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“Laplace” – 11/3/21 – 2<sup>nd</sup> Revision

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## On the shoulders of Laplace

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26        **Abstract**

27        In 1799, Laplace derived the system of differential equations (now called Liouville-Euler) that  
28 fully describes the motions of the rotation axis of any celestial body. Laplace showed that only the  
29 gravitational forces and kinetic moments from other celestial bodies influence the rotation of any  
30 one of them. The equations involve three Euler angles that specify the motions of a body’s rotation  
31 axis; they can be reduced to a system of two equations for the inclination and time derivative of the  
32 declination of the rotation axis. Laplace showed the existence of a forced annual oscillation and the  
33 so-called free Chandler wobble. Most current theories retain only two Euler angles and invoke an  
34 elastic Earth to match observations. We analyze the much longer time series of polar motion  
35 (coordinates  $m_1$  and  $m_2$  of the rotation pole at the Earth’s surface) now available, in order to further  
36 explore phenomena that Laplace could not investigate, given the dearth of data in his time. We use  
37 singular spectral analysis (SSA) to extract components of the time series. The first three  
38 components (trend or Markowitz drift, forced annual oscillation and free Chandler oscillation)  
39 account for 73% of the variance of polar motion. Under the current theory, their modulation is  
40 thought to be a response to reorganization of oceanic and atmospheric masses. However, the periods  
41 of the first six SSA components of polar motion have been encountered in previous studies of  
42 sunspots and in the ephemerids of Jovian planets. We also analyze the derivatives of the envelopes  
43 of the three SSA components of polar motion. Again, most of these components have periods and  
44 modulations that correspond to the ephemeris (periods and combinations of commensurable  
45 periods) of Jovian planets. Examples include 171.5 yr (the Jose cycle linked to Neptune), 90 yr (the  
46 Gleissberg cycle linked to Uranus), 40 yr (a commensurable period linked to the Jovian planets), 22  
47 yr, 11 yr (Jupiter, Sun), 60 yr, 30 yr (Saturn). Figure 3 can be considered as the central result of the  
48 paper. It shows that the sum of forces of the four Jovian planets matches in a striking way the polar  
49 motion reconstructed with SSA components (the Markowitz trend removed). All our results argue  
50 that significant parts of Earth’s polar motion are a consequence (rather than a cause) of the  
51 evolution of planetary ephemerids. The Sun’s activity and many geophysical indices show the same  
52 signatures, including many climate indices. Two different mechanisms (causal chains) are likely at  
53 work: a direct one from the Jovian Planets to Earth, another from planetary motions to the solar  
54 dynamo; variations in solar activity would in turn influence meteorological and climatic  
55 phenomena. Given the remarkable coincidence between the quasi-periods of many of these  
56 phenomena, it is reasonable to assume that both causal chains are simultaneously at work. In that  
57 sense, it is not surprising to find the signatures of the Schwabe, Hale and Gleissberg cycles in many

58 terrestrial phenomena, reflecting the characteristic periods of the combined motions of the Jovian  
59 planets.

60

## 61 **1 - Introduction**

62 On July 5 1687, Isaac Newton published the three volumes of his *Principia Mathematica*, in  
63 which he put on a firm ground the law of universal attraction and the general laws of mutual  
64 attraction of masses. In the following two centuries, a corpus of laws that explained the motions of  
65 celestial bodies was established and vindicated by observations. Foremost among these works,  
66 Laplace published his *Traité de Mécanique Céleste* (Treatise of Celestial Mechanics) in 1799.

67 Based on Newton’s law and the fundamental principle of dynamics, he established the general  
68 equations that govern the motions of material bodies ([Laplace, 1799, book 1, chapter 7, page 74,](#)  
69 [system \(D\)](#)). This system of differential equations of first order was later given the names of  
70 Liouville and Euler. It establishes both the rotation and translation of the rotation axis of any  
71 celestial body, and in particular the Earth. These same equations can be found in [Guinot](#) (in  
72 [Coulomb and Jobert, 1977](#), p. 530) and more recently in the reference book of [Lambeck \(2005](#), p.  
73 31). They are recalled in [Appendices 1](#) and [2](#) in their most general form. When the forces and the  
74 moments that act on Earth are taken to be zero (i.e. the right hand side of the equations is zero), the  
75 solution for the axis is a free oscillation with a Euler period  $1/\sigma$  of 306 days (using the known  
76 values of the mean angular velocity and axial and equatorial moments of inertia ( $\sigma = ((C-A)/A)\Omega$ )).  
77 Based on observations made between June 1884 and November 1885, [Chandler \(1891a,b\)](#) obtained  
78 a value of 427 days for  $1/\sigma$ . Data provided by the *International Earth Rotation and Reference*  
79 *System Service* (IERS) yield a  $1/\sigma$  that has varied between 431 days in 1846 and 434 days in 2020.

80 [Newcomb \(1892\)](#) verified Chandler’s observations and concluded that the Earth should be  
81 viewed as an elastic body submitted to oceanic stresses. For this, Love numbers were introduced  
82 ([Love, 1909](#)). As a result, the Liouville-Euler system (D) of Laplace was made less general. [Hough](#)  
83 [\(1895\)](#) reinforced the idea that what made the Chandler period 121 days longer than the  
84 “theoretical” value was the fact that the Earth behaved as an elastic body. Based on [Poincaré’s](#)  
85 [\(1885\)](#) work on the stability of rotating fluids with a free surface, Hough showed that the period  
86 should decrease rather than increase if one did not take elasticity into account. Works in the  
87 following decades strengthened the notion that the fluid envelopes of Earth (ocean, atmosphere and  
88 mantle) acted on Earth’s rotation axis. An increasingly precise theory was thus proposed, whereas  
89 observations seemed to be increasingly remote from predictions.



90 Two papers ([Peltier and Andrews, 1976](#); [Nakiboglu and Lambeck, 1980](#)) further strengthened  
91 the theory of an elastic Earth whose rotation axis was influenced by both its internal and external  
92 fluid envelopes. An important concept was that of Global Isostatic Adjustment (GIA), in which the  
93 Earth has a visco-elastic response to stress (load) variations, that originated at the onset of the last  
94 ice age. Melting ice would lead to sea level rise and a reorganization of surface masses that  
95 eventually modified the inclination of the rotation pole. Rather than writing in a physically explicit  
96 way the forces implied in system (D), as done by Laplace and Poincaré, more or less complex  
97 “excitation functions” were introduced ([Appendix 1](#)).

