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“Laplace” – 11/3/21 – 2nd Revision

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On the shoulders of Laplace

⁽¹⁾ Lopes, F., ⁽¹⁾ Le Mouél, J.L., ⁽¹⁾ Courtillot, V. and ⁽²⁾ Gibert, D.

⁽¹⁾ Université de Paris, Institut de physique du globe de Paris,

CNRS, 1 rue Jussieu, F-75005 Paris, France

⁽²⁾ Université Lyon 1, ENSL, CNRS, UMS 3721, LGL-TPE, F-69622 Villeurbanne, France

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26 **Abstract**

27 In 1799, Laplace derived the system of differential equations (now called Liouville-Euler) that
28 fully describes the motions of the rotation axis of any celestial body. Laplace showed that only the
29 gravitational forces and kinetic moments from other celestial bodies influence the rotation of any
30 one of them. The equations involve three Euler angles that specify the motions of a body’s rotation
31 axis; they can be reduced to a system of two equations for the inclination and time derivative of the
32 declination of the rotation axis. Laplace showed the existence of a forced annual oscillation and the
33 so-called free Chandler wobble. Most current theories retain only two Euler angles and invoke an
34 elastic Earth to match observations. We analyze the much longer time series of polar motion
35 (coordinates m_1 and m_2 of the rotation pole at the Earth’s surface) now available, in order to further
36 explore phenomena that Laplace could not investigate, given the dearth of data in his time. We use
37 singular spectral analysis (SSA) to extract components of the time series. The first three
38 components (trend or Markowitz drift, forced annual oscillation and free Chandler oscillation)
39 account for 73% of the variance of polar motion. Under the current theory, their modulation is
40 thought to be a response to reorganization of oceanic and atmospheric masses. However, the periods
41 of the first six SSA components of polar motion have been encountered in previous studies of
42 sunspots and in the ephemerids of Jovian planets. We also analyze the derivatives of the envelopes
43 of the three SSA components of polar motion. Again, most of these components have periods and
44 modulations that correspond to the ephemeris (periods and combinations of commensurable
45 periods) of Jovian planets. Examples include 171.5 yr (the Jose cycle linked to Neptune), 90 yr (the
46 Gleissberg cycle linked to Uranus), 40 yr (a commensurable period linked to the Jovian planets), 22
47 yr, 11 yr (Jupiter, Sun), 60 yr, 30 yr (Saturn). Figure 3 can be considered as the central result of the
48 paper. It shows that the sum of forces of the four Jovian planets matches in a striking way the polar
49 motion reconstructed with SSA components (the Markowitz trend removed). All our results argue
50 that significant parts of Earth’s polar motion are a consequence (rather than a cause) of the
51 evolution of planetary ephemerids. The Sun’s activity and many geophysical indices show the same
52 signatures, including many climate indices. Two different mechanisms (causal chains) are likely at
53 work: a direct one from the Jovian Planets to Earth, another from planetary motions to the solar
54 dynamo; variations in solar activity would in turn influence meteorological and climatic
55 phenomena. Given the remarkable coincidence between the quasi-periods of many of these
56 phenomena, it is reasonable to assume that both causal chains are simultaneously at work. In that
57 sense, it is not surprising to find the signatures of the Schwabe, Hale and Gleissberg cycles in many

58 terrestrial phenomena, reflecting the characteristic periods of the combined motions of the Jovian
59 planets.

60

61 **1 - Introduction**

62 On July 5 1687, Isaac Newton published the three volumes of his *Principia Mathematica*, in
63 which he put on a firm ground the law of universal attraction and the general laws of mutual
64 attraction of masses. In the following two centuries, a corpus of laws that explained the motions of
65 celestial bodies was established and vindicated by observations. Foremost among these works,
66 Laplace published his *Traité de Mécanique Céleste* (Treatise of Celestial Mechanics) in 1799.

67 Based on Newton’s law and the fundamental principle of dynamics, he established the general
68 equations that govern the motions of material bodies ([Laplace, 1799, book 1, chapter 7, page 74,](#)
69 [system \(D\)](#)). This system of differential equations of first order was later given the names of
70 Liouville and Euler. It establishes both the rotation and translation of the rotation axis of any
71 celestial body, and in particular the Earth. These same equations can be found in [Guinot](#) (in
72 [Coulomb and Jobert, 1977](#), p. 530) and more recently in the reference book of [Lambeck \(2005](#), p.
73 31). They are recalled in [Appendices 1](#) and [2](#) in their most general form. When the forces and the
74 moments that act on Earth are taken to be zero (i.e. the right hand side of the equations is zero), the
75 solution for the axis is a free oscillation with a Euler period $1/\sigma$ of 306 days (using the known
76 values of the mean angular velocity and axial and equatorial moments of inertia ($\sigma = ((C-A)/A)\Omega$)).
77 Based on observations made between June 1884 and November 1885, [Chandler \(1891a,b\)](#) obtained
78 a value of 427 days for $1/\sigma$. Data provided by the *International Earth Rotation and Reference*
79 *System Service* (IERS) yield a $1/\sigma$ that has varied between 431 days in 1846 and 434 days in 2020.

80 [Newcomb \(1892\)](#) verified Chandler’s observations and concluded that the Earth should be
81 viewed as an elastic body submitted to oceanic stresses. For this, Love numbers were introduced
82 ([Love, 1909](#)). As a result, the Liouville-Euler system (D) of Laplace was made less general. [Hough](#)
83 [\(1895\)](#) reinforced the idea that what made the Chandler period 121 days longer than the
84 “theoretical” value was the fact that the Earth behaved as an elastic body. Based on [Poincaré’s](#)
85 [\(1885\)](#) work on the stability of rotating fluids with a free surface, Hough showed that the period
86 should decrease rather than increase if one did not take elasticity into account. Works in the
87 following decades strengthened the notion that the fluid envelopes of Earth (ocean, atmosphere and
88 mantle) acted on Earth’s rotation axis. An increasingly precise theory was thus proposed, whereas
89 observations seemed to be increasingly remote from predictions.

90 Two papers ([Peltier and Andrews, 1976](#); [Nakiboglu and Lambeck, 1980](#)) further strengthened
91 the theory of an elastic Earth whose rotation axis was influenced by both its internal and external
92 fluid envelopes. An important concept was that of Global Isostatic Adjustment (GIA), in which the
93 Earth has a visco-elastic response to stress (load) variations, that originated at the onset of the last
94 ice age. Melting ice would lead to sea level rise and a reorganization of surface masses that
95 eventually modified the inclination of the rotation pole. Rather than writing in a physically explicit
96 way the forces implied in system (D), as done by Laplace and Poincaré, more or less complex
97 “excitation functions” were introduced ([Appendix 1](#)).

98 We return to the founding work of [Laplace \(1799\)](#) to see how these problems can be tackled
99 further. In what follows, we refer to volumes, chapters, pages and equation numbers in the original
100 edition of the *Traité de Mécanique Céleste*. Throughout the *Treatise*, [Laplace \(1799\)](#) rigorously
101 shows that, whatever the nature of the oceans and atmosphere, the only thing that influences the
102 rotation of celestial bodies is the action of other celestial bodies. On [page 347 \(chapter 1, volume 5\)](#)
103 [Laplace \(1799\)](#) writes (this quotation is given in the original French in [Appendix 4](#)): “*We have*
104 *shown that the mean rotation movement of Earth is uniform, assuming that the planet is entirely*
105 *solid and we have just seen that the fluidity of the sea and of the atmosphere should not alter this*
106 *result. It would seem that the motions that are excited by the Sun’s heat, and from which the*
107 *easterly winds are born should diminish the Earth’s rotation: these winds blow between the tropics*
108 *from west to east and their continued action on the sea, on the continents and on the mountains they*
109 *encounter, should seem to weaken imperceptibly that rotation movement. But the principle of*
110 *conservation of areas, shows to us that the total effect of the atmosphere on this movement must be*
111 *insensible; for the solar heat in dilating equally the air in all directions, should not alter the sum of*
112 *areas covered by the vector radii of each molecule of the Earth and of the atmosphere, and when*
113 *multiplied respectively by the corresponding molecules; which requires that the rotation motion be*
114 *not diminished. We are therefore assured that as the winds analyzed diminish this motion, the other*
115 *movements of the atmosphere that occur beyond the tropics, accelerate it by the same amount. One*
116 *can apply the same reasoning to earthquakes, and in general, to all that can shake the Earth in its*
117 *interior and at its surface. Only the displacement of these parts can alter this motion; if, for*
118 *instance a body placed at the pole, was transported to the equator; since the sum of areas must*
119 *always remain the same, the earth’s motion would be slightly diminished; but for it to be*
120 *noticeable, one should suppose the occurrence of great changes in the Earth’s constitution.”*

121 These views are also shared by [Poincaré \(1899\)](#). They seem to be different from modern

122 views as synthesized for instance by [Lambeck \(2005\)](#). These authors agree on the Liouville-Euler
123 system (D for Laplace) of differential equations, but the forces that act on the Earth are different
124 (and interpreted in a different way, as shown below). In the present paper, we attempt to check
125 Laplace’s full system using the observations that have accumulated and improved since Laplace’s
126 time (time series starting in 1750 for the oldest and no later than 1850 for the shortest ones).

127 We first discuss some of the core ideas of the paper, based on Laplace’s original
128 developments (section 2). We then recall some concepts and tools that we use in the paper and
129 introduce the data, i.e. the coordinates of the Earth’s rotation pole from 1846 to 2020 (section 3). In
130 section 4, we establish a striking result that is central to the paper: the detrended polar motion is
131 highly correlated with the sum of the forces exerted by the four Jovian planets. We next submit the
132 data to Singular Spectral Analysis (SSA) and discuss the first SSA components (section 4): the
133 Markowitz, annual and Chandler rotations. Then, in section 5, we discuss the SSA components of
134 the derivatives of the three components above. In section 6, we give several other examples, such as
135 the excellent correlation of the 40yr SSA component of the derivative of the envelope of the
136 Chandler oscillation with the 40yr SSA component of the combined forces of Uranus and Neptune.
137 We end with a discussion and concluding remarks (section 7).

138

139 **2 –Forces, Moments and the Liouville-Euler System of Equations**

140 In most classical applications of the mechanics of planetary rotation, one uses only the first
141 two components of the trio of Euler angles, i.e. the coordinates of the rotation pole at the Earth’s
142 surface ([Figure A1, Appendix 1](#)). The Earth rotates about the Sun (and so do the other 7 planets) in
143 the ecliptic plane that is almost perpendicular to the rotation axis. The Sun carries more than 99% of
144 the mass of the solar system, and can be considered rather motionless (its center of gravity actually
145 travels along a “small” variable “ellipse”). In addition to the gravitational attractions, one must
146 consider the orbital kinetic moments of all planets (in others words the moment of the momentum,
147 see eq. B1, [Appendix 2](#)), as emphasized by [Laplace \(1799\)](#). Planets carry more than 99% of the
148 total angular momentum of the system (19.3, 7.8, 1.7 and $1.7 \times 10^{42} \text{ kgm}^2\text{s}^{-1}$ respectively for Jupiter,
149 Saturn, Uranus and Neptune). This can be compared to the Sun’s attraction at the Earth’s orbit, 3.5
150 $10^{22} \text{ kg.m.s}^{-2}$, that can be transformed to the dimension of a kinetic moment by multiplying it by the
151 Sun-Earth distance and the orbital revolution period of Earth, yielding $1.7 \cdot 10^{41} \text{ kg.m}^2\text{s}^{-1}$: that is not
152 negligible compared to the order of magnitude of the kinetic moments of the Jovian planets (to 1 or
153 2 orders of magnitude).

154 The central idea of this paper is to analyze variations in the Earth’s rotation axis under the
 155 influence not only, as in many classic treatments (e.g. [Dehant and Mathews, 2015, ch. 2](#)), of
 156 gravitational potentials, but also of kinetic moments. The classical system of differential equations
 157 that describe the pole’s motion (Liouville-Euler) links the sum of simple physical entities with their
 158 time derivatives, hence a first order linear system ([Appendix 1](#), part 1). See [Bode \(1945\)](#) for more
 159 on the definition and consequences that can be drawn from such linear systems. One is that causes
 160 and consequences are similar, up to a constant factor, if the system is not too dissipative and is
 161 maintained: this implies that gravitational potentials, kinetic moments (of Jovian planets) and polar
 162 motions should share characteristic features.

163 Polar motion is described by three coordinates, usually labeled m_1 , m_2 and m_3 ([Appendix 1](#)). If
 164 one only wants to study the perturbations due to the gravitational potential of a planet in rotation
 165 about itself, two coordinates, m_1 and m_2 , are sufficient to describe the motion. In the case of our
 166 Solar system, planets revolve about the Sun in (or close to) the ecliptic plane; the moments they
 167 generate are perpendicular to that plane ([Appendix 2](#)). They act on the *inclinations* of the rotation
 168 axes of all planets, including Earth’s. This is the well-known phenomenon of interaction of spinning
 169 tops and is adequately described by the Liouville-Euler equations. This was known to Laplace who
 170 chose not to use the three Euler angles, but gave all the analytic formulas that allow one to compute
 171 the *inclination* θ of the rotation axis ([Figure A1](#) and [Appendix 3](#)) as a function of time, under the
 172 influence of the Moon and Sun ([Laplace, 1799; book 5, page 317, number 5](#)), and the time
 173 derivative of the *declination* ψ of the rotation axis ([Laplace, 1799; book 5, page 318, number 6](#)). θ
 174 and ψ are defined in [Laplace \(1799; book 1, page 73, number 26\)](#). [Laplace \(1799; book 5, pages](#)
 175 [352-355, number 14\)](#) deduces that, when neither the Moon nor the Sun act on Earth (conjunction
 176 nodes), the time derivative of the declination (which in modern terms is the Euler period $1/\sigma$) has a
 177 value of 306 days ([Appendix 3](#)). This value is fully determined by the Earth’s moments of inertia
 178 (i.e. the internal distribution of masses).

179 The equations derived by Laplace are:

$$\theta = h + \frac{3m}{4n} \cdot \left(\frac{2C - A - B}{C} \right) \cdot \left\{ \begin{array}{l} \frac{1}{2} \cdot \sin(\theta) \cdot \left\{ \cos(2\nu) + \frac{\lambda m}{m'} \cdot \cos(2\nu') \right\} \\ -(1 + \lambda) \cdot m \cdot \cos(\theta) \cdot \sum \cdot \frac{c}{f} \cdot \cos(ft + \varsigma) \\ + \frac{\lambda c'}{f'} \cdot \cos(\theta) \cdot \cos(f't + \varsigma') \end{array} \right.$$

180

$$\frac{d\psi}{dt} = \frac{3m}{4n} \cdot \left(\frac{2C - A - B}{C} \right) \cdot \left\{ \begin{array}{l} (1 + \lambda) \cdot m \cdot \cos(\theta) - \frac{\cos(\theta)}{2dt} \cdot \left\{ d \cdot \sin(2\nu) + \frac{\lambda m}{m'} \cdot d \cdot \sin(2\nu') \right\} \\ + (1 + \lambda) \cdot m \cdot \frac{\cos^2(\theta) - \sin^2(\theta)}{\sin(\theta)} \cdot \sum .c \cdot \cos(ft + \varsigma) \\ + \lambda \cdot m \cdot \frac{\cos^2(\theta) - \sin^2(\theta)}{\sin(\theta)} \cdot c' \cdot \cos(f't + \varsigma') \end{array} \right.$$

181

182 All celestial and terrestrial parameters in these equations are defined in [Appendix 2](#). The time
183 variation of declination of the Earth’s rotation pole is a function of inclination. Since the (θ, ψ) and
184 (m_1, m_2) couples represent the same physics, the pattern of the sum of planetary kinetic moments
185 that “force” part of the Earth’s polar motions should be found in m_1 and m_2 (see below). Laplace
186 obtained these equations taking into account “only” the Moon and Sun.

