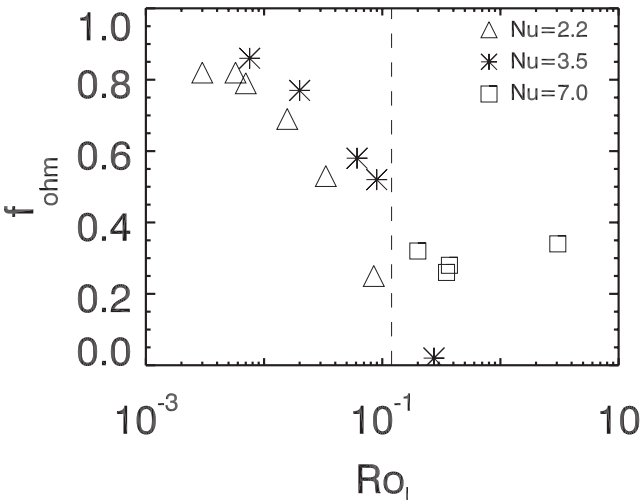

Figure 3. The dynamo efficiency f_{ohm} versus the Rossby number for sequences of models with $\mathrm{Pm} = 3$ and Nusselt numbers $\mathrm{Nu} = 2.2$, 3.5 and 7. The vertical dashed line indicates the boundary between the dipolar ($\mathrm{Ro}_\ell < 0.12$) and multipolar dynamo regime.

amplified convective energy flux shifts the sequence towards higher Rossby numbers. For an even higher Nusselt number, $\mathrm{Nu} = 7$, the sequence of models falls entirely in the multipolar regime and a systematic increase of f_{ohm} with rotation rate is no longer observed.

4 DISCUSSION AND CONCLUSIONS

In an equilibrium state, the energy released by buoyancy in our models is dissipated by viscous dissipation and ohmic diffusion. The latter requires that a magnetic field is built up by dynamo action and the rate of ohmic dissipation determines the fraction of the available power used for the magnetic field generation. For fast rotators f_{ohm} increases, this means that a larger fraction of the available power is converted to magnetic energy and dynamo action becomes more efficient. According to relation (1), the growth of f_{ohm} visible in Fig. 2 leads to an increase of the average magnetic field strength by an order of magnitude. Because f_{ohm} is bound by 1, the field strength saturates for even higher rotation rates and then becomes independent of rotation. Both, the increase at $\mathrm{Ro}_\ell \approx 0.1$ and the saturation of the field strength are in good agreement with observations. Yet, not much is known about the saturation level of the magnetic field in slowly rotating stars, except that it falls below the value predicted by (1) with $f_{\mathrm{ohm}} = 1$ (Christensen et al. 2009). With a Rossby number of $\mathrm{Ro}_\ell > 0.5$ (Reiners, Joshi & Goldman 2012), the Sun is an example for a slow rotator. Assuming an energy flux of $q_{\mathrm{c}} = 63\,\mathrm{MW\,m^{-2}}$, a mean density of $\varrho = 1.4\,\mathrm{kg\,m^{-3}}$ and an average internal field of $B = 0.063\,\mathrm{T}$ (Christensen et al. 2009), relation (1) requires $f_{\mathrm{ohm}} \approx 0.07$. This would be consistent with the range of f_{ohm} presented in this study. We note, however, that the flux-based scaling law is somewhat at odds with the estimate of $f_{\mathrm{ohm}} = 10^{-3}$ derived by Rempel (2006) from dynamic flux-transport solar dynamo models.

The decline of f_{ohm} at $\mathrm{Ro}_\ell \approx 0.1$ visible in Figs 2 and 3 is related to a rotational dynamo threshold. It is characterized by a minimum magnetic Prandtl number, $\mathrm{Pm}_{\mathrm{crit}}$, below which self-sustained dynamo action does not occur. Christensen, Olson & Glatzmaier (1999) found that $\mathrm{Pm}_{\mathrm{crit}}$ is a function of only the Ekman number and varies in the dipolar regime as:

$$\mathrm{Pm}_{\mathrm{crit}} = 450\, E^{0.75}. \tag{11}$$

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Models with given diffusivities approach this dynamo threshold if their rotation rate is decreased and $f_{\rm ohm}$ drops to zero.

Equation (11) is independent of any velocity amplitude and does not relate the rotational threshold to a given Rossby number. Hence, particular low values for $f_{\rm ohm}$ could in principle be found at any Rossby number (and at any Rm). However, we could not confirm equation (11) for the multipolar dynamo regime where $f_{\rm ohm}$ remains low and does not change significantly with rotation rate (see Fig. 3). Attempts to identify a similar decline of $f_{\rm ohm}$ close to a rotational threshold at much lower Rossby numbers also failed. Models in this parameter regime with a lower Ekman and magnetic Prandtl often bifurcate subcritically (Morin & Dormy 2009). Consequently, the magnetic field strength and $f_{\rm ohm}$ remain high and collapse abruptly to zero only beyond the dynamo threshold.