98 We return to the founding work of [Laplace \(1799\)](#) to see how these problems can be tackled  
99 further. In what follows, we refer to volumes, chapters, pages and equation numbers in the original  
100 edition of the *Traité de Mécanique Céleste*. Throughout the *Treatise*, [Laplace \(1799\)](#) rigorously  
101 shows that, whatever the nature of the oceans and atmosphere, the only thing that influences the  
102 rotation of celestial bodies is the action of other celestial bodies. On [page 347 \(chapter 1, volume 5\)](#)  
103 [Laplace \(1799\)](#) writes (this quotation is given in the original French in [Appendix 4](#)): “*We have*  
104 *shown that the mean rotation movement of Earth is uniform, assuming that the planet is entirely*  
105 *solid and we have just seen that the fluidity of the sea and of the atmosphere should not alter this*  
106 *result. It would seem that the motions that are excited by the Sun’s heat, and from which the*  
107 *easterly winds are born should diminish the Earth’s rotation: these winds blow between the tropics*  
108 *from west to east and their continued action on the sea, on the continents and on the mountains they*  
109 *encounter, should seem to weaken imperceptibly that rotation movement. But the principle of*  
110 *conservation of areas, shows to us that the total effect of the atmosphere on this movement must be*  
111 *insensible; for the solar heat in dilating equally the air in all directions, should not alter the sum of*  
112 *areas covered by the vector radii of each molecule of the Earth and of the atmosphere, and when*  
113 *multiplied respectively by the corresponding molecules; which requires that the rotation motion be*  
114 *not diminished. We are therefore assured that as the winds analyzed diminish this motion, the other*  
115 *movements of the atmosphere that occur beyond the tropics, accelerate it by the same amount. One*  
116 *can apply the same reasoning to earthquakes, and in general, to all that can shake the Earth in its*  
117 *interior and at its surface. Only the displacement of these parts can alter this motion; if, for*  
118 *instance a body placed at the pole, was transported to the equator; since the sum of areas must*  
119 *always remain the same, the earth’s motion would be slightly diminished; but for it to be*  
120 *noticeable, one should suppose the occurrence of great changes in the Earth’s constitution.”*

121 These views are also shared by [Poincaré \(1899\)](#). They seem to be different from modern

122 views as synthesized for instance by [Lambeck \(2005\)](#). These authors agree on the Liouville-Euler  
123 system (D for Laplace) of differential equations, but the forces that act on the Earth are different  
124 (and interpreted in a different way, as shown below). In the present paper, we attempt to check  
125 Laplace’s full system using the observations that have accumulated and improved since Laplace’s  
126 time (time series starting in 1750 for the oldest and no later than 1850 for the shortest ones).

127 We first discuss some of the core ideas of the paper, based on Laplace’s original  
128 developments (section 2). We then recall some concepts and tools that we use in the paper and  
129 introduce the data, i.e. the coordinates of the Earth’s rotation pole from 1846 to 2020 (section 3). In  
130 section 4, we establish a striking result that is central to the paper: the detrended polar motion is  
131 highly correlated with the sum of the forces exerted by the four Jovian planets. We next submit the  
132 data to Singular Spectral Analysis (SSA) and discuss the first SSA components (section 4): the  
133 Markowitz, annual and Chandler rotations. Then, in section 5, we discuss the SSA components of  
134 the derivatives of the three components above. In section 6, we give several other examples, such as  
135 the excellent correlation of the 40yr SSA component of the derivative of the envelope of the  
136 Chandler oscillation with the 40yr SSA component of the combined forces of Uranus and Neptune.  
137 We end with a discussion and concluding remarks (section 7).

138

## 139 **2 –Forces, Moments and the Liouville-Euler System of Equations**

140 In most classical applications of the mechanics of planetary rotation, one uses only the first  
141 two components of the trio of Euler angles, i.e. the coordinates of the rotation pole at the Earth’s  
142 surface ([Figure A1, Appendix 1](#)). The Earth rotates about the Sun (and so do the other 7 planets) in  
143 the ecliptic plane that is almost perpendicular to the rotation axis. The Sun carries more than 99% of  
144 the mass of the solar system, and can be considered rather motionless (its center of gravity actually  
145 travels along a “small” variable “ellipse”). In addition to the gravitational attractions, one must  
146 consider the orbital kinetic moments of all planets (in others words the moment of the momentum,  
147 see eq. B1, [Appendix 2](#)), as emphasized by [Laplace \(1799\)](#). Planets carry more than 99% of the  
148 total angular momentum of the system (19.3, 7.8, 1.7 and  $1.7 \times 10^{42} \text{ kgm}^2\text{s}^{-1}$  respectively for Jupiter,  
149 Saturn, Uranus and Neptune). This can be compared to the Sun’s attraction at the Earth’s orbit,  $3.5$   
150  $10^{22} \text{ kg.m.s}^{-2}$ , that can be transformed to the dimension of a kinetic moment by multiplying it by the  
151 Sun-Earth distance and the orbital revolution period of Earth, yielding  $1.7 \cdot 10^{41} \text{ kg.m}^2\text{s}^{-1}$ : that is not  
152 negligible compared to the order of magnitude of the kinetic moments of the Jovian planets (to 1 or  
153 2 orders of magnitude).

154 The central idea of this paper is to analyze variations in the Earth’s rotation axis under the  
 155 influence not only, as in many classic treatments (e.g. [Dehant and Mathews, 2015, ch. 2](#)), of  
 156 gravitational potentials, but also of kinetic moments. The classical system of differential equations  
 157 that describe the pole’s motion (Liouville-Euler) links the sum of simple physical entities with their  
 158 time derivatives, hence a first order linear system ([Appendix 1](#), part 1). See [Bode \(1945\)](#) for more  
 159 on the definition and consequences that can be drawn from such linear systems. One is that causes  
 160 and consequences are similar, up to a constant factor, if the system is not too dissipative and is  
 161 maintained: this implies that gravitational potentials, kinetic moments (of Jovian planets) and polar  
 162 motions should share characteristic features.

163 Polar motion is described by three coordinates, usually labeled  $m_1$ ,  $m_2$  and  $m_3$  ([Appendix 1](#)). If  
 164 one only wants to study the perturbations due to the gravitational potential of a planet in rotation  
 165 about itself, two coordinates,  $m_1$  and  $m_2$ , are sufficient to describe the motion. In the case of our  
 166 Solar system, planets revolve about the Sun in (or close to) the ecliptic plane; the moments they  
 167 generate are perpendicular to that plane ([Appendix 2](#)). They act on the *inclinations* of the rotation  
 168 axes of all planets, including Earth’s. This is the well-known phenomenon of interaction of spinning  
 169 tops and is adequately described by the Liouville-Euler equations. This was known to Laplace who  
 170 chose not to use the three Euler angles, but gave all the analytic formulas that allow one to compute  
 171 the *inclination*  $\theta$  of the rotation axis ([Figure A1](#) and [Appendix 3](#)) as a function of time, under the  
 172 influence of the Moon and Sun ([Laplace, 1799; book 5, page 317, number 5](#)), and the time  
 173 derivative of the *declination*  $\psi$  of the rotation axis ([Laplace, 1799; book 5, page 318, number 6](#)).  $\theta$   
 174 and  $\psi$  are defined in [Laplace \(1799; book 1, page 73, number 26\)](#). [Laplace \(1799; book 5, pages](#)  
 175 [352-355, number 14\)](#) deduces that, when neither the Moon nor the Sun act on Earth (conjunction  
 176 nodes), the time derivative of the declination (which in modern terms is the Euler period  $1/\sigma$ ) has a  
 177 value of 306 days ([Appendix 3](#)). This value is fully determined by the Earth’s moments of inertia  
 178 (i.e. the internal distribution of masses).