187 When the Moon and Sun act with maximum effect (conjunction bellies) $1/\sigma$ reaches a value
188 of 578 days. $1/\sigma$ therefore oscillates between 306 and 578 days; [Chandler \(1891\)](#) found a value of
189 427 days and today one observes values of 432-434 days. Both inclination θ and declination ψ drift.

190

191 **3 –The Toolbox: Rotation Pole Data, Ephemerids, Commensurability and Singular** 192 **Spectrum analysis**

193 Some of the tools and data needed to pursue our goal are listed in this section. Of course, we
194 require knowledge of planetary *ephemerids*, that are given by the IMCCE. Then we need/use:

195 *3-1 Rotation pole data:* Laplace did not have sufficient observations to demonstrate the
196 influence of planets, though he certainly did not deny their possible role. We now have sufficiently
197 long series of observations to test his full theory.

198 The rotation pole is defined by its components m_1 and m_2 , respectively on the Greenwich (0°)
199 and 90°E meridians ([Figure A1](#)). Two series of measurements of (m_1, m_2) are provided by IERS¹
200 under the codes EOP-C01-IAU1980 and EOP-14-C04. The first one runs from 1846 to July 1st 2020
201 with a sampling rate of 18.26 days, and the second runs from 1962 to July 1st 2020 with daily
202 sampling (also giving access to the length of day). [Figure 1](#) shows the components m_1 and m_2 of the
203 longer series (data are given in milli arc second - mas - and converted here in radians per second -
204 rad.s⁻¹). [Figure 2](#) shows the Fourier spectrum of component m_1 , corrected for a degree 2 non linear
205 trend. The forced annual oscillation and the free Chandler oscillation with period close to 1.19 yr
206 are conspicuous.

1 <https://www.iers.org/IERS/EN/DataProducts/EarthOrientationData/eop.html>

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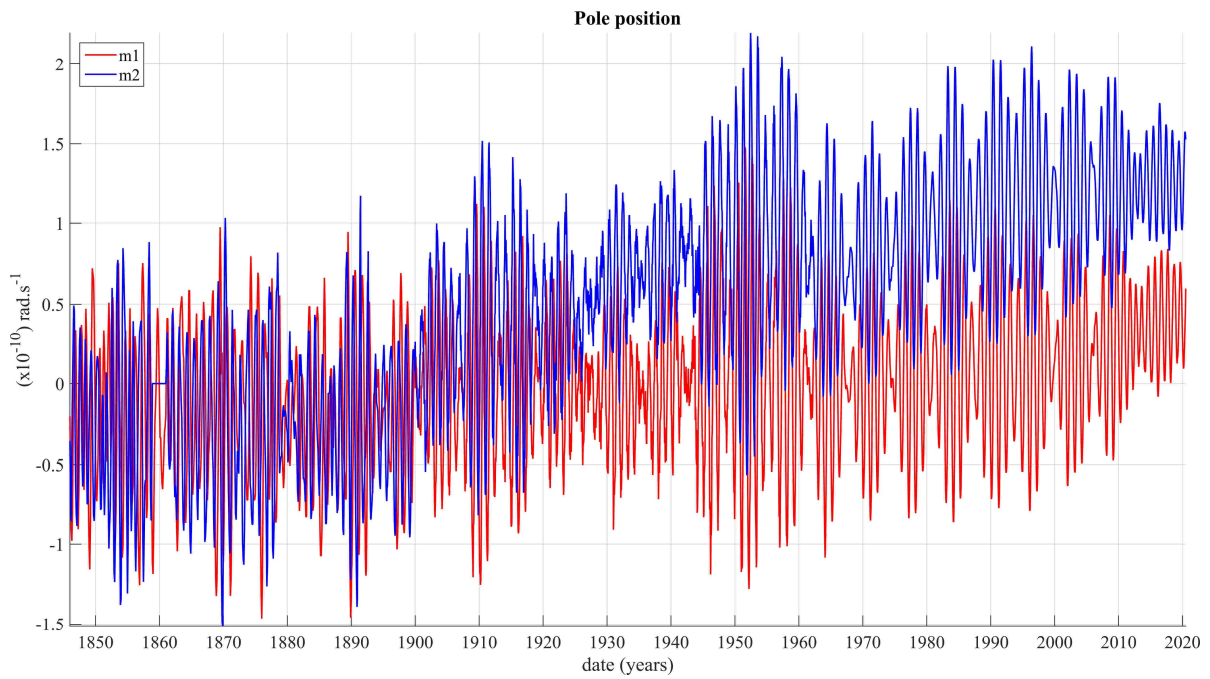
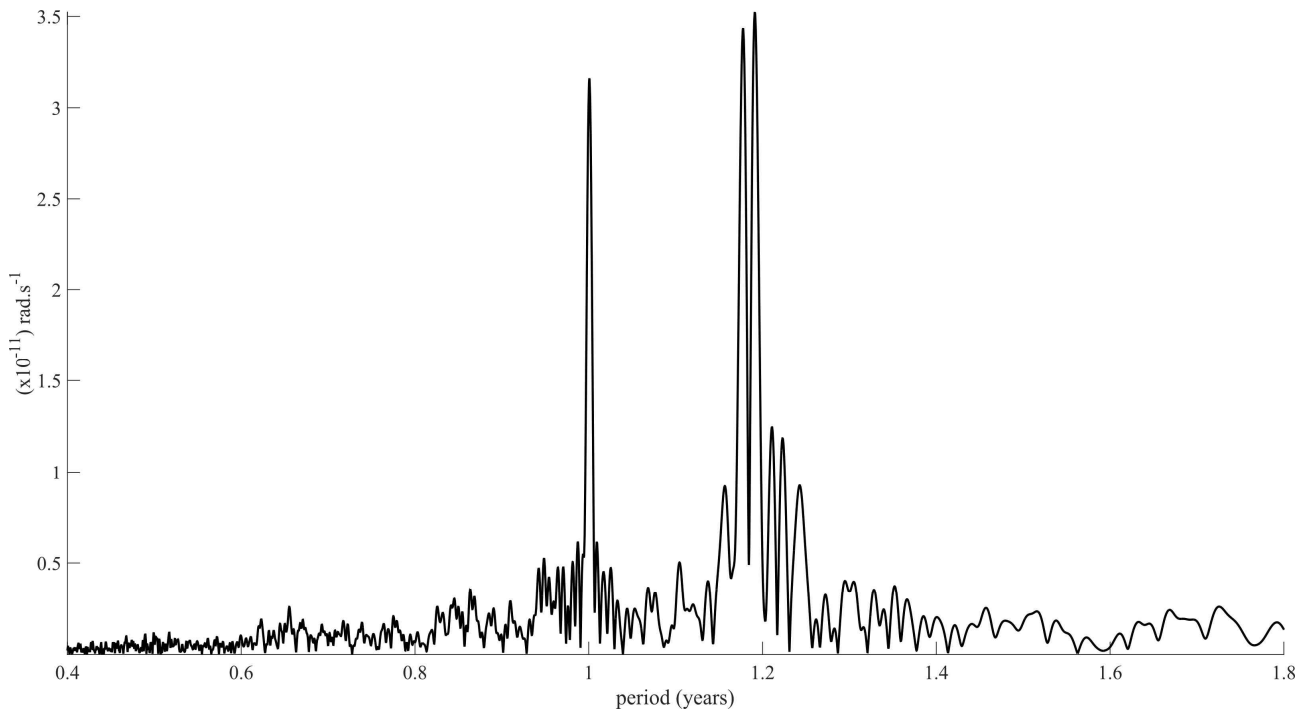


Figure 1: Components (m_1, m_2) of polar motion since 1846 (time series EOP-C01-IAU1980)



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Figure 2: Fourier spectrum of component m_1 , corrected for a degree 2 non linear trend

224 3-2 Commensurability: Then we use the concept of *commensurability*. One talks about
225 commensurability when the ratio of the periods of 2 planets can be expressed as a fraction with

226 integer numerator and denominator less than 9 (Mörth and Schlamming, 1979; Okhlopov, 2016;
227 Scafetta, 2020). Planets encounter a resonance and can be paired, and each pair can be considered
228 as a single object (an egregor or aggregate). Jupiter/Saturn and Uranus/Neptune form two pairs.
229 Pairs of pairs can also be considered, thus the set (Jupiter/Saturn)/(Uranus/Neptune). Many analyses
230 of sunspot series (Lassen and Friis-Christensen, 1995; Hataway, 2015; Usoskin et al., 2016; Le
231 Mouël et al., 2017; Stefani et al., 2019; Courtillot et al., 2021; Le Mouël et al., 2020a; Stefani et al.,
232 2020) and of a number of geophysical phenomena (Courtillot et al., 2013; Scafetta, 2016; Lopes et
233 al., 2017; Scafetta et al., 2019; Bignami et al., 2020; Le Mouël et al., 2019a; Le Mouël et al., 2019b;
234 Hilgen et al., 2020; Le Mouël et al., 2020b; Zaccagnino et al., 2020; Le Mouël et al., 2021) contain
235 components with periods that can be attributed to Jovian planets to first order, and all planets
236 including the telluric ones to second order (Courtillot et al., 2021). Table 1 lists planetary
237 commensurabilities following Mörth and Schlamming (1979). The periods found in our analysis
238 of the SSA components of polar motion (section 4) and of the derivatives of their envelopes (section
239 5) are labeled in red (there are 8, ranging from 1.2 to 165 years).

240 *Note:* Inspection of Table 1 may give the impression that there is a risk of “cherry picking”.
241 But certain periods that could have been reconstructed are not present, such as 103 yr that could
242 have been obtained with Neptune. Commensurabilities are built from two consecutive planets and
243 once their effect has been aggregated, they can be used in the next step of
244 aggregation/commensurability. The concept of commensurability is used by astronomers in order to
245 discriminate between planets and other objects. The corresponding periods are not random: they are
246 directly related to the revolutions of these bodies, and result from calculating means or subtracting
247 periods two by two. Thus what can be obtained is not random. Moreover, as already pointed out by
248 Mörth and Schlamming (1979) or more recently Scafetta (2016), uncovering a limited number of
249 common periods in a number of geophysical observables including sunspots cannot be due to
250 chance. The action of kinetic moments of Jovian planets on the Sun’s surface is what has allowed us
251 to predict the next solar cycle from the ephemerids in a previous paper (Courtillot et al., 2021).

252 *3-3 Singular spectral analysis:* Finally we extract the relevant components of polar motion
253 and ephemerids, and other long time series, with the help of *Singular spectral analysis* (SSA;
254 Vautard and Ghil, 1989; Vautard et al., 1992; for more up to date versions of the technique by the St
255 Petersburg school of mathematics, see Golyandina and Zhigljavsky, 2013). We have described and
256 used our own version of SSA in a number of previous papers (e.g. Lopes et al., 2017; Le Mouël et
257 al., 2021; Courtillot et al., 2021).

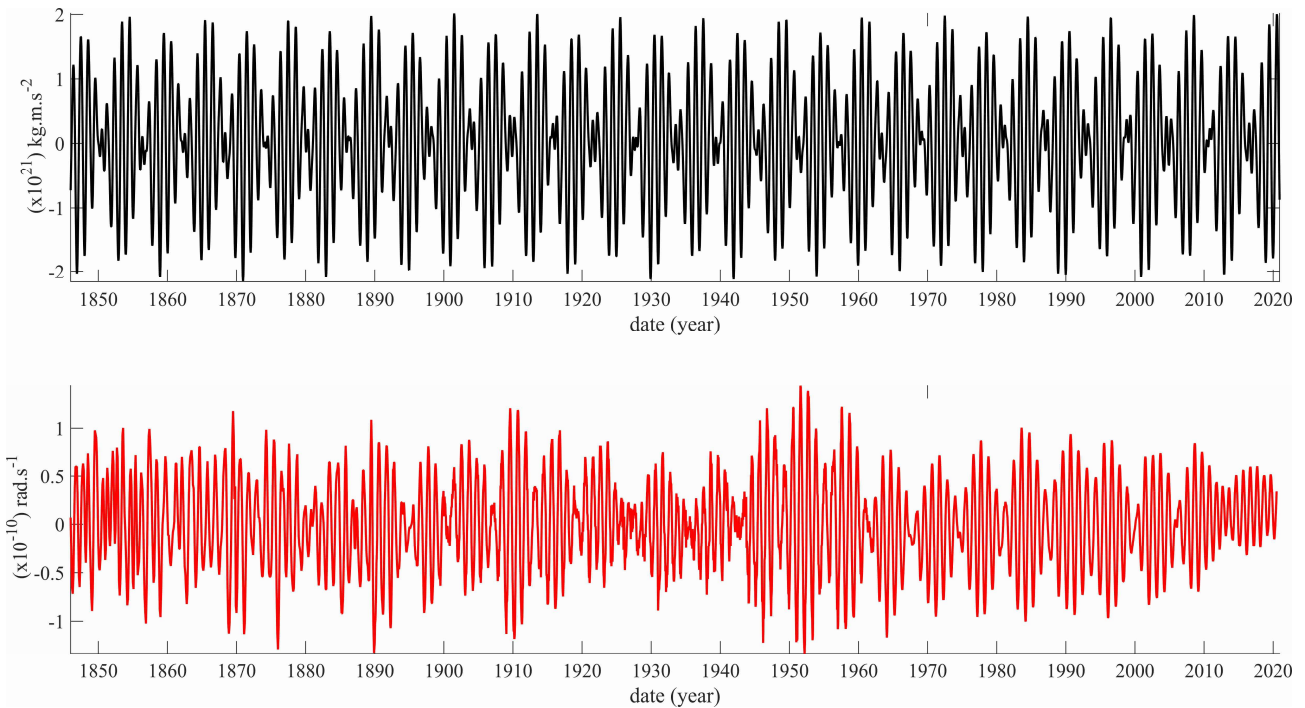
258 We discuss here a point that often comes up. An important factor in any time series analysis is
259 the size of the window used in classical (Fourier) filters, to avoid erroneous interpretations ([Kay and](#)
260 [Marple, 1981](#)). In SSA, the lagged-vector analysis window L should be sufficiently large so that
261 each eigen vector carries a large part of the information contained in the original time series. In
262 more mathematical words, one should work in the frame of Structural Total Least Squares (STLS)
263 for a Hankel Matrix ([Lemmerling and Van Huffel, 2001](#)). A second issue is the separability of
264 components. Many solutions are available, an exhaustive list being given by [Golyandina and](#)
265 [Zhigljavsky \(2013, chap. 2.5.3, page 75\)](#). In this paper, we have used the *sequential SSA*. The
266 window width L is variable, but remains close to 145 years.