What explains the rotational threshold and the high rotational sensitivity of $f_{\rm ohm}$ for models in the dipolar regime close to $\mathrm{Ro}_\ell = 0.1$? Dipolar models typically exhibit different characteristic length-scales for their velocity and their magnetic field with $\ell_{\rm V} < \ell_{\rm B}$. The balance of the advection and the diffusion term in the induction equation then leads to a modified magnetic Reynolds number, $\widetilde{\mathrm{Rm}} = v_{\rm rms}\,\ell_{\rm B}^2/(\eta\,\ell_{\rm V})$, which needs to exceed a critical value for the onset of dynamo action. For given diffusivities, $\ell_{\rm B}^2$ is inversely proportional to $v_{\rm rms}$ (Christensen & Tilgner 2004). Therefore, an increase of the velocity amplitude is compensated by smaller $\ell_{\rm B}$ and does not change $\widetilde{\mathrm{Rm}}$. This heuristic argument might explain why equation (11) is independent of any velocity amplitude and holds only in the dipolar regime. The strong dependence of the dynamo efficiency on rotation rate, however, requires some further explanation.

Dynamo models in the dipolar regime with $\mathrm{Ro}_\ell \lesssim 0.1$ may be adequately described in the framework of mean-field theory (Krause & Rädler 1980; Schrinner et al. 2007; Schrinner 2011; Schrinner, Schmitt & Hoyng 2011). The mean-field formalism provides useful concepts to better understand the influence of rotation on the dynamo processes in our models. It is usually set up by splitting the velocity and the magnetic field in a mean and a residual component varying on different length-scales, $\boldsymbol{v} = \overline{\boldsymbol{V}} + \boldsymbol{v}'$ and $\boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{b}'$. Mean quantities denoted here by an overbar may be thought of as combined azimuthal and time averages (Schrinner 2011). They vary on a scale similar to the system size L. Fluctuating quantities, on the other hand, vary on the scale of the velocity field, $\ell_{\rm V}$. We emphasize that this is not an assumption but a result obtained from direct numerical simulations in a particular parameter regime. The mean flow $\overline{\boldsymbol{V}}$ is negligible for the models considered here (Olson et al. 1999; Schrinner et al. 2007). Therefore, the induction equation (2) separated for the mean and the residual component may be written as:

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} - \frac{1}{\mathrm{Pm}}\nabla^2\overline{\boldsymbol{B}} = \nabla\times\overline{\boldsymbol{v}\times\boldsymbol{b}'} \tag{12}$$

$$\wedge\ \frac{\partial \boldsymbol{b}'}{\partial t} - \nabla\times(\boldsymbol{v}\times\boldsymbol{b}')' - \frac{1}{\mathrm{Pm}}\nabla^2\boldsymbol{b}' = \nabla\times(\boldsymbol{v}\times\overline{\boldsymbol{B}}). \tag{13}$$

The residual magnetic field $\boldsymbol{b}'$ is diffusive on the length-scale $\ell_{\rm V}$ and would rapidly decay without the source term on the right-hand side of equation (13). Thus, the magnetic field generation depends decisively on $\overline{\boldsymbol{B}}$ and in particular on the so-called mean electromotive force, $\boldsymbol{\mathcal{E}} = \overline{\boldsymbol{v}\times\boldsymbol{b}'}$. Similarly, the magnetic energy density is dominated by the mean field. The energy equation for $\overline{\boldsymbol{B}}$ reads

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_\infty \frac{\overline{\boldsymbol{B}}^2}{2}\,\mathrm{d}v = -\int_{\mathcal{V}} \overline{\boldsymbol{j}}\cdot\overline{\boldsymbol{E}}\,\mathrm{d}v, \tag{14}$$

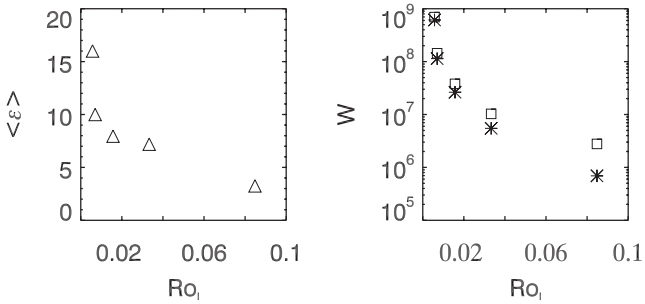

Figure 4. Left: time-averaged rms values of the electromotive force in units of $[\nu/L(\varrho\eta\mu\Omega)^{1/2}]$ for a sequence of models with $\mathrm{Nu}\approx 2.2$ and $\mathrm{Pm} = 4$ versus the Rossby number. Right: the rate of ohmic dissipation (stars) and total dissipation (squares) in units of $\varrho\nu^3/L$ versus Ro_ℓ for the same sequence of models.