179 The equations derived by Laplace are:

$$\theta = h + \frac{3m}{4n} \cdot \left( \frac{2C - A - B}{C} \right) \cdot \left\{ \begin{array}{l} \frac{1}{2} \cdot \sin(\theta) \cdot \left\{ \cos(2\nu) + \frac{\lambda m}{m'} \cdot \cos(2\nu') \right\} \\ -(1 + \lambda) \cdot m \cdot \cos(\theta) \cdot \sum \cdot \frac{c}{f} \cdot \cos(ft + \varsigma) \\ + \frac{\lambda c'}{f'} \cdot \cos(\theta) \cdot \cos(f't + \varsigma') \end{array} \right.$$

180

$$\frac{d\psi}{dt} = \frac{3m}{4n} \cdot \left( \frac{2C - A - B}{C} \right) \cdot \left\{ \begin{array}{l} (1 + \lambda) \cdot m \cdot \cos(\theta) - \frac{\cos(\theta)}{2dt} \cdot \left\{ d \cdot \sin(2\nu) + \frac{\lambda m}{m'} \cdot d \cdot \sin(2\nu') \right\} \\ + (1 + \lambda) \cdot m \cdot \frac{\cos^2(\theta) - \sin^2(\theta)}{\sin(\theta)} \cdot \sum .c \cdot \cos(ft + \varsigma) \\ + \lambda \cdot m \cdot \frac{\cos^2(\theta) - \sin^2(\theta)}{\sin(\theta)} \cdot c' \cdot \cos(f't + \varsigma') \end{array} \right.$$

181

182 All celestial and terrestrial parameters in these equations are defined in [Appendix 2](#). The time  
183 variation of declination of the Earth’s rotation pole is a function of inclination. Since the  $(\theta, \psi)$  and  
184  $(m_1, m_2)$  couples represent the same physics, the pattern of the sum of planetary kinetic moments  
185 that “force” part of the Earth’s polar motions should be found in  $m_1$  and  $m_2$  (see below). Laplace  
186 obtained these equations taking into account “only” the Moon and Sun.

187 When the Moon and Sun act with maximum effect (conjunction bellies)  $1/\sigma$  reaches a value  
188 of 578 days.  $1/\sigma$  therefore oscillates between 306 and 578 days; [Chandler \(1891\)](#) found a value of  
189 427 days and today one observes values of 432-434 days. Both inclination  $\theta$  and declination  $\psi$  drift.  
190

### 191 3 –The Toolbox: Rotation Pole Data, Ephemerids, Commensurability and Singular 192 Spectrum analysis

193 Some of the tools and data needed to pursue our goal are listed in this section. Of course, we  
194 require knowledge of planetary *ephemerids*, that are given by the IMCCE. Then we need/use:

195 *3-1 Rotation pole data:* Laplace did not have sufficient observations to demonstrate the  
196 influence of planets, though he certainly did not deny their possible role. We now have sufficiently  
197 long series of observations to test his full theory.

198 The rotation pole is defined by its components  $m_1$  and  $m_2$ , respectively on the Greenwich (0°)  
199 and 90°E meridians ([Figure A1](#)). Two series of measurements of  $(m_1, m_2)$  are provided by IERS<sup>1</sup>  
200 under the codes EOP-C01-IAU1980 and EOP-14-C04. The first one runs from 1846 to July 1<sup>st</sup> 2020  
201 with a sampling rate of 18.26 days, and the second runs from 1962 to July 1<sup>st</sup> 2020 with daily  
202 sampling (also giving access to the length of day). [Figure 1](#) shows the components  $m_1$  and  $m_2$  of the  
203 longer series (data are given in milli arc second - mas - and converted here in radians per second -  
204 rad.s<sup>-1</sup>). [Figure 2](#) shows the Fourier spectrum of component  $m_1$ , corrected for a degree 2 non linear  
205 trend. The forced annual oscillation and the free Chandler oscillation with period close to 1.19 yr  
206 are conspicuous.

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1 <https://www.iers.org/IERS/EN/DataProducts/EarthOrientationData/eop.html>