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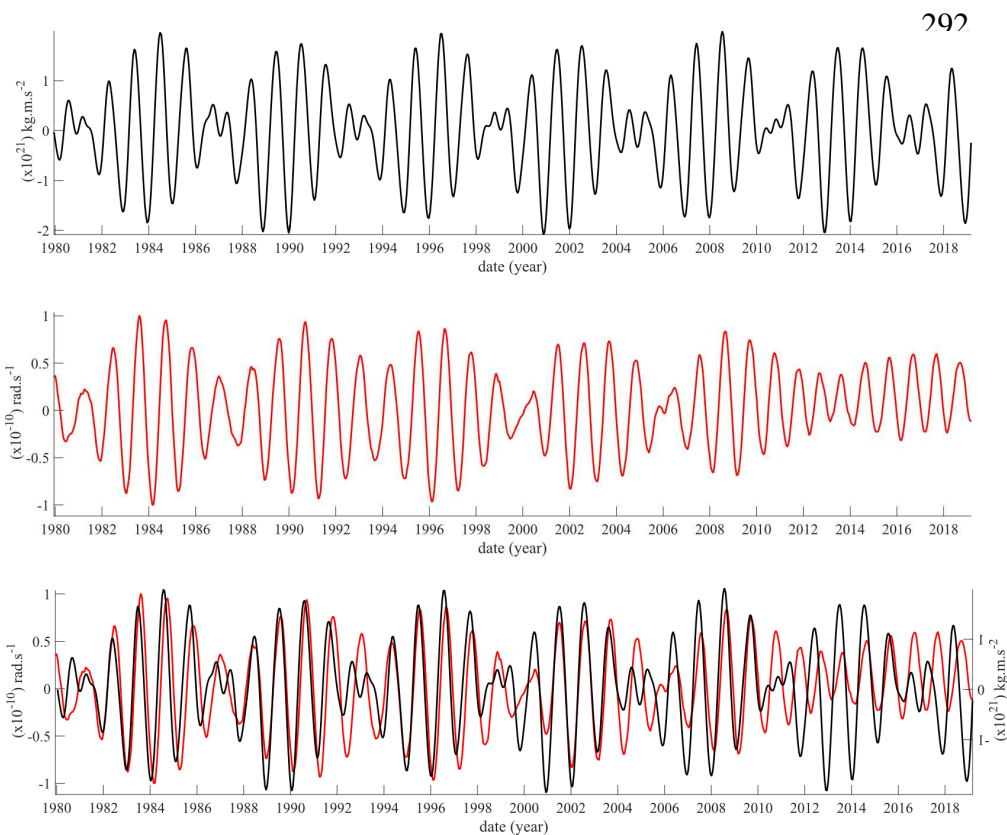
268 4 – First Results

269 *4.1 A striking reconstruction – Planets action on Earth’s rotation:* We start to test the ideas
270 and use the tools summarized above by comparing the sum of the forces exerted by the four Jovian
271 planets (using the IMCCE ephemerids) and the m_I component of polar motion (1846 to 2020) as
272 reconstructed from its SSA components, with the trend removed.

273 The polar coordinates m_I and m_2 are related to the forces acting on Earth ([Appendix 1](#)). To
274 first order, we can consider that the total “force” is simply proportional to the sum of individual
275 (Jovian) planetary kinetic moments, plus the Solar kinetic moment. We have computed these
276 moments from the planetary ephemerids, revolution periods and masses; their sum is given as the
277 top black curve in [Figure 3a](#). The red curve below is the reconstructed m_I polar coordinate from
278 [Figure 1](#), after it has been decomposed in its SSA components, then reconstructed from them, but
279 with the first component (the trend, see sub-section 4.2) removed. [Figure 3b](#) shows an enlargement
280 of the 1980-2019 part of [Figure 3a](#). The correlation is quite striking. It is indeed expected, as
281 already pointed out by [Laplace \(1799, book 5 in whole\)](#), that the Earth’s rotation axis should
282 undergo motions with components that carry the periods (and combinations of periods) of the
283 Moon, Sun, and planets, particularly the Jovian planets as far as their kinetic moments are
284 concerned (see also [Mörth and Schlaminger, 1979](#), and [Courtilot et al, 2021](#)). This first exercise
285 demonstrates that one should indeed consider planetary kinetic moments when describing the
286 motions of the Earth’s rotation axis. Based on this remarkable result, the aim of the rest of this
287 paper is to see whether characteristic components of the ephemerids are also found in Earth’s polar
288 motion and other related (or not obviously related) phenomena.



289 **Figure 3a:** Upper curve (in black) the sum of the forces of the four Jovian planets affecting Earth.
 290 Ephemerids from the IMCCE. Lower curve (in red) the m_l component of polar motion (1846-2020)
 291 reconstructed with SSA and with the trend (Markowitz) removed.



307
 308 **Figure 3b:** Enlargement of **Figure 3a** (1980-2019) and superposition of the 2 curves.

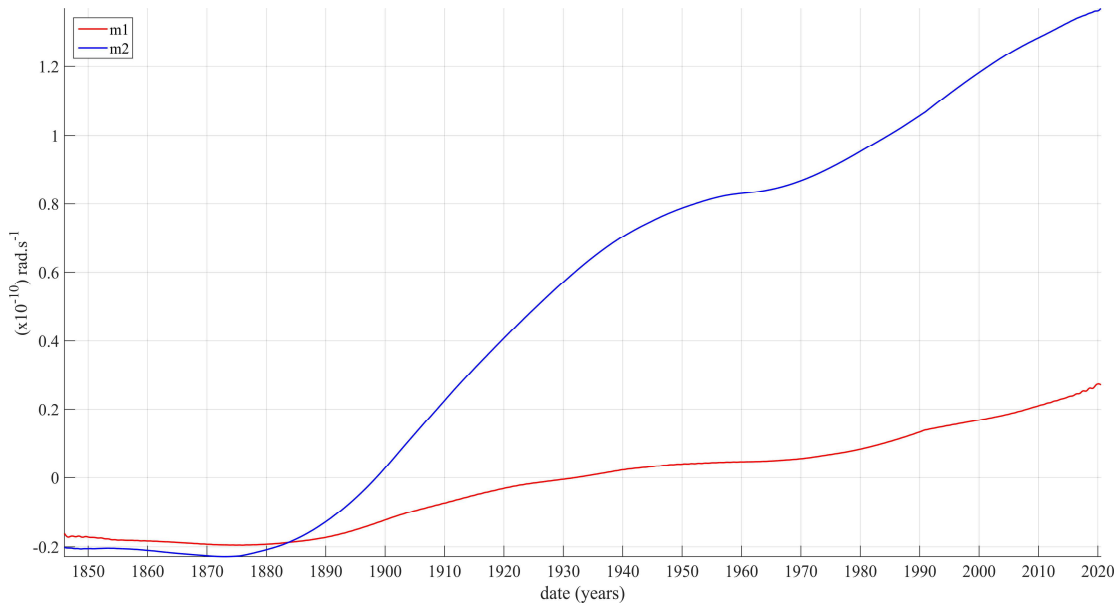
Planet		Commensurability (ϵ)	I/J	Associated periods (yr)	
1 {	Mercury	Venus	-6.6×10^{-3}	2/5	0.18, 0.42
		Earth	10×10^{-3}	1/4	0.38, 0.62
		Mars	8.3×10^{-3}	1/8	0.78, 1.02
2 {	Venus	Earth	10×10^{-3}	3/5	0.19, 0.80
		Mars	5.6×10^{-3}	1/3	0.59, 1.20
3 {	Jupiter	Saturn	1.9×10^{-3}	3/8	9.79, 21.64
		Uranus	-1.7×10^{-3}	1/7	36.06, 47.91
4 {	Uranus	Neptune	9.6×10^{-3}	1/2	40.40 , 124.37
		Pluto	6.0×10^{-3}	1/3	81.75, 165.72
5	Pluto	Neptune	-8.3×10^{-4}	2/3	41.35, 206.13
1	2	-6.4×10^{-3}	1/4	0.29, 0.49	
		-1.3×10^{-3}	1/5	0.41, 0.60	
		2.5×10^{-3}	1/2	0.01, 0.79	
		6.5×10^{-3}	7/9	0.11, 0.91	
3	4	-7.7×10^{-3}	1/4	15.30, 25.09	
		-3.2×10^{-3}	1/9	57.29 , 67.08	
		1.9×10^{-3}	5/9	9.38, 31.02	
		6.5×10^{-3}	1/6	51.36, 73.00	
3	5	1.3×10^{-3}	1/4	15.78, 25.57	
		-6.3×10^{-3}	1/9	98.17, 107.96	
		2.3×10^{-3}	5/2	9.85, 31.49	
		6.5×10^{-3}	1/6	92.24 , 113.88	

310

311 **Table 1:** Commensurable periods of pairs and pairs of pairs of planets computed following
 312 [Mörth and Schlamming \(1979\)](#). The periods encountered in the present paper are printed in red.

313

314



326 **Figure 4:** First SSA components of polar motion (trend) since 1846, called the Markowitz
 327 drift. Component m_1 is in red and m_2 in blue.

328

329 We next analyze one by one the leading SSA components of the Earth’s rotation pole
 330 coordinates.

331 *4.2 First SSA Component (Markowitz)*

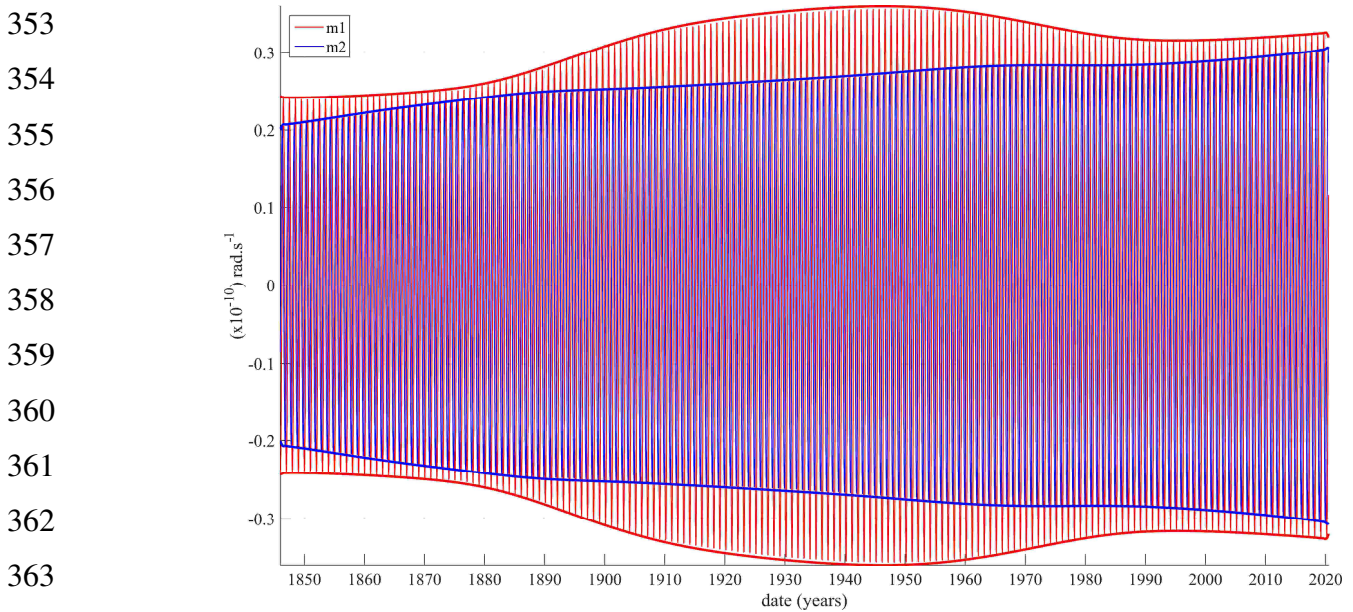
332 The first SSA components, shown in **Figure 4**, correspond to the mean trend of polar motion
 333 called the Markowitz drift (**Markowitz, 1968**). The drift velocity is on the order of 13 cm/yr and is
 334 principally carried by the E-W component m_2 . As noted by a reviewer, these curves show changes
 335 in slope and inflection points that are reminiscent of the recent evolution of the Earth’s global
 336 surface temperature (**Le Mouél et al., 2019b**, Figure 20). This important point is not discussed
 337 further in the present paper.

338 *4.3 Second SSA Component (Annual)*

339 The second SSA component is the forced annual oscillation (**Figure 5**). On that annual
 340 oscillation, **Lambeck (2005, chapter 7, page 146)** writes “*The seasonal oscillation in the wobble is*
 341 *the annual term which has generally been attributed to a geographical distribution of mass*
 342 *associated with meteorological causes. Jeffreys in 1916 first attempted a detailed quantitative*
 343 *evaluation of this excitation function by considering the contributions from atmospheric and*
 344 *oceanic motion, of precipitation, of vegetation, and of a polar ice. Jeffreys concluded that these*
 345 *factors explain the observed annual polar motion, a conclusion that is still valid today”.*

346 **Figure 5** shows that the annual components of m_1 and m_2 are significantly modulated, and in

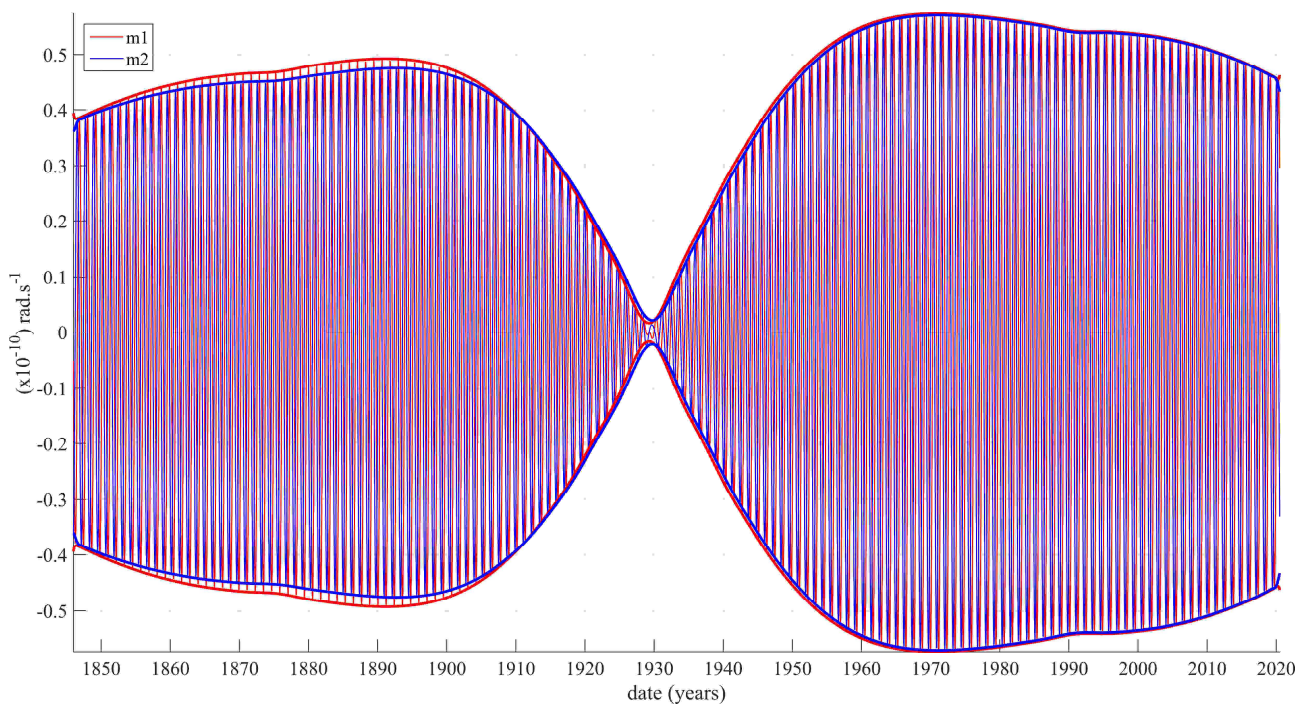
347 different ways (recall that the excitation functions are sums of sines and cosines with constant
 348 weights; [Lambeck, 2005, page 153, equations 7.1.9](#)). In the generally accepted theory, modulation
 349 is thought to be a response to reorganization of oceanic and atmospheric masses. We note in the
 350 modulation of m_1 the suggestion of a periodicity on the order of 150 years or more that could
 351 correspond to the [Jose \(1965\) 171.5 yr cycle](#). Note that, given uncertainties, the Jose cycle could
 352 actually be the [Suess- de Vries ~200 yr cycle \(Stefani et al. 2020\)](#).



364 **Figure 5:** 2nd SSA comp. of polar motion (annual oscillation) since 1846. (m_1 red, m_2 blue).