where $\overline{\boldsymbol{E}}$ is the mean electrical field, $\overline{\boldsymbol{j}}$ is the mean current density and $\mathcal{V}$ denotes the volume of the fluid shell. With Ohm's law,

$$\overline{\boldsymbol{j}} = \mathrm{Pm}\,(\overline{\boldsymbol{E}} + \boldsymbol{\mathcal{E}}), \tag{15}$$

equation (14) yields

$$\frac{1}{\mathrm{Pm}}\int_{\mathcal{V}} \overline{\boldsymbol{j}}^2\,\mathrm{d}v = \int_{\mathcal{V}} \overline{\boldsymbol{j}}\cdot\boldsymbol{\mathcal{E}}\,\mathrm{d}v, \tag{16}$$

for an equilibrium state. Hence, also the mean ohmic diffusion is controlled by the electromotive force, i.e. by the correlation of the residual velocity and the residual magnetic field, and thus linked to rotation. It is expected that rotation strengthens the correlation between $\boldsymbol{v}$ and $\boldsymbol{b}'$. Indeed, $\boldsymbol{\mathcal{E}}$ grows with rotation rate for a sequence of models with fixed Nu, as shown in Fig. 4. Therefore, also W_J and eventually the dynamo efficiency $f_{\rm ohm}$ increase with decreasing Rossby number. We note, however, that also $W_{\rm b}$ (in units of $\varrho\nu^3/L$) increases for this sequence of models, though somewhat slower than W_J.

For clarification, we stress that the decrease of the dynamo efficiency with increasing Rossby number in our models is not caused by an Rm-dependent quenching of the electromotive force, which is sometimes called catastrophic quenching (see Brandenburg & Subramanian 2005, and references therein). In contrast to the catastrophic quenching scenario, the mean electromotive force increases with Rm in our simulations (see also Appendix A).

In summary, the field strength of our models is set by the available energy flux and via $f_{\rm ohm}$ by the rotation rate. The dynamo efficiency $f_{\rm ohm}$ increases strongly with rotation rate at $\mathrm{Ro}_\ell \approx 0.1$ and saturates at smaller Rossby numbers. The high rotational sensitivity of $f_{\rm ohm}$ is related to a rotational dynamo threshold and finally to the strong dependence of the mean electromotive force on rotation in this parameter regime. For multipolar dynamos at higher Rossby number, however, the dynamo efficiency seems to be almost independent of rotation. Similarities with the observed activity–rotation relation are encouraging and need to be further assessed.

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APPENDIX A: SCALING OF RA/RA$_c$ AND Rm AT CONSTANT NUSSELT NUMBER

We use scaling laws given by Christensen & Aubert (2006) to show that the ratio $\mathrm{Ra}/\mathrm{Ra_c}$ decreases slightly with rotation rate for a sequence of models at constant Nusselt and Prandtl numbers, whereas the magnetic Reynolds number increases.

The Nusselt number scaling proposed by Christensen & Aubert (2006) may be written as:

$$\mathrm{Nu} - 1 \sim \mathrm{Ra}\, E, \tag{A1}$$

provided that $\mathrm{Nu} > 1$ and convection is sufficiently supercritical. Moreover, the critical Rayleigh number varies as $\mathrm{Ra_c} \sim E^{-4/3}$ (Busse 1970), and we finally find $\mathrm{Ra}/\mathrm{Ra_c} \sim E^{1/3}$ for models at constant Nusselt number. For $\mathrm{Nu} \approx 2.2$, 3.5 and 7, we considered a maximum Rayleigh number of 6, 15 and 50 times its critical value.

The Rossby number scaling from Christensen & Aubert (2006) together with the Nusselt number scaling yields

$$\mathrm{Ro} \sim ((\mathrm{Nu} - 1)\, E/\mathrm{Pr})^{0.77}. \tag{A2}$$

With $\mathrm{Rm} = \mathrm{Ro}\,\mathrm{Pm}/E$ we conclude that the magnetic Reynolds number increases slightly with increasing rotation rate, $\mathrm{Rm} \sim E^{-0.23}$. For $\mathrm{Nu} \approx 2.2$ and $\mathrm{Pm} = 4$, for instance, we obtained $\mathrm{Rm} \approx 130$ at $E = 10^{-3}$ and $\mathrm{Rm} \approx 270$ at $E = 10^{-5}$.

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