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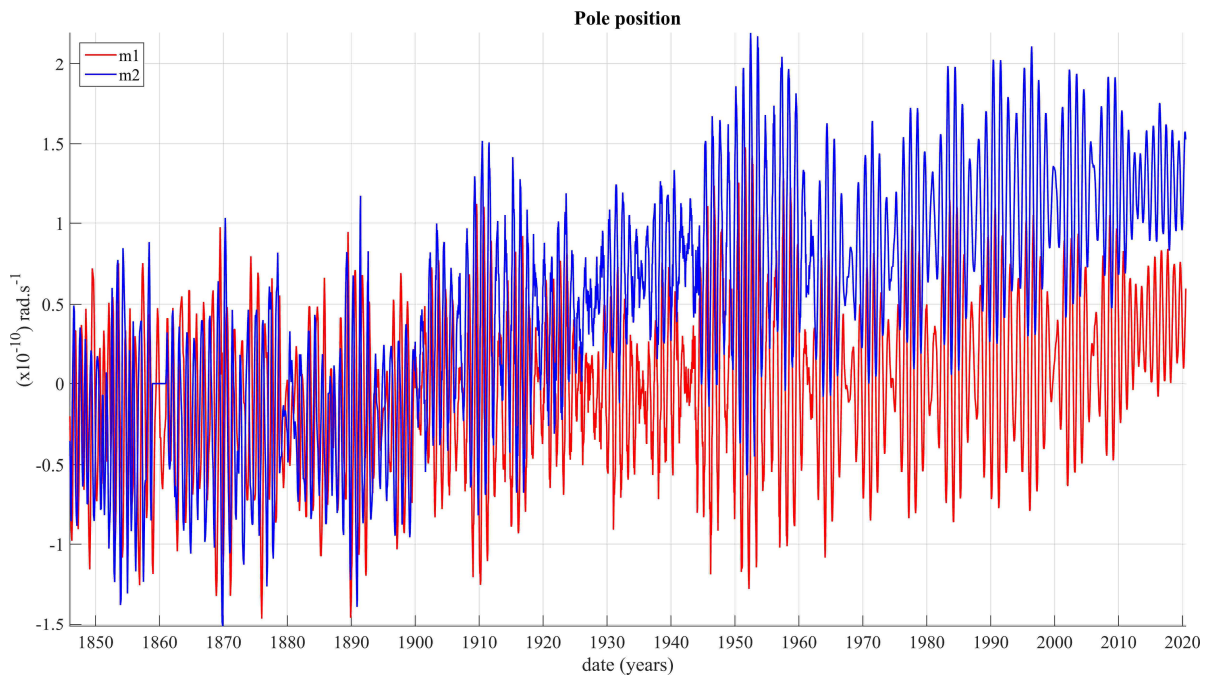
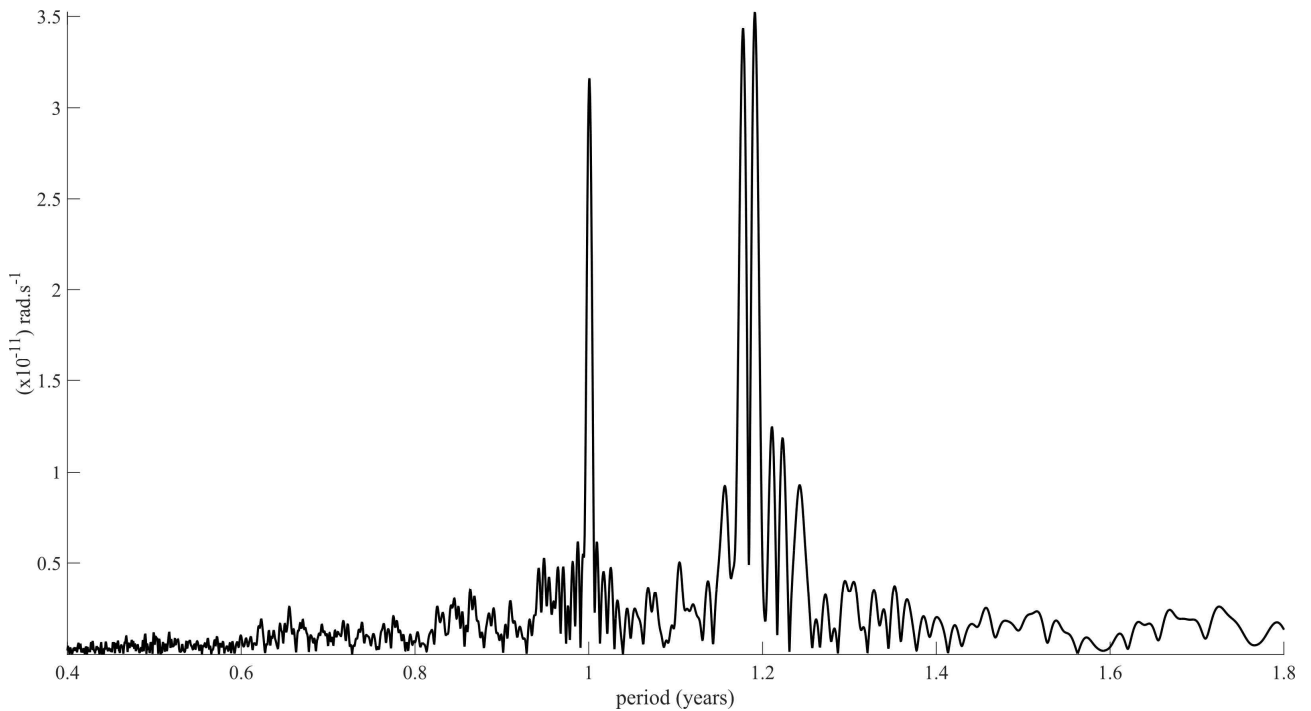


Figure 1: Components ( $m_1, m_2$ ) of polar motion since 1846 (time series EOP-C01-IAU1980)



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Figure 2: Fourier spectrum of component  $m_1$ , corrected for a degree 2 non linear trend

224 3-2 Commensurability: Then we use the concept of *commensurability*. One talks about  
225 commensurability when the ratio of the periods of 2 planets can be expressed as a fraction with

226 integer numerator and denominator less than 9 (Mörth and Schlamminger, 1979; Okhlopkov, 2016;  
227 Scafetta, 2020). Planets encounter a resonance and can be paired, and each pair can be considered  
228 as a single object (an egregor or aggregate). Jupiter/Saturn and Uranus/Neptune form two pairs.  
229 Pairs of pairs can also be considered, thus the set (Jupiter/Saturn)/(Uranus/Neptune). Many analyses  
230 of sunspot series (Lassen and Friis-Christensen, 1995; Hataway, 2015; Usoskin et al., 2016; Le  
231 Mouël et al., 2017; Stefani et al., 2019; Courtillot et al., 2021; Le Mouël et al., 2020a; Stefani et al.,  
232 2020) and of a number of geophysical phenomena (Courtillot et al., 2013; Scafetta, 2016; Lopes et  
233 al., 2017; Scafetta et al., 2019; Bignami et al., 2020; Le Mouël et al., 2019a; Le Mouël et al., 2019b;  
234 Hilgen et al., 2020; Le Mouël et al., 2020b; Zaccagnino et al., 2020; Le Mouël et al., 2021) contain  
235 components with periods that can be attributed to Jovian planets to first order, and all planets  
236 including the telluric ones to second order (Courtillot et al., 2021). Table 1 lists planetary  
237 commensurabilities following Mörth and Schlamminger (1979). The periods found in our analysis  
238 of the SSA components of polar motion (section 4) and of the derivatives of their envelopes (section  
239 5) are labeled in red (there are 8, ranging from 1.2 to 165 years).

240 *Note:* Inspection of Table 1 may give the impression that there is a risk of “cherry picking”.  
241 But certain periods that could have been reconstructed are not present, such as 103 yr that could  
242 have been obtained with Neptune. Commensurabilities are built from two consecutive planets and  
243 once their effect has been aggregated, they can be used in the next step of  
244 aggregation/commensurability. The concept of commensurability is used by astronomers in order to  
245 discriminate between planets and other objects. The corresponding periods are not random: they are  
246 directly related to the revolutions of these bodies, and result from calculating means or subtracting  
247 periods two by two. Thus what can be obtained is not random. Moreover, as already pointed out by  
248 Mörth and Schlamminger (1979) or more recently Scafetta (2016), uncovering a limited number of  
249 common periods in a number of geophysical observables including sunspots cannot be due to  
250 chance. The action of kinetic moments of Jovian planets on the Sun’s surface is what has allowed us  
251 to predict the next solar cycle from the ephemerids in a previous paper (Courtillot et al., 2021).