365 4.4 Third SSA Component (Chandler)

366 **Figure 6** shows the third SSA component, that is the Chandler component. Its amplitude is
 367 twice that of the annual component and its behavior is very different. The modulations are very
 368 large, similar for m_1 and m_2 , and undergo a sharp and simultaneous change in phase and amplitude
 369 in 1930. Many scientists have studied this phase change ([Hinderer et al., 1987](#); [Runcorn et al., 1988](#);
 370 [Gibert et al, 1998](#); [Bellanger et al, 2001](#); [Bellanger et al, 2002](#); [Gibert and Le Mouél, 2008](#)). The

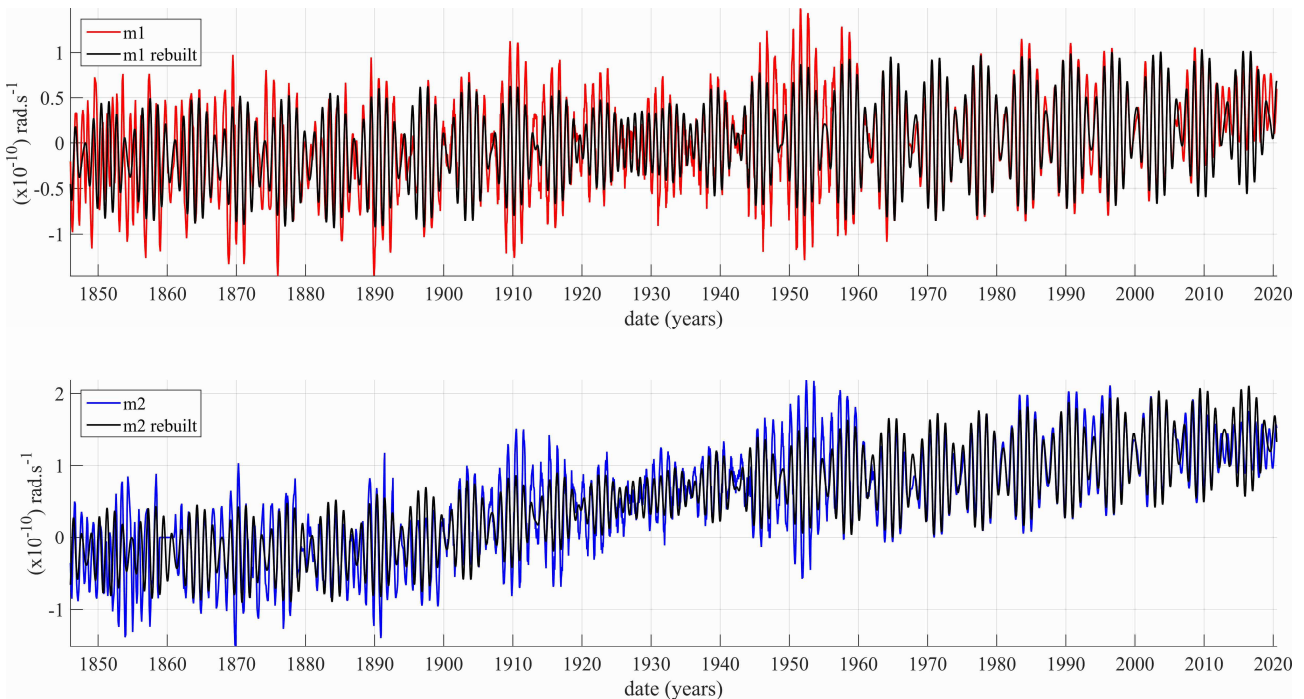


371 **Figure 6:** Third SSA components of polar motion (Chandler oscillation) since 1846.
 372 Component m_1 is in red and m_2 in blue.

373
 374 Chandler oscillation extracted by SSA is similar to that obtained with wavelets by [Gibert et al. \(1998\)](#). It is also as regular as that obtained with SSA by [Gorshkov et al. \(2012\)](#).

375
 376 When the first three SSA components of m_1 and m_2 are added, they account for 73% of the
 377 original variance. The quality of that incomplete reconstruction is shown in [Figure 7](#).

378 Pushing the SSA analysis further reveals an oscillation with period 1.22 yr with an 18.6 yr
 379 modulation (the nutation), one with period 1.15 yr with a symmetrical modulation as in the case of
 380 the Chandler term, one with period 1.10 yr. Some of these (quasi-) periods have already been found
 381 using SSA on time series of sunspots ([Le Mouél et al, 2020a](#)). These periods seem to be linked to
 382 the ephemerids of solar system planets, which has been used by [Courtilot et al \(2021\)](#) to predict the



383
 384 **Figure 7:** Reconstruction of polar motion since 1846 using only its first three SSA
 385 components. Top: observed component m_1 in black and reconstructed in red; bottom: observed
 386 component m_2 in black and reconstructed in blue.

387
 388 date and amplitude of the next solar (Cycle 25) maximum. Other components are compatible with
 389 the Schwabe (11 yr) and Hale (22 yr) cycles. The 5.5 yr cycle is often associated with the Schwabe
 390 cycle ([Usoskin, 2017](#)), but not all authors agree. Moreover, these components are found only in m_2

391 and are much smaller in amplitude, on the order of 10^{-13} to 10^{-14} rad.s⁻¹ vs 10^{-10} to 10^{-11} rad.s⁻¹ for
 392 the first three (Lopes et al, 2017; Japaridze et al, 2020). When all components from the trend to the
 393 Hale cycle are added, they account for 95% of the total variance of the original series. Except for
 394 the 1.10 and 1.15 yr components, all others are found in the table of planetary interactions (Table
 395 1).

396

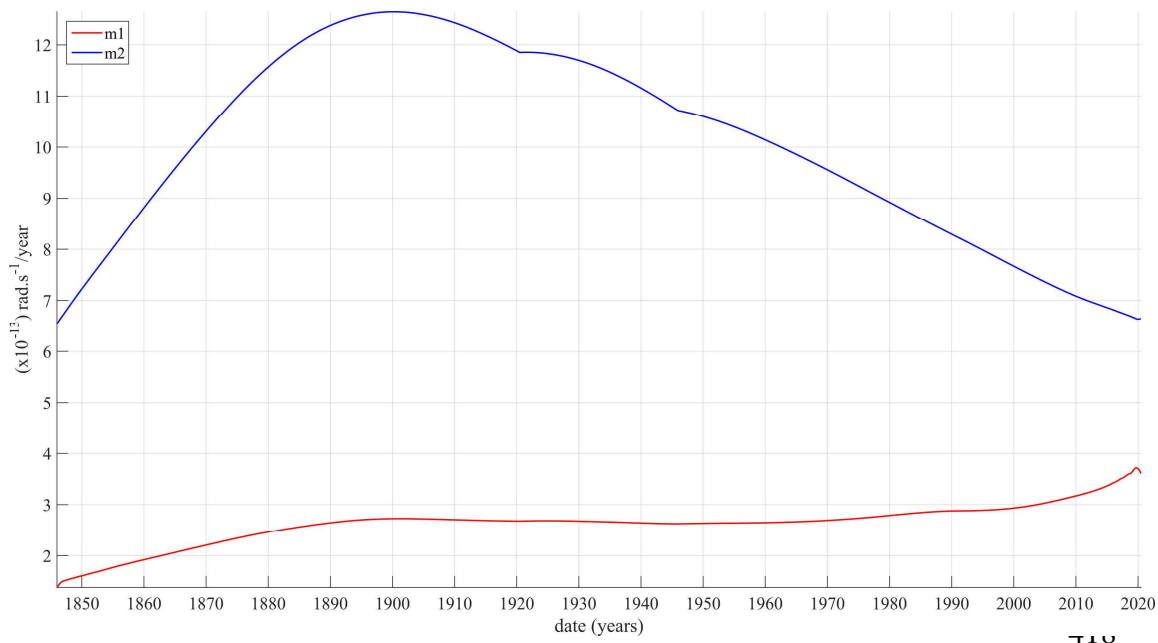
397 5 - On Some Derivatives of SSA Components of Polar Motion

398 System (D) expresses that there is a link between a force and the derivative of the resulting
 399 polar motion (Appendix 1, equation 2). In other words Earth acts as a natural integrator (Appendix
 400 1, equation 2 implies that m is an integral of ξ ; see Le Mouél et al, 2010). This leads us to analyze
 401 the derivatives of the first three (largest) SSA components identified in the previous section.

402 5.1 Markowitz Drift

403 We first calculate the derivative of the Markowitz drift (Figure 4), and analyze its major SSA
 404 components. They are a trend (Figure 8a), a 90 yr pseudo-cycle (Figure 8b), a 40 yr pseudo-cycle
 405 (Figure 8c), a 22 yr period and an 11 yr period (Figure 8d).

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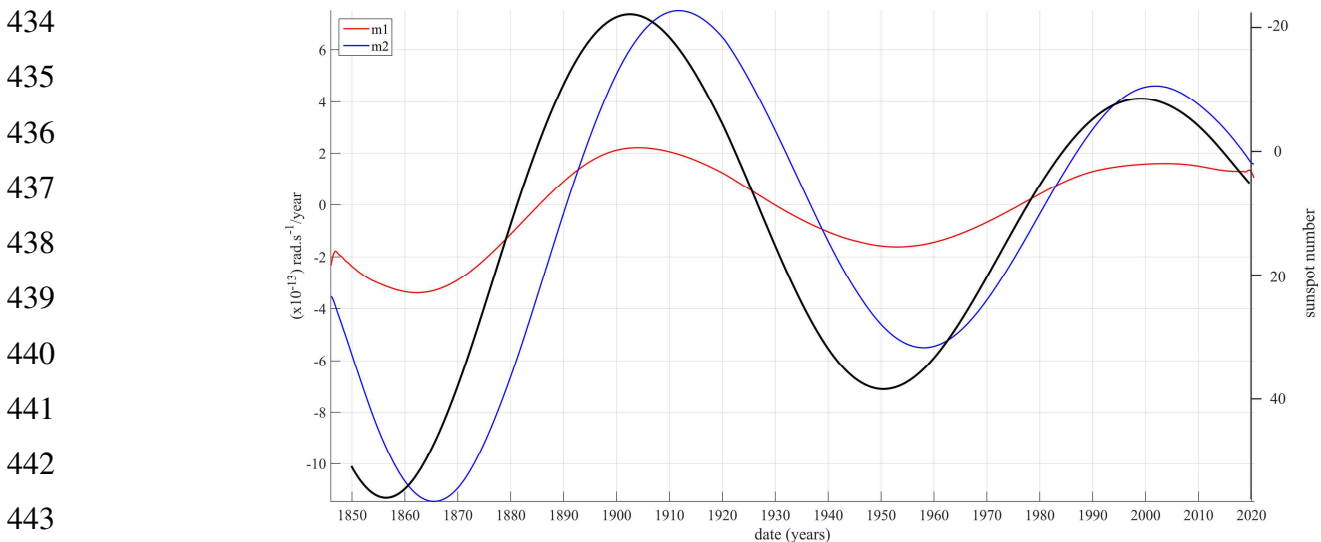


419 **Figure 8a:** First SSA component (trend) of the derivative of the Markowitz drift (first SSA
 420 component of polar motion). Component m_1 in red and m_2 in blue.

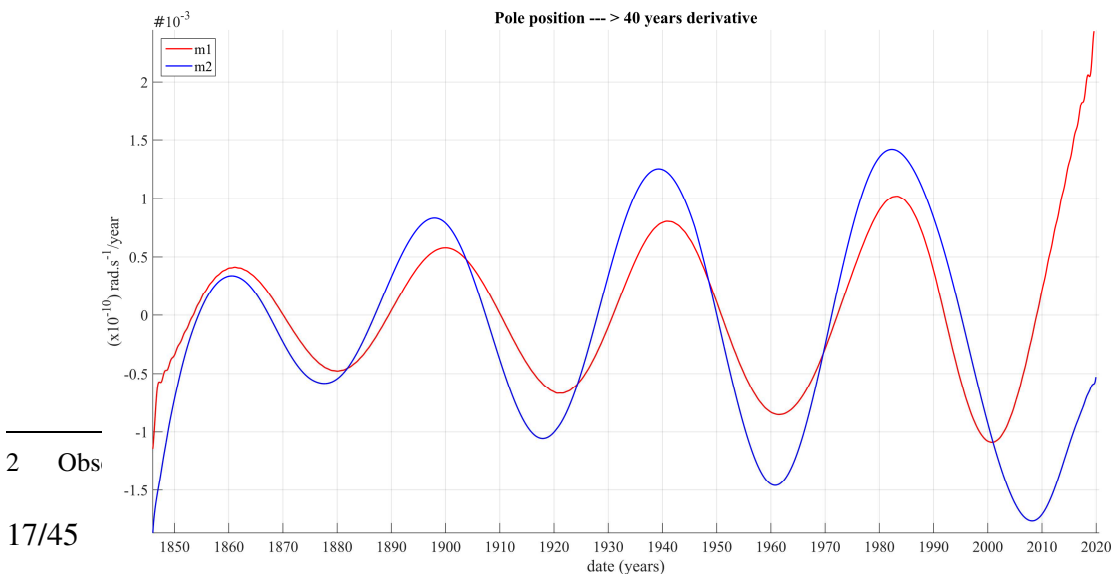
421

422 The 90 yr cycle is strongly correlated with the Gleissberg (1939) solar cycle (Figure 8b). Le

423 [Mouël et al \(2017\)](#) have obtained a period of 90 ± 3 yr from sunspot series `SN_m_tot_V2.0`². It
 424 corresponds to a characteristic period in the ephemeris of Uranus ([Table 1](#)). There is also a close
 425 correspondence of periods for the 11 yr oscillation ([Figure 8d](#)). The drift could be linked to the
 426 modulation and varying “periodicity” of sunspots (8 to 13 yr). This is close to a characteristic
 427 period of Jupiter’s ephemeris. The trend ([Figure 8a](#)) could be linked to the [Jose \(1965\)](#) 171.5 yr
 428 cycle, attributed to Neptune ([Table 1](#)) or to the Suess-de Vries ~200 yr cycle ([Stefani et al., 2020](#)).
 429 Finally, the 40 yr component has been shown by [Mörth and Shlamminger \(1979\)](#); see also [Courtillot](#)
 430 [et al, 2021](#)) to correspond to a commensurable revolution period of the four Jovian planets. It is
 431 interesting to point out that in both terrestrial polar motion and solar activity (as studied through the
 432 proxy of sunspots) the first 3 components that emerge from SSA are a trend, then the Gleissberg
 433 and Schwabe quasi-cycles.



444 **Figure 8b:** Second SSA component (90 yr period)) of the derivative of the Markowitz drift (first
 445 SSA component of polar motion). Component m_1 in red and m_2 in blue. In black: Gleissberg cycle
 446 extracted from sunspots (sign reversed).



454 **Figure 8c:** Third SSA component (40 yr period)) of the derivative of the Markowitz drift (first
 455 SSA component of polar motion). Component m_1 in red and m_2 in blue.