252 *3-3 Singular spectral analysis:* Finally we extract the relevant components of polar motion  
253 and ephemerids, and other long time series, with the help of *Singular spectral analysis* (SSA;  
254 Vautard and Ghil, 1989; Vautard et al., 1992; for more up to date versions of the technique by the St  
255 Petersburg school of mathematics, see Golyandina and Zhigljavsky, 2013). We have described and  
256 used our own version of SSA in a number of previous papers (e.g. Lopes et al., 2017; Le Mouël et  
257 al., 2021; Courtillot et al., 2021).



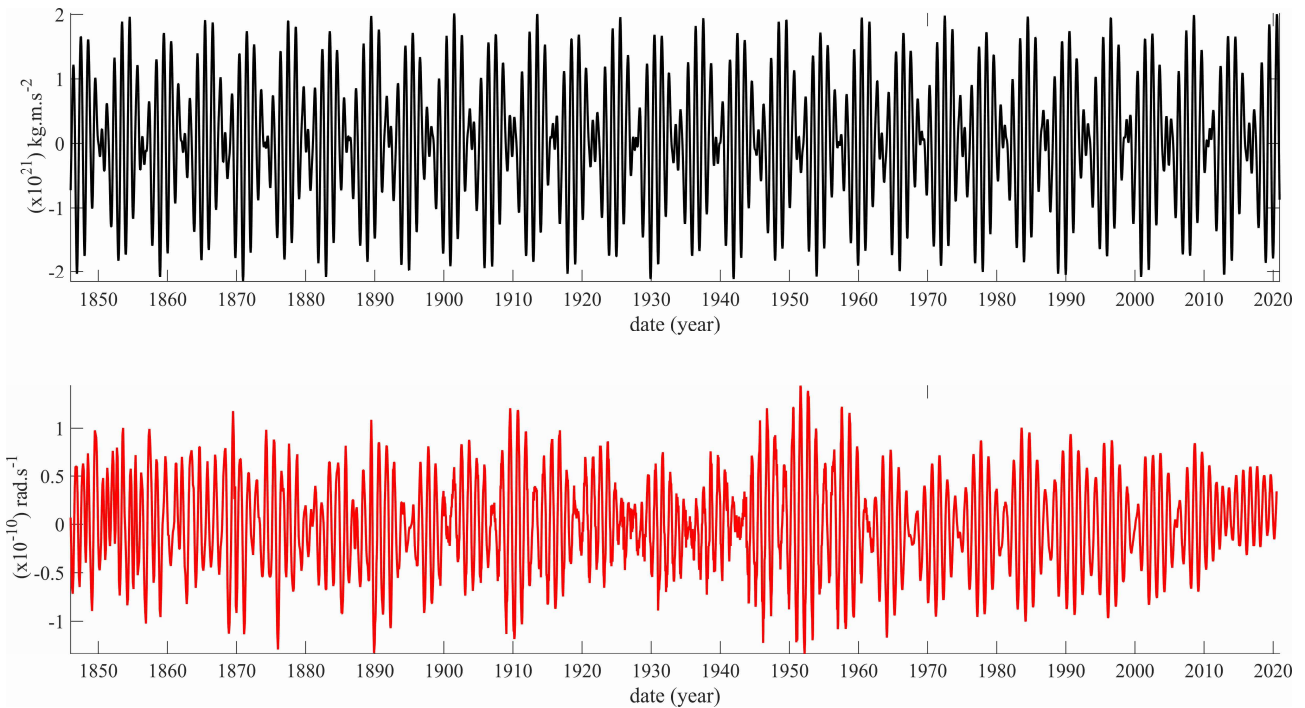
258 We discuss here a point that often comes up. An important factor in any time series analysis is  
259 the size of the window used in classical (Fourier) filters, to avoid erroneous interpretations (Kay and  
260 Marple, 1981). In SSA, the lagged-vector analysis window  $L$  should be sufficiently large so that  
261 each eigen vector carries a large part of the information contained in the original time series. In  
262 more mathematical words, one should work in the frame of Structural Total Least Squares (STLS)  
263 for a Hankel Matrix (Lemmerling and Van Huffel, 2001). A second issue is the separability of  
264 components. Many solutions are available, an exhaustive list being given by Golyandina and  
265 Zhigljavsky (2013, chap. 2.5.3, page 75). In this paper, we have used the *sequential SSA*. The  
266 window width  $L$  is variable, but remains close to 145 years.

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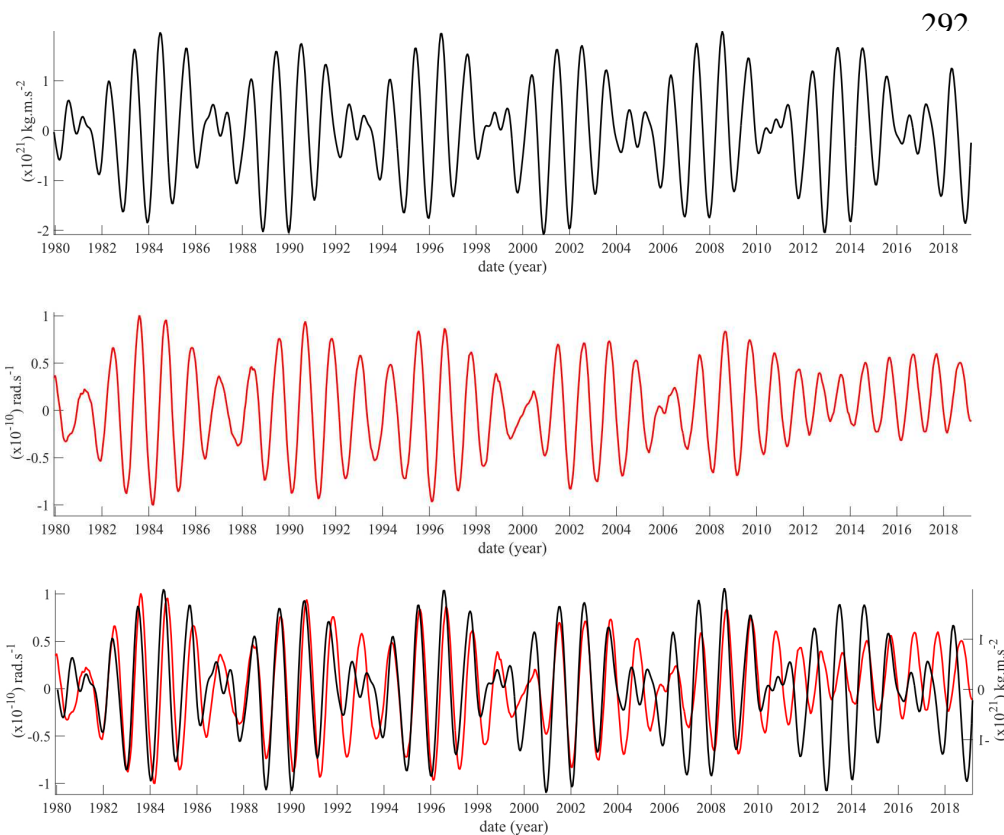
## 268 4 – First Results

269 *4.1 A striking reconstruction – Planets action on Earth’s rotation:* We start to test the ideas  
270 and use the tools summarized above by comparing the sum of the forces exerted by the four Jovian  
271 planets (using the IMCCE ephemerids) and the  $m_I$  component of polar motion (1846 to 2020) as  
272 reconstructed from its SSA components, with the trend removed.