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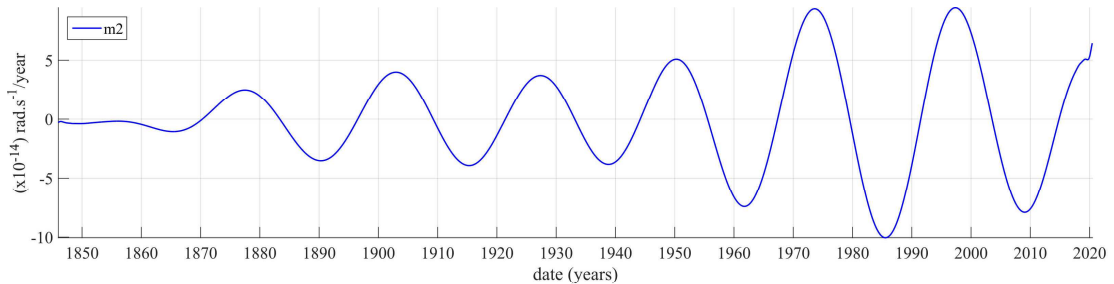
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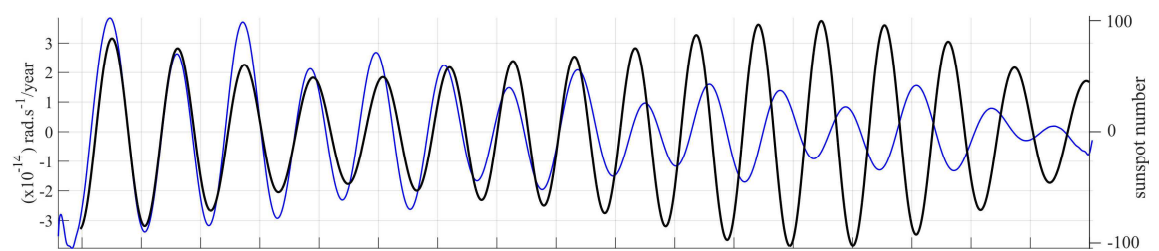
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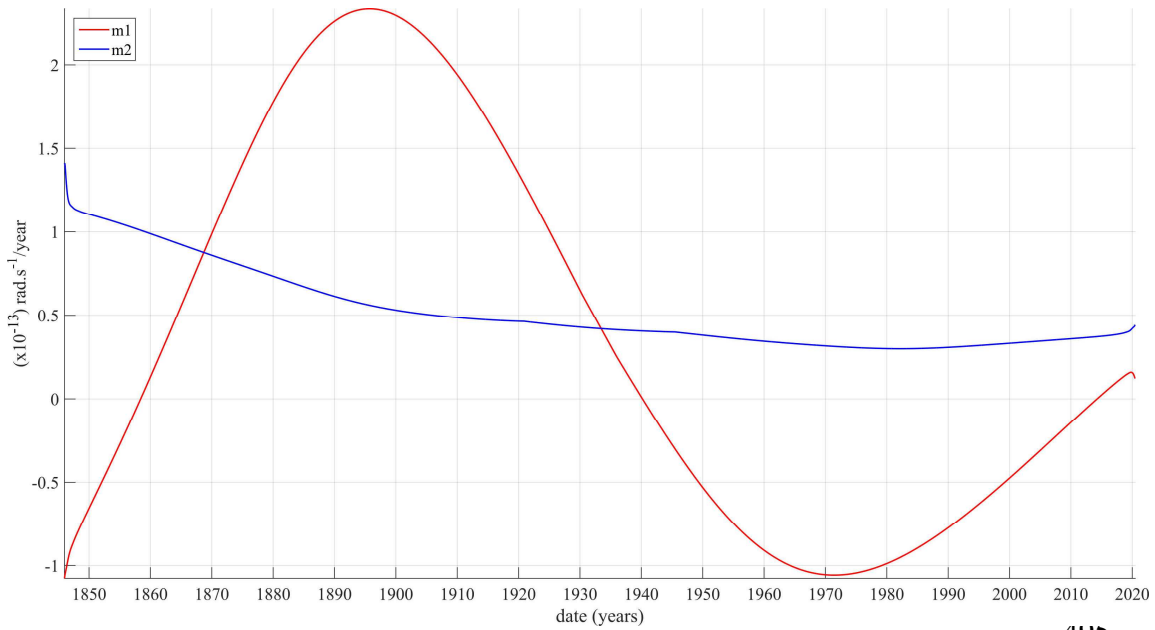
468 **Figure 8d:** 22 yr SSA component (top, component m_2 in blue) and 11 yr SSA component
 469 (bottom, component m_2 in blue) of the derivative of the Markowitz drift (first SSA component of
 470 polar motion). Bottom, black curve: the 11 yr Schwabe cycle extracted by SSA from the sunspot
 471 series (sign reversed).

472

473 5.3 Envelope of the Forced Annual Oscillation

474 We next turn to the derivative of the envelope of the forced annual oscillation (**Figure 5**). Its
 475 first SSA component, the trend, is shown in **Figure 9a**. For m_1 this trend is compatible with a little
 476 more than one period of a sine curve with a period close to 170 yr, that is the Jose (**1965**) solar
 477 cycle, corresponding to the ephemeris of Neptune (or again given uncertainties to the Suess-de
 478 Vries ~200 yr cycle). The next SSA component is a 70 yr cycle for m_1 and a 60 yr cycle for m_2
 479 (**Figure 9b**). These periods, or pseudo-periods, are among those resulting from combinations of
 480 ephemerids of the Jovian planets (**Mörth and Schlaminger, 1979; Scafetta, 2020; Table 1**). The 60
 481 yr cycle had already been found in sunspot series by **Scafetta (2010)** and **Le Mouél et al (2020a)**.
 482 We had also seen it as an important component of series of global temperature and PDO and AMO
 483 oceanic indices (**Courtilot et al, 2013**).

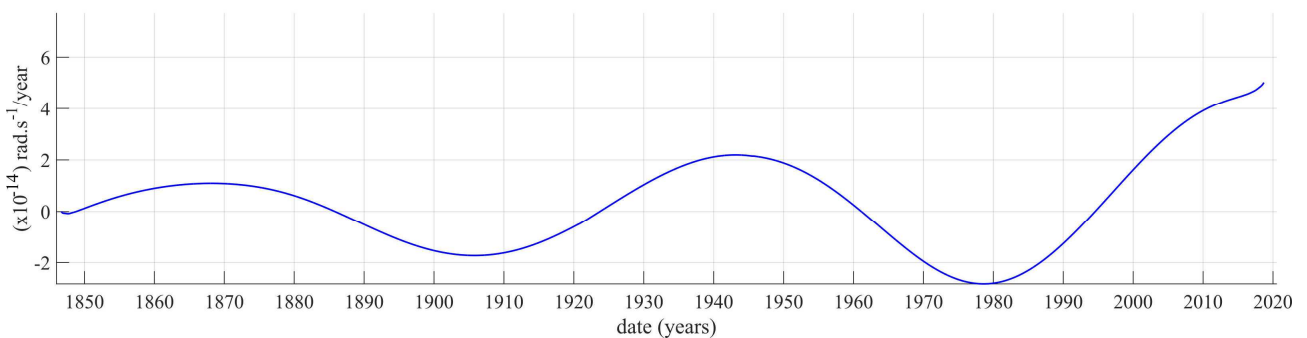
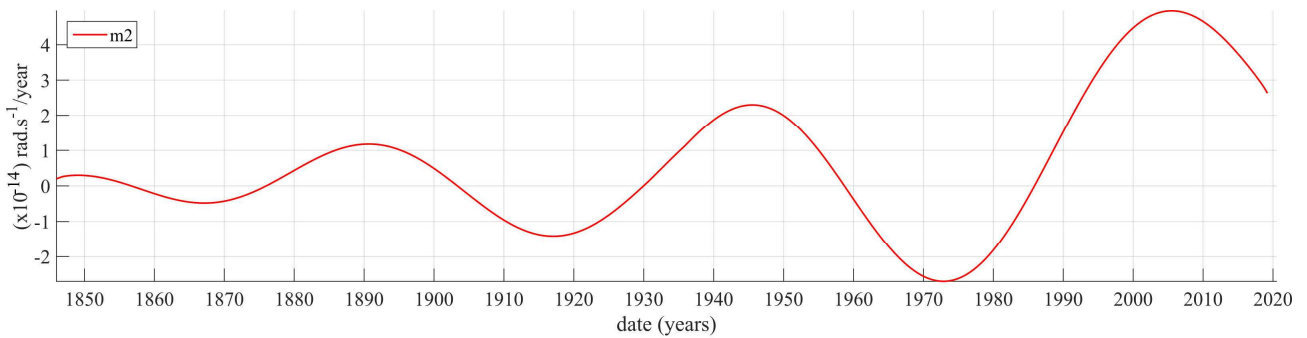
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495

496 **Figure 9a:** First SSA component (trend) of the derivative of the envelope of SSA component 2
497 (annual oscillation) of polar motion.

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501 **Figure 9b:** Second SSA component (60 years and 70 years) of the derivative of the envelope
502 of SSA component 2 (annual oscillation) of polar motion.

503 *5.4 Free Chandler Oscillation*

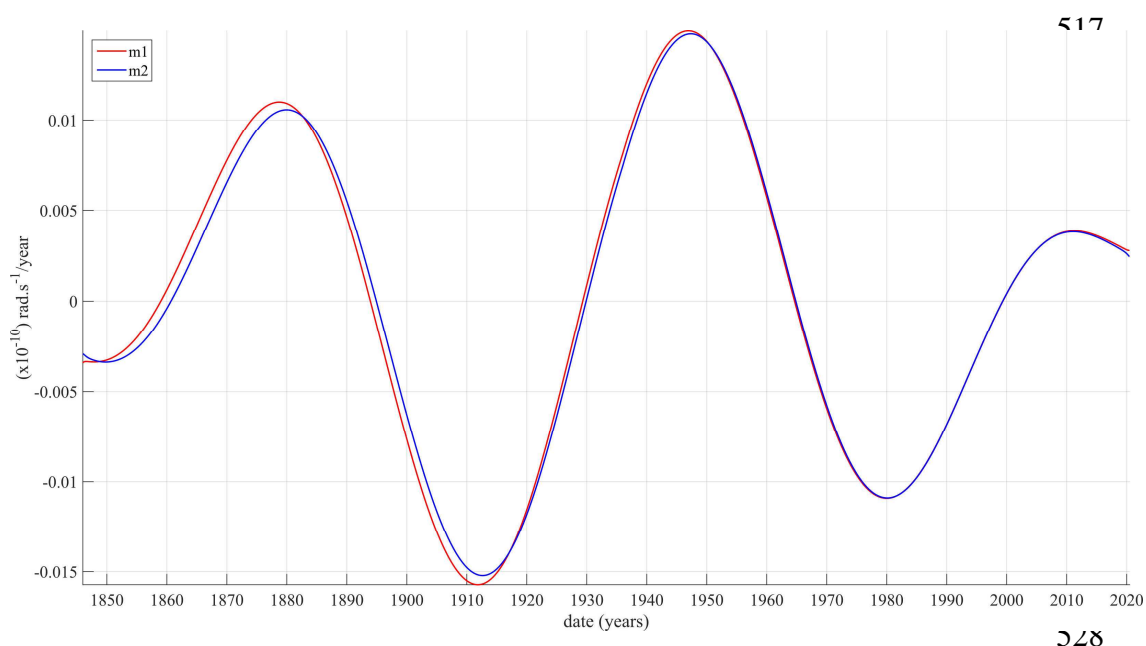
504 We now undertake the SSA analysis of the derivative of the envelope of the Chandler
 505 oscillation (Figure 6). We find components with periods 70, 40, 30 and 22 yr (Figures 10a to 10d).
 506 It is remarkable that the components for m_1 and m_2 are quasi-identical and have a very regular
 507 behavior, close to sine functions but with some slower modulation: they could be described as
 508 “astronomical” (as opposed to “astrophysical”, as defined by [Mayaud, 1980](#)).

509

510 **6 – Further Examples**

511 We can illustrate further how Jovian planets influence polar motion with the combined effects
 512 of the pair Uranus (84 yr) – Neptune (165 yr): this pair has revolution periods compatible with the
 513 envelopes in Figures 5, 6 and 8b. Figure 11a shows the sum of the kinetic moments of these two
 514 Jovian planets as seen by Earth, which explains the annual oscillation whose envelope is of interest
 515 to us. Using again SSA, we have extracted the first four components of that signal; we show three

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529 **Figure 10a:** First SSA component (70 yr quasi-period) of the derivative of the envelope of the
 530 Chandler oscillation.

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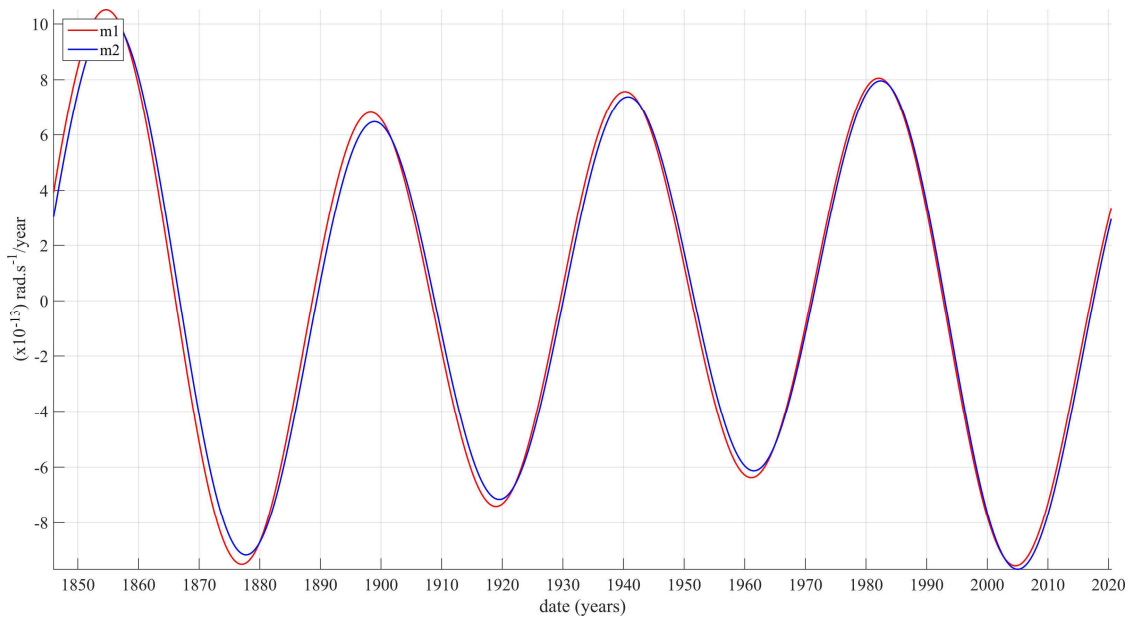
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546 **Figure 10b:** Second SSA component (40 yr quasi-period) of the derivative of the envelope of
547 the Chandler oscillation.

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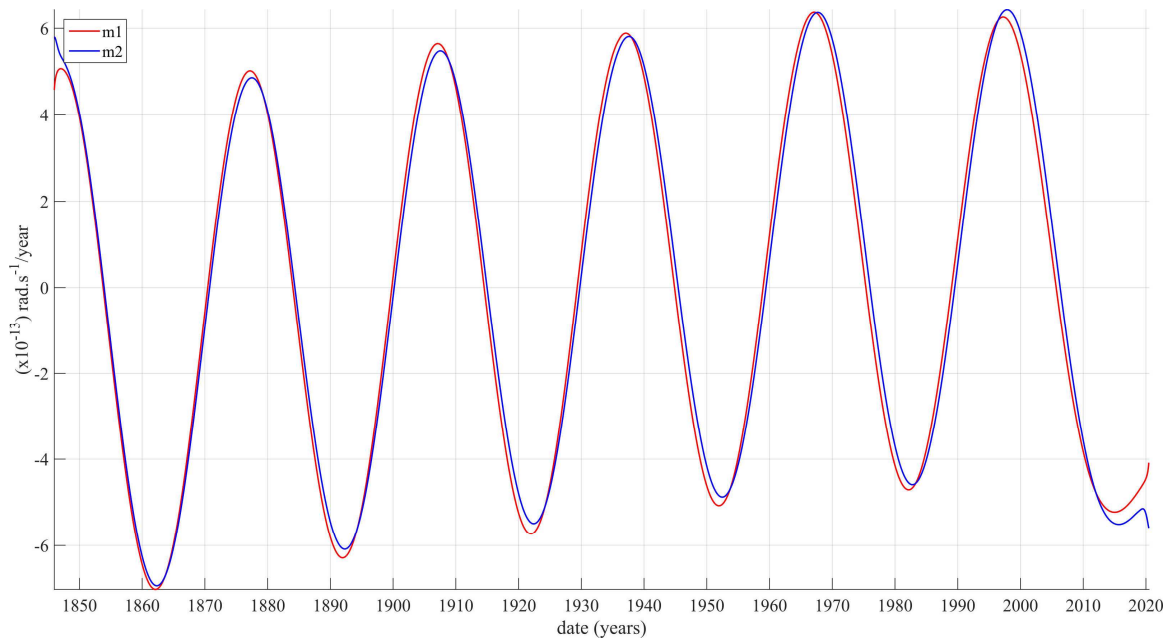
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563 **Figure 10c:** Third SSA component (30 yr quasi-period) of the derivative of the envelope of
564 the Chandler oscillation

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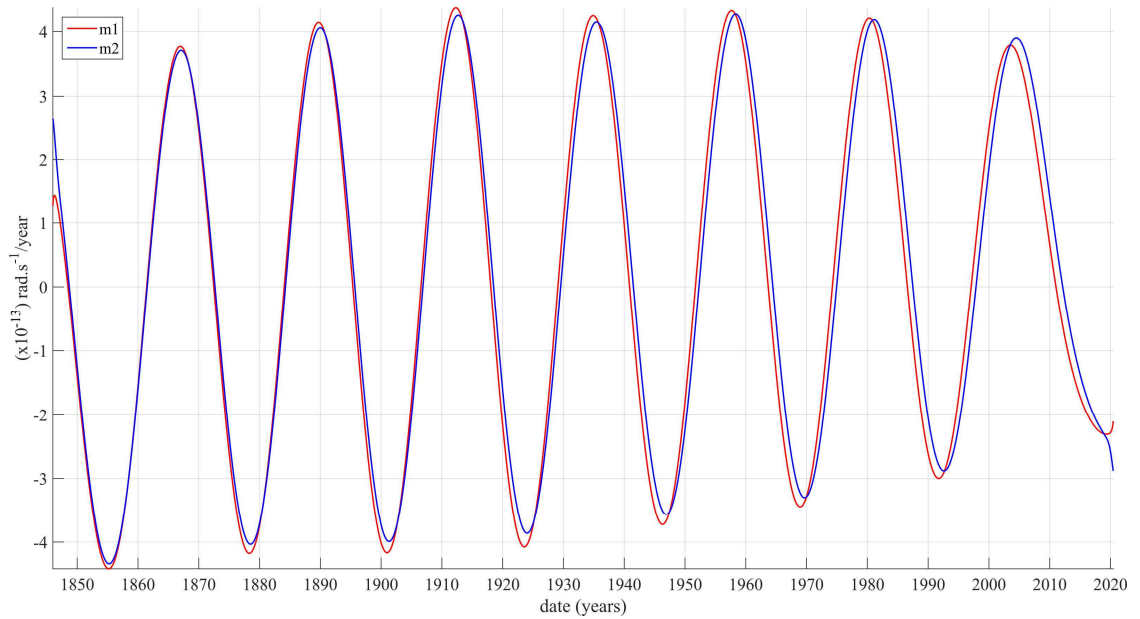
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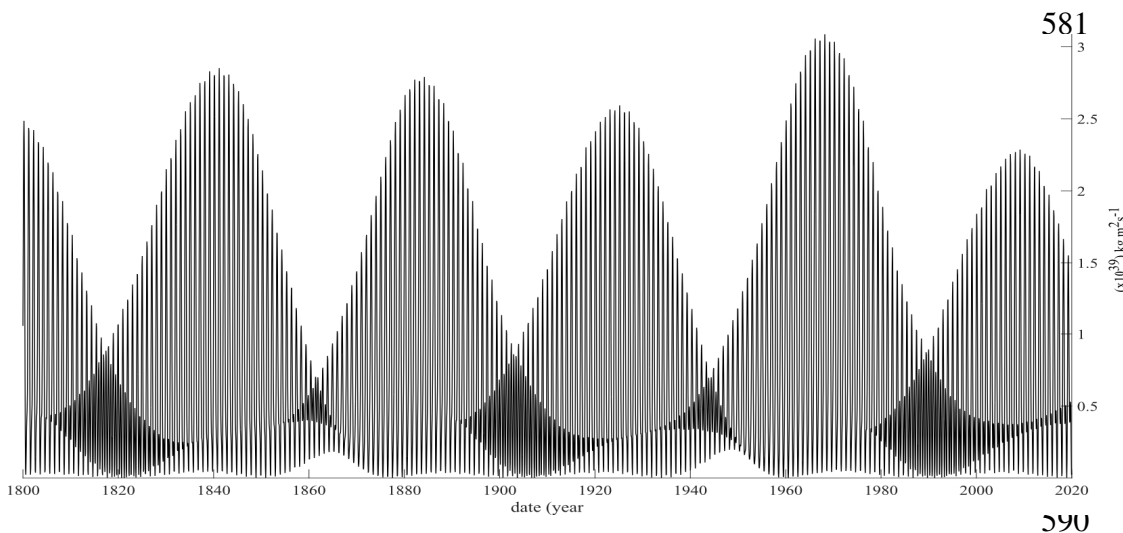


578

578 **Figure 10d:** Fourth SSA component (22 yr quasi-period) of the derivative of the envelope of

579 the Chandler oscillation

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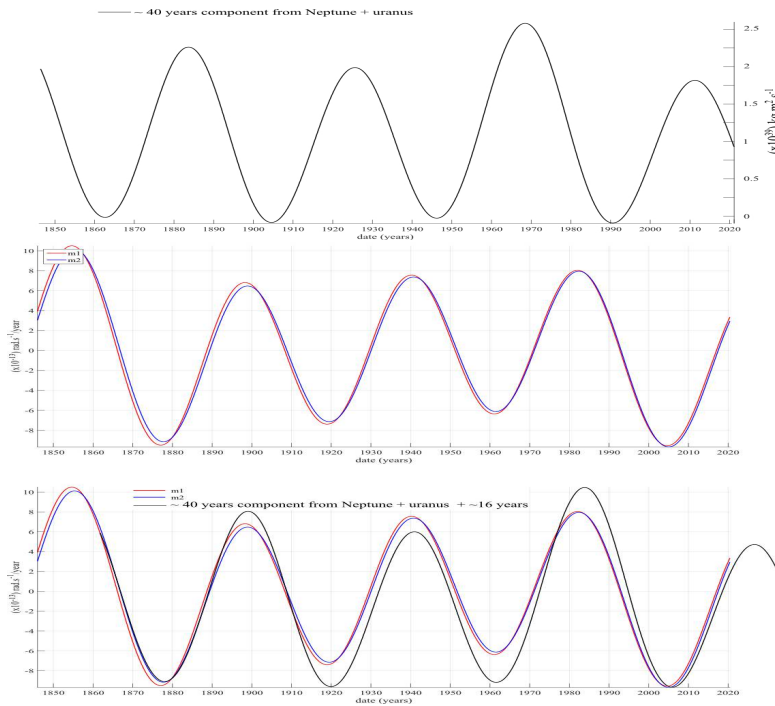


591 **Figure 11a:** The sum of the kinetic moments of Uranus and Neptune in a geocentric reference

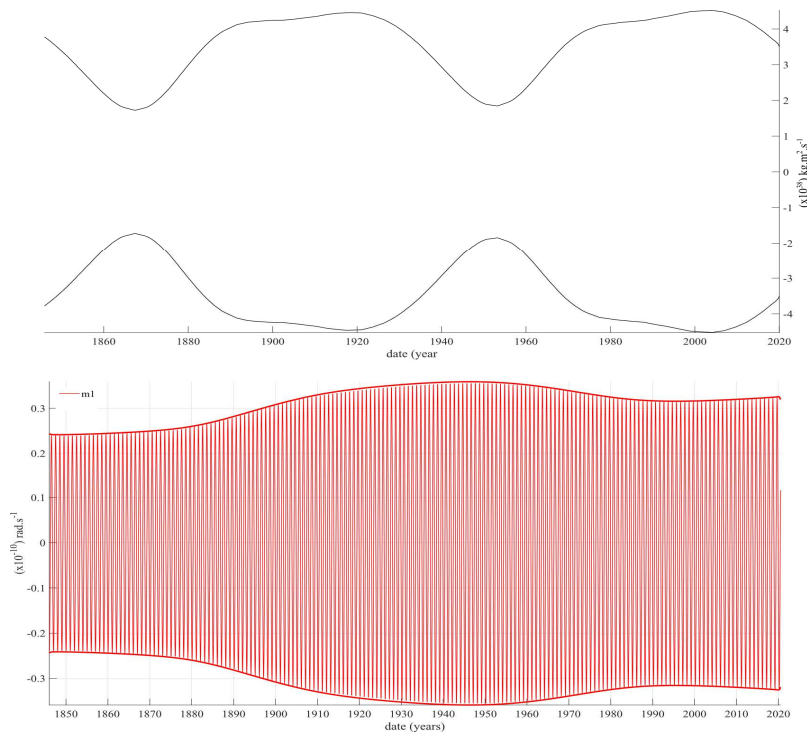
592 frame.

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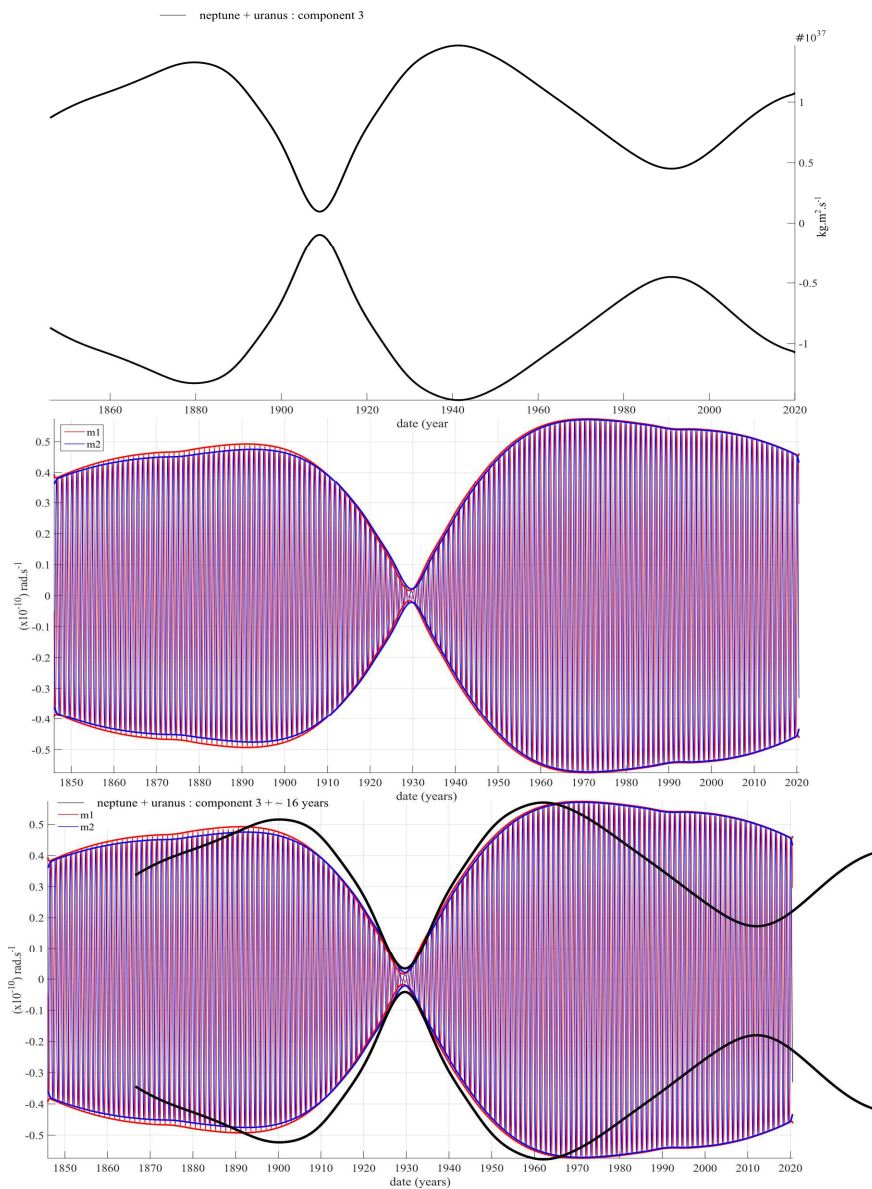


607 **Figure 11b:** Superposition (bottom) of the 40yr SSA (first) component of the curve shown in
 608 **Figure 11a** (top) and the component of polar motion with similar pseudo period in **Figure 10b**
 609 (middle). The top curve has been shifted by 16 yr with respect to the middle one in the bottom
 610 comparison.



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 624 **Figure 11c:** Second component of the Uranus-Neptune pair (top) and forced annual oscillation
 625 of the polar motion m_1 (bottom).

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649 **Figure 11d:** Third SSA component of the Uranus-Neptune couple (top), Chandler oscillation (third
650 SSA component of polar motion; middle) and comparison of the two with a 16 yr offset.

651

652 of them, from which the annual and semi-annual oscillations are absent. The first one (**Figure 11b**)
653 has a pseudo period of about 40yr, similar to the envelope of the component of polar motion shown
654 in **Figure 10b**. When a 16yr phase shift is taken into account, the correlation is remarkable (**Fig 11b**,
655 bottom frame). The second component has nodes and bellies that are reminiscent of the envelope of
656 the annual forced m_1 component of polar motion (**Figure 11c**). The third component of the Uranus-
657 Neptune pair compares well with the Chandler free oscillation, particularly so when a 16 yr offset is

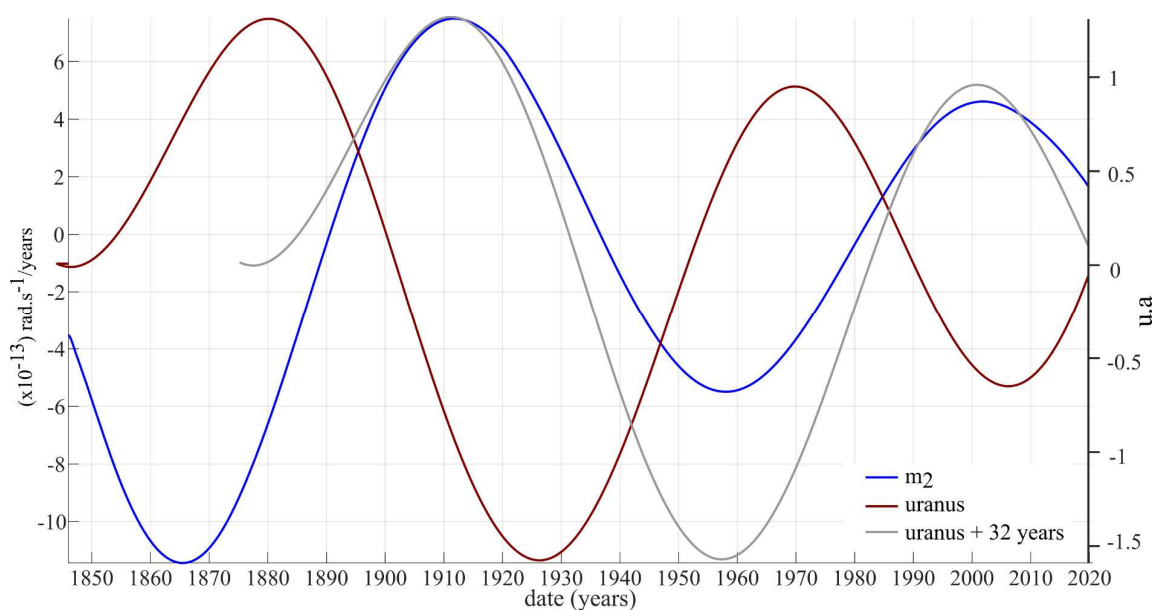
658 taken into account (Figure 11d).