273 The polar coordinates  $m_1$  and  $m_2$  are related to the forces acting on Earth (Appendix 1). To  
274 first order, we can consider that the total “force” is simply proportional to the sum of individual  
275 (Jovian) planetary kinetic moments, plus the Solar kinetic moment. We have computed these  
276 moments from the planetary ephemerids, revolution periods and masses; their sum is given as the  
277 top black curve in Figure 3a. The red curve below is the reconstructed  $m_I$  polar coordinate from  
278 Figure 1, after it has been decomposed in its SSA components, then reconstructed from them, but  
279 with the first component (the trend, see sub-section 4.2) removed. Figure 3b shows an enlargement  
280 of the 1980-2019 part of Figure 3a. The correlation is quite striking. It is indeed expected, as  
281 already pointed out by Laplace (1799, book 5 in whole), that the Earth’s rotation axis should  
282 undergo motions with components that carry the periods (and combinations of periods) of the  
283 Moon, Sun, and planets, particularly the Jovian planets as far as their kinetic moments are  
284 concerned (see also Mörth and Schlaminger, 1979, and Courtillot et al, 2021). This first exercise  
285 demonstrates that one should indeed consider planetary kinetic moments when describing the  
286 motions of the Earth’s rotation axis. Based on this remarkable result, the aim of the rest of this  
287 paper is to see whether characteristic components of the ephemerids are also found in Earth’s polar  
288 motion and other related (or not obviously related) phenomena.



289 **Figure 3a:** Upper curve (in black) the sum of the forces of the four Jovian planets affecting Earth.  
 290 Ephemerids from the IMCCE. Lower curve (in red) the  $m_l$  component of polar motion (1846-2020)  
 291 reconstructed with SSA and with the trend (Markowitz) removed.



307  
 308 **Figure 3b:** Enlargement of **Figure 3a** (1980-2019) and superposition of the 2 curves.



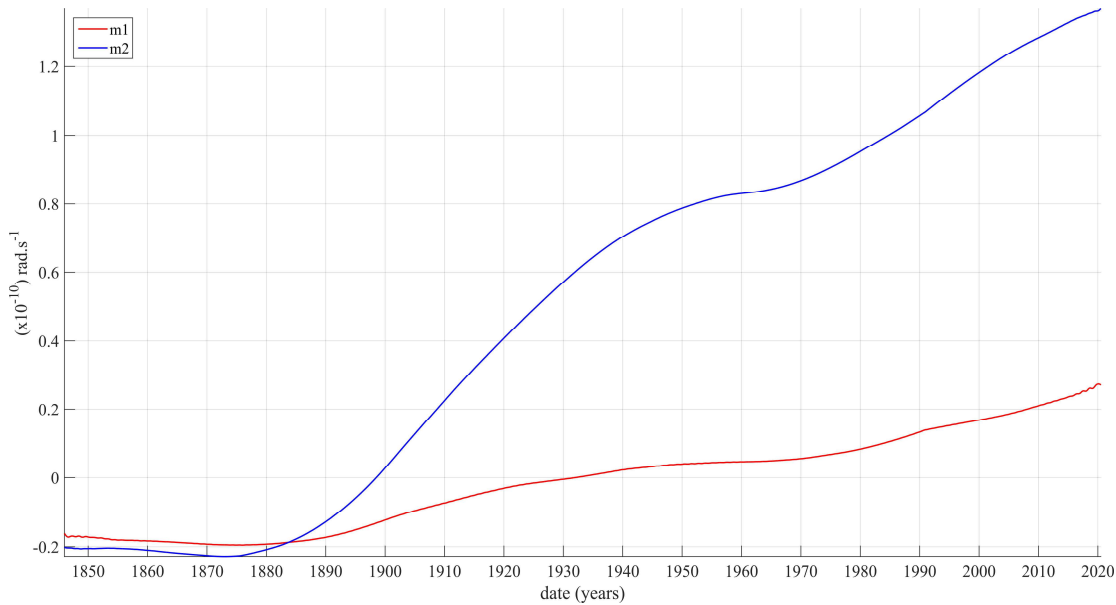
Planet		Commensurability ( $\epsilon$ )	I/J	Associated periods (yr)	
1 {	Mercury	Venus	$-6.6 \times 10^{-3}$	2/5	0.18, 0.42
		Earth	$10 \times 10^{-3}$	1/4	0.38, 0.62
		Mars	$8.3 \times 10^{-3}$	1/8	0.78, 1.02
2 {	Venus	Earth	$10 \times 10^{-3}$	3/5	0.19, 0.80
		Mars	$5.6 \times 10^{-3}$	1/3	0.59, <b>1.20</b>
3 {	Jupiter	Saturn	$1.9 \times 10^{-3}$	3/8	9.79, <b>21.64</b>
		Uranus	$-1.7 \times 10^{-3}$	1/7	36.06, 47.91
4 {	Uranus	Neptune	$9.6 \times 10^{-3}$	1/2	<b>40.40</b> , 124.37
		Pluto	$6.0 \times 10^{-3}$	1/3	81.75, <b>165.72</b>
5	Pluto	Neptune	$-8.3 \times 10^{-4}$	2/3	41.35, 206.13
1	2	$-6.4 \times 10^{-3}$	1/4	0.29, 0.49	
		$-1.3 \times 10^{-3}$	1/5	0.41, 0.60	
		$2.5 \times 10^{-3}$	1/2	0.01, 0.79	
		$6.5 \times 10^{-3}$	7/9	0.11, 0.91	
3	4	$-7.7 \times 10^{-3}$	1/4	15.30, 25.09	
		$-3.2 \times 10^{-3}$	1/9	<b>57.29</b> , 67.08	
		$1.9 \times 10^{-3}$	5/9	9.38, 31.02	
		$6.5 \times 10^{-3}$	1/6	51.36, <b>73.00</b>	
3	5	$1.3 \times 10^{-3}$	1/4	15.78, 25.57	
		$-6.3 \times 10^{-3}$	1/9	98.17, 107.96	
		$2.3 \times 10^{-3}$	5/2	9.85, <b>31.49</b>	
		$6.5 \times 10^{-3}$	1/6	<b>92.24</b> , 113.88	

310

311 **Table 1:** Commensurable periods of pairs and pairs of pairs of planets computed following  
 312 [Mörth and Schlamming \(1979\)](#). The periods encountered in the present paper are printed in red.