659 Finally, in Figures 12a to 12c, we superimpose the signatures (components) of the ephemeris
 660 of Jovian planets on the components of polar motion. In Figure 12a, the 90 yr component of the
 661 envelope of m_2 matches the ephemeris of Uranus offset by 32 years. In Figure 12b, the 165 yr
 662 component of the envelope of m_1 matches the ephemeris of Neptune, also offset by 32 years. In
 663 Figure 12c, the 30 yr component of the envelope of m_1 of the Chandler oscillation matches the
 664 ephemeris of Saturn offset by 15 years. The 11 yr component detected in the m_2 component of the
 665 derivative of the Markowitz drift (Figure 8d) has a variable phase drift with respect to the
 666 ephemeris. But, whereas “solar” components (periodicities) do appear at 22 and 11 yr (and 5.5 yr?)
 667 in polar motions, they are 3 to 4 orders of magnitude smaller than the leading components we
 668 discuss here.

669 We have seen that the sum of the Markowitz drift, annual oscillation and Chandler oscillation
 670 explain some 70% of polar motion. The same is true for the leading components of sunspots, i.e. the
 671 sum of the trend (Jose ~171.5 yr cycle), Schwabe cycle (~11 yr) and Gleissberg cycle (~90 yr) (on
 672 the same time range). These periods correspond to those of Neptune (~165 yr), Uranus (84 yr) and
 673 Jupiter (11.8 yr).

674 Many if not most of the (quasi-)periods found in the SSA components of polar motion, of
 675 their modulations, of their derivatives can be associated with the Jovian planets. Only one, the 432-
 676 434 day period is due to the Earth’s mass and moments of inertia and not to the Jovian planets, as
 677 predicted by Laplace (1799).

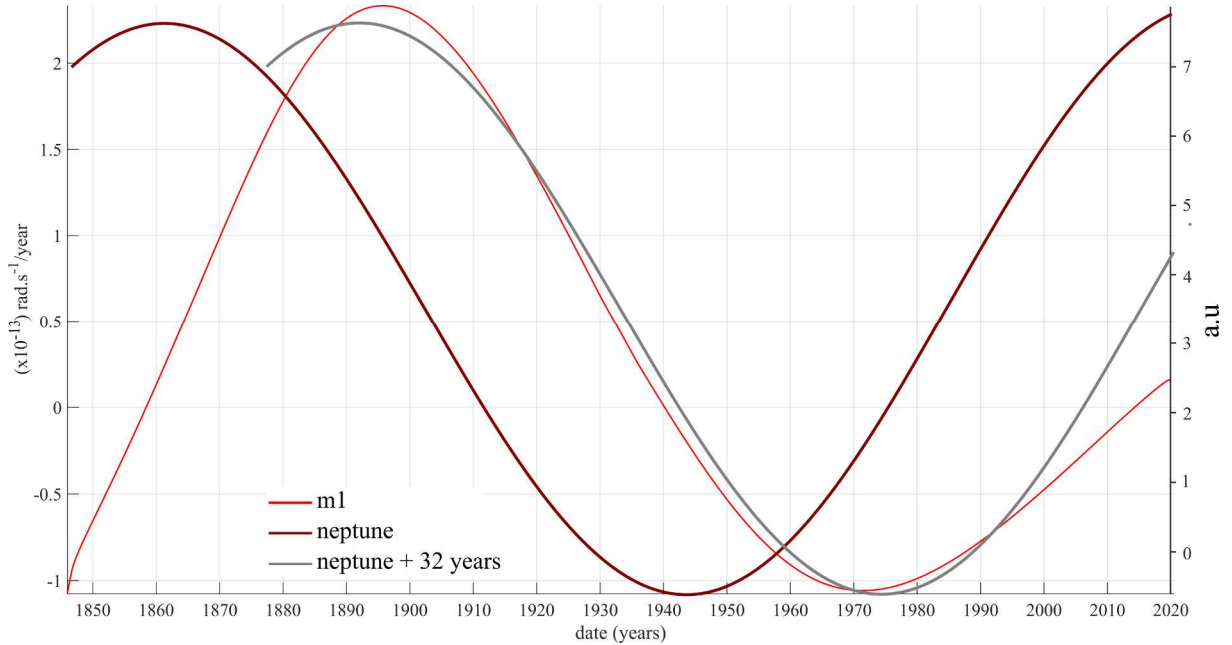
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679
 680 Figure 12a: Superimposition of the ~90 yr SSA component of the envelope of m_2 (blue curve; see

681 **Figure 8b)** with the ephemerids of Uranus (red curve). Grey curve: Uranus curve delayed by 32
 682 years.

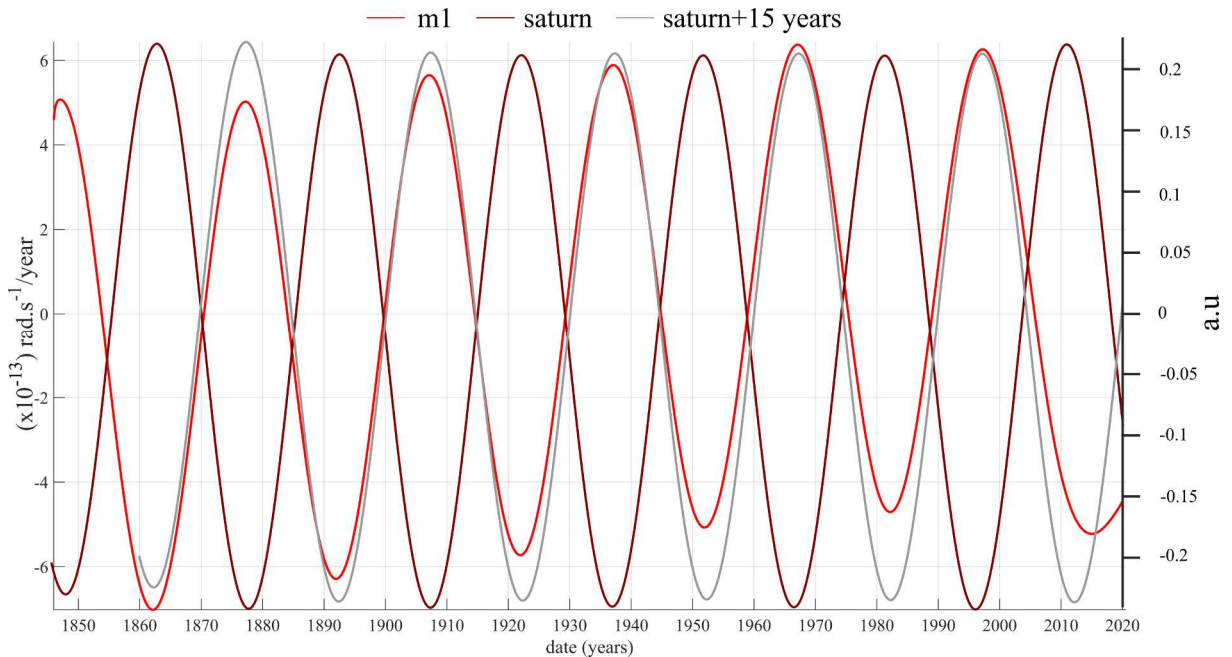
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685 **Figure 12b:** Superimposition of the ~165 yr SSA component of the envelope of m_1 (red curve;
 686 see **Figure 10a)** with the ephemerids of Neptune (dark red curve). Grey curve: Neptune curve
 687 delayed by 32 years.

688



689

690 **Figure 12c:** Superimposition of the ~30 yr SSA component of the envelope of m_1 (red curve; see
 691 **Figure 10c)** with the ephemerids of Saturn (dark red curve). Grey curve: Saturn curve delayed by 15

692 years.

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694

695 **7 – Summary, Discussion and Conclusion**

696

697 The general laws that govern the motions of celestial bodies have been derived and discussed
698 by Laplace (1799) in his remarkable *Traité de Mécanique Céleste*. Laplace established the system
699 of linear differential equations now known as the Liouville-Euler equations. He provided the full set
700 of equations for the three Euler angles that specify the motions of a body’s axis of rotation. Laplace
701 differs from most later authors in the way he uses the Liouville-Euler system. Laplace makes full
702 use of the system (D) for a rotating body that undergoes both rotation and translation, and solves the
703 algebraic transcendent equations of Appendix 3, given all astronomical parameters. Most others use
704 a simplified version with the formalism of excitation functions (Appendix 1, equation 2; a second
705 order system) in which the possibility of a translation of the body’s rotation axis is denied.

706 When Laplace obtains system (D) on page 74 of Chapter 7 of Book I, after 7 chapters that led
707 him to these equations, he recognizes the fact that the system accounts for rotation as well as
708 translation of a rotating body’s polar axis. When Lambeck (for instance) follows the same route, his
709 Chapter 3 (entitled «*Rotational Dynamic*») on page 30 begins with the following sentence : «*The*
710 *fundamental equations governing the rotation of a body are Euler’s dynamical equations*».
711 Lambeck links the angular momentum to the torque that generated it. One means only rotation: that
712 would be valid if the Earth’s inclination were zero or a constant. The equations are the same, but
713 one soon forgets that the momentum that is the source of the torque (Lambeck’s system (3.1), page
714 30) is a 3D vector (with no reason to be restricted to 2D, since the Earth is neither flat, nor is its
715 inclination constant; its rotation axis revolves about the Sun and is therefore subjected at least to our
716 star’s kinetic momentum). This oversight has some consequences. Since one only considers
717 rotations, not translations, then the (Chandler) free rotation is obtained by zero-ing all torques and
718 disregarding the third equation for the m_3 polar coordinate (that is assumed constant). Then, the
719 forced annual oscillation cannot be due to the revolution of Earth about the Sun and one must find
720 causes for these forced oscillations (the excitation functions). Laplace of course knew that polar
721 coordinates m_1 and m_2 were connected to m_3 . Therefore, Laplace did not constrain polar motion to
722 the two surface components (m_1, m_2) but represented it by two meaningful components, the axis’
723 inclination θ and the time derivative of its declination ψ depends on the inclination

724 (previously calculated as a solution of the first Liouville-Euler equation). Laplace showed that there
725 existed a free oscillation that would drift with a period between 306 (conjunction nodes) and 578
726 days (conjunction bellies), fully determined by the Earth’s moments of inertia. This free oscillation,
727 the Chandler oscillation, has a current value of 432-434 days. We now have long time series, up to a
728 couple of centuries long, available and we use series of coordinates of the rotation pole m_1 and m_2
729 (Figure 1) to extend some of Laplace’s (1799) results. A simple Fourier transform (Figure 2) shows
730 the dominant spectral lines at 1 yr (forced annual oscillation) and 1.19 yr (free Chandler
731 oscillation).

732 Singular spectral analysis (SSA) allows to better characterize the three leading components,
733 the trend (~13cm/yr) called the (free) Markowitz drift (Figure 4), then the (forced) annual
734 oscillation (showing different modulations for m_1 and m_2 , Figure 5) and the Chandler oscillation
735 (with a very large modulation and a phase change in 1930, similar for m_1 and m_2 , Figure 6). Under
736 the current theory, modulation is thought to be a response to reorganization of oceanic and
737 atmospheric masses (e.g. Lambeck, 2005, chapter 7). Taken together, the first three SSA
738 components explain 73% of the signal’s total variance (Figure 7). The smaller components that
739 follow have (pseudo-) periods of 1.22 (with an 18.6 yr modulation), 1.15 and 1.10 yr. Some of these
740 periods have been encountered in sunspot series and in the ephemerids of Jovian planets (Le Mouél
741 et al., 2020a; Courtillot et al., 2021).

742 We have next analyzed in the same way the envelopes of the derivatives of the first three SSA
743 components of polar motion (Figure 8). We find a trend in the derivative of the Markowitz drift,
744 that could also correspond to the 171.5 yr Jose cycle (associated with the ephemeris of Neptune) or
745 the ~200 yr Suess-de Vries cycle). Next, a 90 yr component, reminiscent of the Gleissberg solar
746 cycle (associated with the ephemeris of Uranus), a 40 yr component, corresponding to a
747 commensurable revolution period of the four Jovian planets, a 22 yr and an 11 yr component, that
748 can be associated with Jupiter and/or the Sun. For the modulation of the annual component of polar
749 motion, SSA finds periods of 165, 70 and 60 years (Figure 9). The 60 yr component has been found
750 in sunspots, global temperature of Earth’s surface, and the oceanic oscillation patterns PDO and
751 AMO (and Saturn). Finally, for the Chandler component, excellent matches are found for m_1 and m_2
752 with periods of 70, 40, 30 and 22 years (Figure 10).

753 One can think in Laplace’s terms that the kinetic moments of planets act directly on Earth, or
754 that these moments act on the external layers of the Sun (which is a fluid mass) and perturb its
755 rotation, hence its revolution and kinetic moment M (Appendix 1), eventually affecting the Earth’s

756 axis of rotation: classical mechanics allows the hypothesis that planets influence the Earth’s rotation
757 axis (hence climate and other global phenomena). Laplace (1799) has shown that one should
758 consider the orbital kinetic moments of all planets and that the Earth’s rotation axis should undergo
759 motions with the signatures/periods of the Sun and planets: the moments of the Jovian planets range
760 from 1.7 to 19.3 10^{42} kg.m².s⁻¹, and for the Sun, an equivalent is 1.7 10^{41} kg.m².s⁻¹. To first order,
761 the total kinetic moment applied to the Earth’s rotation axis is simply the sum of individual (Jovian)
762 planetary kinetic moments plus the Solar kinetic moment.

763 We have shown (Figure 3) that the m_1 component of polar motion reconstructed with SSA,
764 with the Markowitz trend removed, matches remarkably well the sum of kinetic moments of the
765 four Jovian planets. We have also computed these kinetic moments from the planetary ephemerids
766 of Uranus and Neptune (Figure 11a). They “predict” remarkably well (Figure 11b) the 40 yr SSA
767 component of the derivative of the envelope of the Chandler oscillation (Figure 11d).

768 We have previously determined the characteristic SSA components of solar activity, using
769 sunspot numbers as a proxy (Le Mouél et al., 2019b). The sum of the Markowitz drift, annual
770 oscillation and Chandler oscillation explain over 70% of polar motion. The same is true for
771 sunspots, on the same time range, regarding the sum of the trend (Jose ~171.5 yr), Schwabe (~11
772 yr) and Gleissberg (~90 yr) cycles. These periods correspond to those of Neptune (~165 yr), Uranus
773 (~90 yr) and Jupiter (~11 yr). We have superimposed the signatures (components) of the
774 ephemerids of Jovian planets on the components of polar motion. The 90 yr component of the
775 envelope of m_2 matches the ephemerids of Uranus, offset by 32 years (Figure 12a). The 165 yr
776 component of the envelope of m_1 matches the ephemerids of Neptune, also offset by 32 years
777 (Figure 12b). And the 30 yr component of the envelope of m_1 of the Chandler oscillation matches
778 the ephemerids of Saturn, offset by 15 years (Figure 12c).

779 We have followed Mörth and Schlamming (1979), who determined the commensurable
780 periods of pairs and pairs of pairs of Jovian planets (Table 1): we find that 8 of them, ranging from
781 1.2 to 165 years, correspond to the SSA components of polar motion identified in sections 4 to 6 of
782 this paper. Taken together, the curves shown in Figures 3 to 12 strongly argue that many
783 components of Earth’s polar motion can be explained by Laplace’s classical celestial mechanics,
784 with the combined instantaneous action of gravitational forces, and the longer, time-integrated
785 action of kinetic moments. All celestial bodies act on the (m_1, m_2) pair of polar coordinates, a list
786 that starts with the Sun and Moon, and continues with the Jovian planets. It would be satisfying to
787 undertake a rigorous demonstration of the influence of all planets on the Sun and on the Earth’s

788 rotation axis, that is to calculate the inclination θ and declination ψ following Laplace’s (1799) full
789 treatment of the equations (Appendix 2). Since the Liouville-Euler equations are linear differential
790 equations of first order, we have been able to use the frame of small perturbations and we have
791 considered that the influence of planets can be taken as the sum of individual influences.