313

314



326 **Figure 4:** First SSA components of polar motion (trend) since 1846, called the Markowitz  
 327 drift. Component  $m_1$  is in red and  $m_2$  in blue.

328

329 We next analyze one by one the leading SSA components of the Earth’s rotation pole  
 330 coordinates.

331 *4.2 First SSA Component (Markowitz)*

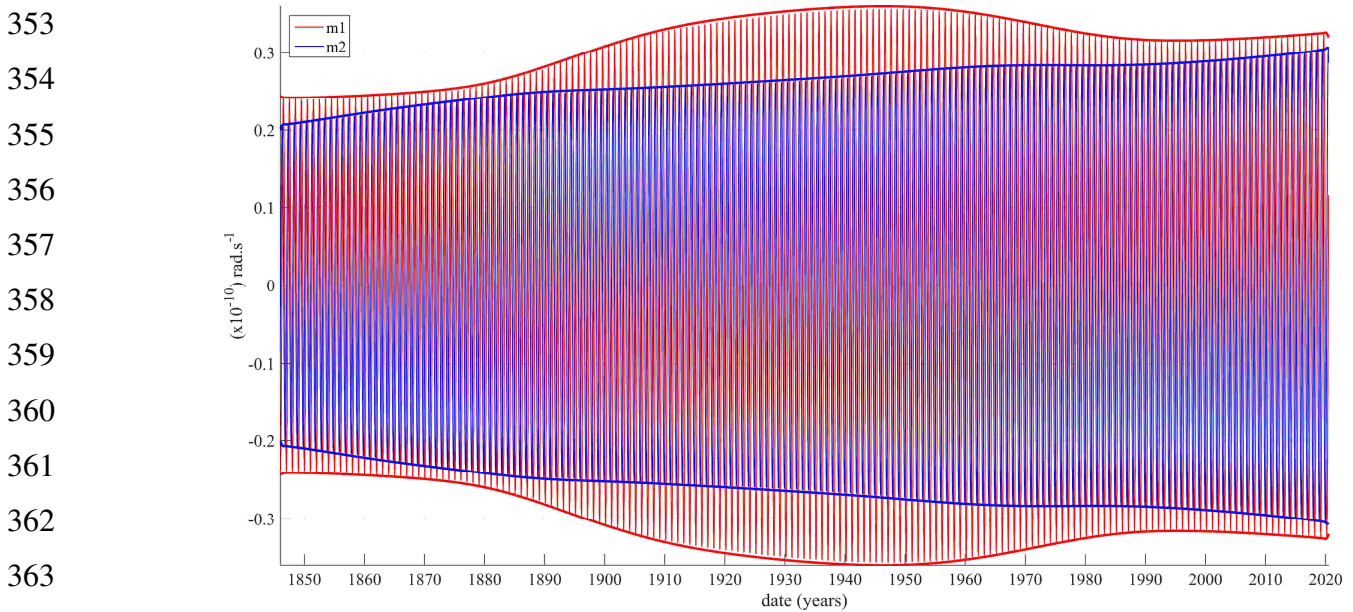
332 The first SSA components, shown in **Figure 4**, correspond to the mean trend of polar motion  
 333 called the Markowitz drift (**Markowitz, 1968**). The drift velocity is on the order of 13 cm/yr and is  
 334 principally carried by the E-W component  $m_2$ . As noted by a reviewer, these curves show changes  
 335 in slope and inflection points that are reminiscent of the recent evolution of the Earth’s global  
 336 surface temperature (**Le Mouél et al., 2019b**, Figure 20). This important point is not discussed  
 337 further in the present paper.

338 *4.3 Second SSA Component (Annual)*

339 The second SSA component is the forced annual oscillation (**Figure 5**). On that annual  
 340 oscillation, **Lambeck (2005, chapter 7, page 146)** writes “*The seasonal oscillation in the wobble is*  
 341 *the annual term which has generally been attributed to a geographical distribution of mass*  
 342 *associated with meteorological causes. Jeffreys in 1916 first attempted a detailed quantitative*  
 343 *evaluation of this excitation function by considering the contributions from atmospheric and*  
 344 *oceanic motion, of precipitation, of vegetation, and of a polar ice. Jeffreys concluded that these*  
 345 *factors explain the observed annual polar motion, a conclusion that is still valid today”.*

346 **Figure 5** shows that the annual components of  $m_1$  and  $m_2$  are significantly modulated, and in

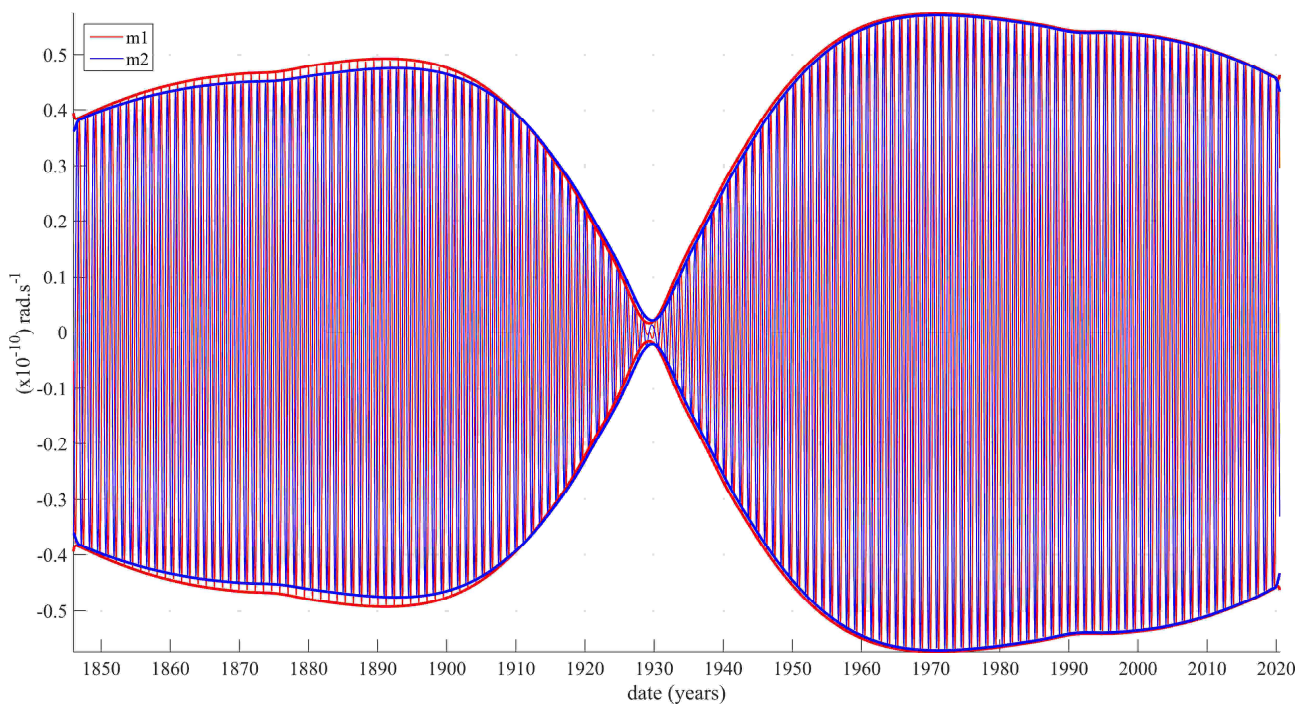
347 different ways (recall that the excitation functions are sums of sines and cosines with constant  
 348 weights; [Lambeck, 2005, page 153, equations 7.1.9](#)). In the generally accepted theory, modulation  
 349 is thought to be a response to reorganization of oceanic and atmospheric masses. We note in the  
 350 modulation of  $m_1$  the suggestion of a periodicity on the order of 150 years or more that could  
 351 correspond to the [Jose \(1965\) 171.5 yr cycle](#). Note that, given uncertainties, the Jose cycle could  
 352 actually be the [Suess- de Vries ~200 yr cycle \(Stefani et al. 2020\)](#).