792 We have recalled that Lambeck (2005) writes: ‘*The seasonal oscillation in the wobble is the*
793 *annual term which has generally been attributed to a geographical distribution of mass associated*
794 *with meteorological causes [...], a conclusion that is still valid today.*’ When one works within this
795 theoretical frame, there remain unexplained observations such as the 434 day value of the current
796 period of the Chandler wobble or the 6 month component of oceanic indices (Le Mouël et al.,
797 2019d). This has led to attempts to increase the complexity of the model, such as the forcing by
798 climate or the visco-elastic response to glacial isostatic rebound. We have seen that this theory uses
799 only 2 of the 3 Euler angles. By using the full system of equations in the Liouville-Euler system (D
800 for Laplace), Laplace (1799) was able to go beyond the synthetic treatments of (for instance) Guinot
801 (1977) or Lambeck (2005). We have seen in this paper numerous applications of this theory that
802 explain many pseudo-periodic components of a number of geophysical (and solar) phenomena,
803 making the leading role of planetary ephemeris clear.

804 The shorter periods (months to a few decades) often show as modulations of even shorter
805 variations. And trends, with about 200 years of data, are possibly due to periods in the ephemeris
806 comparable to or longer than the range of available observations. Still, these 200 years allow us to
807 test Laplace’s work further than he himself could. We have for instance been able to use this
808 formalism to predict the future evolution of solar Cycle 25 (Courtillot et al, 2021).

809 It is widely assumed that both forced and free oscillations of Earth can, at least in part, be
810 associated with climate forcings. Such has been the case from Jeffreys (1916) to Lambeck (2005),
811 and recently to Zotov and Bizouard (2012) and Zotov et al. (2016). In all these works, causality is
812 absent, be it from a time perspective or based on the orders of magnitude of the forces required to
813 perturb the Earth’s rotation. The periods that for instance Zotov et al. (2016) associate with an
814 interaction between Earth’s fluid and rigid envelopes are found in other geophysical phenomena
815 such as the Earth’s magnetic field or sunspots (Le Mouël et al., 2019a,b,c,d ; Le Mouël et al.,
816 2020a,b ; Le Mouël et al., 2021; Courtillot et al., 2021 ; and references therein). We have come to
817 the same conclusion regarding many climatic indices (Le Mouël et al., 2019d). If there is a good
818 correlation of many characteristic periods, pseudo-periods and components extracted with SSA, for
819 instance between Earth’s rotation and many features of climate, it is reasonable to assume that this

820 is because they are subject to some common forcings. This is not an overly speculative hypothesis:
821 with the views of Laplace on tides, we know that the fluid envelopes react on short time scales (to
822 changes in the Moon’s declination for 2/3rds and the Sun for 1/3rd). On longer time scales, the
823 whole (including solid) Earth responds (e.g. [Dehant et Mathieus, 2015](#)), all being governed by the
824 Liouville-Euler equations.

825 In the present study, we have been able to find planetary signatures in polar motions, strictly
826 based on observational data and using only classical mechanics. A possible causal chain thus
827 emerges that has gravity potential and kinetic moments of planets acting directly or modulating
828 motions of the fluid parts of celestial bodies, i.e. the Sun’s outer layers (sunspots) and the Earth’s
829 atmosphere and ocean. These effects are in general not yet modeled: this is a domain where climate
830 modeling warrants significant research advances.

831 In summary and conclusion of this work, two different mechanisms (causal chains) are likely
832 at work. One is illustrated by the spectacular and direct effect of the kinetic moments of the (Jovian)
833 planets on the Chandler wobble, whose intrinsic period (somewhere between 306 and 578 days) is
834 synchronized to 433 days (a value that depends on Earth properties). The causal chain is directly
835 from the Jovian Planets to Earth. Another causal chain would be an effect of planetary motions on
836 the solar dynamo; variations in solar activity would in turn influence meteorological and climatic
837 phenomena, such as mass transport between the equator and the poles, length of day, sea-level,...
838 Given the remarkable coincidence between the quasi-periods of many of these phenomena, it is
839 reasonable to assume that both causal chains are simultaneously at work. In that sense, it is not
840 surprising to find the signatures of the Schwabe, Hale and Gleissberg cycles in many terrestrial
841 phenomena, reflecting the characteristic periods of the combined motions of the Jovian planets.

842

843

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845

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849

Appendix 1: Polar Coordinates and Excitation Functions

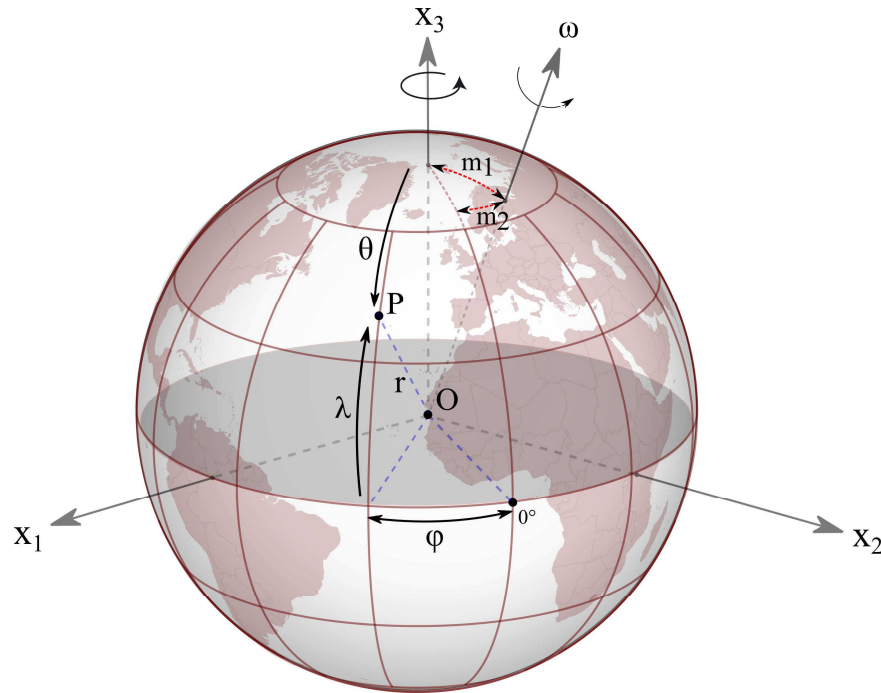


Figure A1 The reference system for the pole (m_1 and m_2).

Figure A1 gives the notations for the reference system that we use. The rotation pole is defined by its components m_1 and m_2 , respectively on the Greenwich (0°) and 90° E meridians. We follow Lambeck's (2005, chapter 3) formalism. The rotation of the pole ω can be decomposed into three Euler angles (ω_1 , ω_2 , ω_3) associated with the axes (X_1 , X_2 , X_3) of the fixed reference frame. These Euler angles are a function of the Earth's mean angular velocity Ω and of the apparent position of the pole at the Earth's surface m_1 , m_2 , m_3 :

$$\begin{aligned} \omega_1 &= \Omega m_1 \\ \omega_2 &= \Omega m_2 \\ \omega_3 &= \Omega(1 + m_3) \end{aligned} \tag{1}$$

Noting that the Earth rotates about its axis and that its radius is constant, the Liouville-Euler system of equations (D for Laplace, 1799) becomes ($1/\sigma$ is the Euler period):

$$\begin{aligned} j\left(\frac{\dot{\mathbf{m}}}{\sigma}\right)\Omega + \mathbf{m} &= \boldsymbol{\xi} \\ \dot{m}_3 &= \xi_3 \end{aligned} \tag{2}$$

880 where $\mathbf{m} = m_1 + jm_2$ and $\boldsymbol{\xi} = \xi_1 + j\xi_2$. (ξ_1, ξ_2, ξ_3) are the excitation functions (forces and
881 moments).

882

883 **Appendix 2: Orientation of the Classical Orbit of a Planet Moving in the Field of a**
884 **Central Body**

885 The orientation of the classical orbit of a particle of mass μ (the Earth) around a large object
886 of mass m (the Sun; $m \gg \mu$) is determined by two conservative vectors, the kinetic moment :

887

$$\vec{M} = \vec{r} \times \vec{p} \tag{B1}$$

889

890 and the vector:

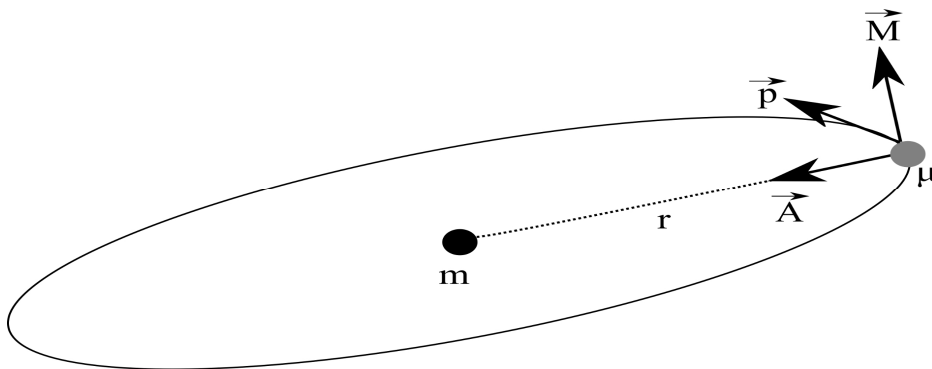
$$\vec{A} = \frac{\vec{p}}{\mu} \times \vec{M} - k \frac{m \vec{r}}{\mu r} \tag{B2}$$

893

894 whose conservation characterizes the Newtonian field (potential $\phi = -km/r$; k being the constant of
895 universal gravitation). Vector \vec{M} is perpendicular to the orbital plane and vector \vec{A} is oriented along
896 the major semi-axis towards perihelion (Figure A2); the modulus of \vec{A} is $km\mu e$, where e is the
897 orbit’s eccentricity. If m (the Sun) is rotating, then \vec{M} is perturbed following the equation:

$$\frac{d\vec{M}}{dt} = \vec{\Omega} \times \vec{M}$$

900 $\vec{\Omega}$ is the angular velocity of vector \vec{A} , thus the revolution velocity of the perihelion.



901

902

903 **Figure B1:** Particle μ (the Earth) rotates about the Sun ($m \gg \mu$). \vec{M} is its kinetic moment and \vec{A}
904 derives from the Newtonian potential.

905

906 **Appendix 3: Equations 5 and 6 from Laplace (1799), Book 5, Chapter 1, Pages 317-319**

907 [Laplace \(1799\)](#) establishes the two equations that determine the joint influence of the Moon
908 and the Sun on the inclination θ of the rotation (polar) axis ([Figure A1](#)) as a function of time and
909 the time derivative of its declination ψ , that is of its rotation (chapter 1 of book 5 of his treatise, in
910 sections 5 and 6 to which the interested reader is invited to [turn](#)).

911 The first equation reads (in Laplace’s terms):

$$\theta = h + \frac{3m}{4n} \cdot \left(\frac{2C - A - B}{C} \right) \cdot \left\{ \begin{array}{l} \frac{1}{2} \cdot \sin(\theta) \cdot \left\{ \cos(2\nu) + \frac{\lambda m}{m'} \cdot \cos(2\nu') \right\} \\ -(1 + \lambda) \cdot m \cdot \cos(\theta) \cdot \sum \cdot \frac{c}{f} \cdot \cos(ft + \varsigma) \\ + \frac{\lambda c'}{f'} \cdot \cos(\theta) \cdot \cos(f't + \varsigma') \end{array} \right.$$

912
913 h corresponds to the inclination of the rotation axis, that was 26.0796° ([Laplace, 1799](#), V (1), page
914 349), and is now 23.4333° . m is the Sun’s mass and m' is the Moon’s mass. n is the mean angular
915 rotation velocity of Earth about its third principal axis, that is the length of day. A, B, C are the
916 Earth’s principal moments of inertia ($A=B$). Parameter λ is an integration constant obtained for the
917 free oscillation of the Liouville-Euler system of partial differential equations, equal to:

$$n \sqrt{\frac{C - A}{C - B} AB} = \frac{C - A}{A} = \frac{\sigma}{\Omega}$$

918
919 $1/\sigma$ is the Euler period mentioned in the paper. ν is the Sun’s and ν' the Moon’s angular motion,
920 that is their longitudes measured from the Spring mobile equinox or their right ascensions relative
921 to Earth. One must also take into account the inclinations of Earth and Moon with respect to the
922 Sun, represented by c and c' . They are associated to the longitudes of ascending nodes of Earth and
923 Moon with respect to the Sun, also measured from the Spring mobile equinox and noted $(ft + \zeta)$ and
924 $(f't + \zeta')$.

925 The second equation expresses the time variations of the rotation and uses the same notations
926 and parameters:

$$\frac{d\psi}{dt} = \frac{3m}{4n} \cdot \left(\frac{2C - A - B}{C} \right) \cdot \left\{ \begin{array}{l} (1 + \lambda) \cdot m \cdot \cos(\theta) - \frac{\cos(\theta)}{2dt} \cdot \left\{ d \cdot \sin(2\nu) + \frac{\lambda m}{m'} \cdot d \cdot \sin(2\nu') \right\} \\ + (1 + \lambda) \cdot m \cdot \frac{\cos^2(\theta) - \sin^2(\theta)}{\sin(\theta)} \cdot \sum \cdot c \cdot \cos(ft + \varsigma) \\ + \lambda \cdot m \cdot \frac{\cos^2(\theta) - \sin^2(\theta)}{\sin(\theta)} \cdot c' \cdot \cos(f't + \varsigma') \end{array} \right.$$

927

928 One sees the rotation as the sum of two oscillations, one intrinsic to the Earth linked to the
929 constant $3m/4n$, that varies like $(1+\lambda).m.\cos \theta$ for all the nodes of luni-solar orbits, and is therefore
930 a function of inclination θ , and another one forced by the Moon and Sun, linked to longitudes $(ft+\zeta)$
931 and $(f't+\zeta')$ and to right ascensions ν and ν' of these two orbs. Laplace therefore can estimate that
932 the rotation period varies from 306 and 578 days.

933

934 **Appendix 4: Original Text in French of the Quotation from Laplace**

935 *(Traité de Mécanique Céleste, vol. 5, cap. 1, page 347, 1799)*

936 “Nous avons fait voir (n°8), que le moyen mouvement de rotation de la Terre est uniforme,
937 dans la supposition que cette planète est entièrement solide, et l'on vient de voir que la fluidité de la
938 mer et de l'atmosphère ne doit point altérer ce résultat. Les mouvements que la chaleur du Soleil
939 excite dans l'atmosphère, et d'où naissent les vents alizés semblent devoir diminuer la rotation de
940 la Terre: ces vents soufflent entre les tropiques, d'occident en orient, et leur action continuelle sur
941 la mer, sur les continents et les montagnes qu'ils rencontrent, paraît devoir affaiblir insensiblement
942 ce mouvement de rotation. Mais le principe de conservation des aires, nous montre que l'effet total
943 de l'atmosphère sur ce mouvement doit être insensible; car la chaleur solaire dilatant également
944 l'air dans tous les sens, elle ne doit point altérer la somme des aires décrites par les rayons
945 vecteurs de chaque molécule de la Terre et de l'atmosphère, et multipliées respectivement par leur
946 molécules correspondantes; ce qui exige que le mouvement de rotation ne soit point diminué. Nous
947 sommes donc assurés qu'en même temps que les vents analysés diminuent ce mouvement, les autres
948 mouvements de l'atmosphère qui ont lieu au-delà des tropiques, l'accélèrent de la même quantité.
949 On peut appliquer le même raisonnement aux tremblements de Terre, et en général, à tous ce qui
950 peut agiter la Terre dans son intérieur et à sa surface. Le déplacement de ces parties peut seul
951 altérer ce mouvement; si, par exemple un corps placé au pôle, était transporté à l'équateur ; la
952 somme des aires devant toujours rester la même, le mouvement de la Terre en serait un peu
953 diminué; mais pour que cela fut sensible, il faudrait supposer de grands changements dans la
954 constitution de la Terre”.

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