364 **Figure 5:** 2nd SSA comp. of polar motion (annual oscillation) since 1846. ( $m_1$  red,  $m_2$  blue).

#### 365 4.4 Third SSA Component (Chandler)

366 **Figure 6** shows the third SSA component, that is the Chandler component. Its amplitude is  
 367 twice that of the annual component and its behavior is very different. The modulations are very  
 368 large, similar for  $m_1$  and  $m_2$ , and undergo a sharp and simultaneous change in phase and amplitude  
 369 in 1930. Many scientists have studied this phase change ([Hinderer et al., 1987](#); [Runcorn et al., 1988](#);  
 370 [Gibert et al, 1998](#); [Bellanger et al, 2001](#); [Bellanger et al, 2002](#); [Gibert and Le Mouél, 2008](#)). The

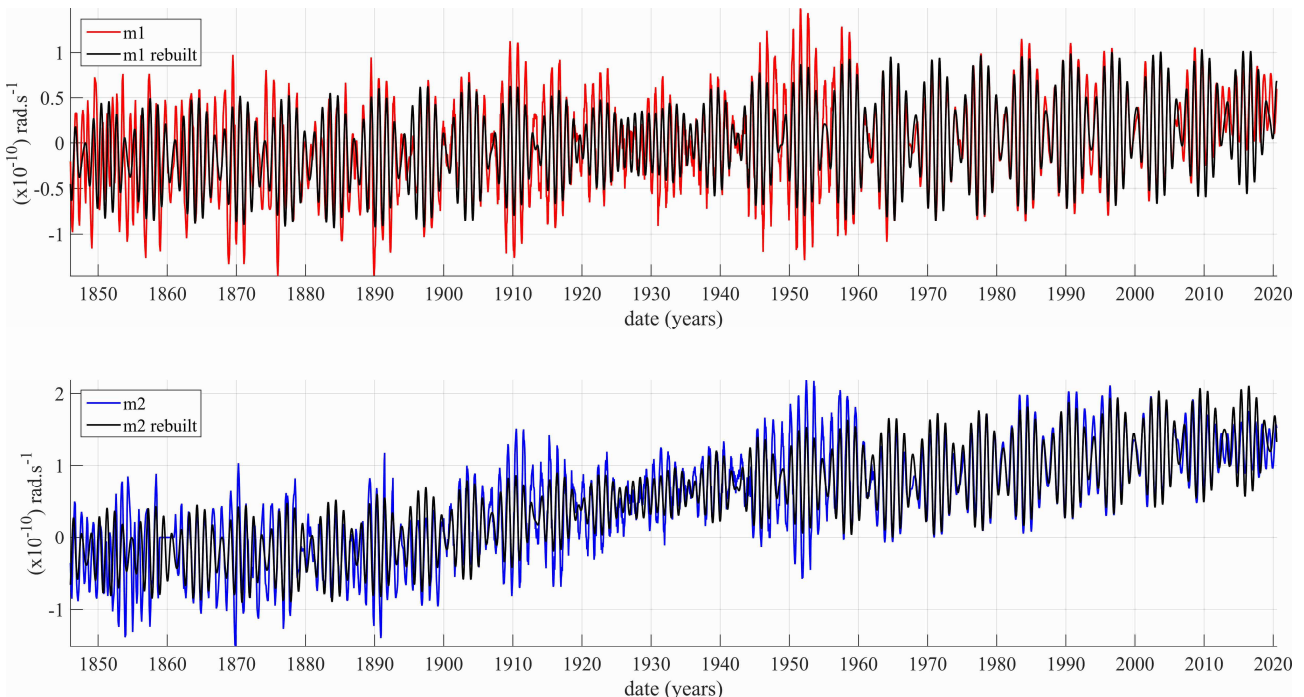


371 **Figure 6:** Third SSA components of polar motion (Chandler oscillation) since 1846.  
 372 Component  $m_1$  is in red and  $m_2$  in blue.

373  
 374 Chandler oscillation extracted by SSA is similar to that obtained with wavelets by [Gibert et al. \(1998\)](#). It is also as regular as that obtained with SSA by [Gorshkov et al. \(2012\)](#).

375  
 376 When the first three SSA components of  $m_1$  and  $m_2$  are added, they account for 73% of the  
 377 original variance. The quality of that incomplete reconstruction is shown in [Figure 7](#).

378 Pushing the SSA analysis further reveals an oscillation with period 1.22 yr with an 18.6 yr  
 379 modulation (the nutation), one with period 1.15 yr with a symmetrical modulation as in the case of  
 380 the Chandler term, one with period 1.10 yr. Some of these (quasi-) periods have already been found  
 381 using SSA on time series of sunspots ([Le Mouél et al, 2020a](#)). These periods seem to be linked to  
 382 the ephemerids of solar system planets, which has been used by [Courtilot et al \(2021\)](#) to predict the



383  
 384 **Figure 7:** Reconstruction of polar motion since 1846 using only its first three SSA  
 385 components. Top: observed component  $m_1$  in black and reconstructed in red; bottom: observed  
 386 component  $m_2$  in black and reconstructed in blue.

387  
 388 date and amplitude of the next solar (Cycle 25) maximum. Other components are compatible with  
 389 the Schwabe (11 yr) and Hale (22 yr) cycles. The 5.5 yr cycle is often associated with the Schwabe  
 390 cycle ([Usoskin, 2017](#)), but not all authors agree. Moreover, these components are found only in  $m_2$



391 and are much smaller in amplitude, on the order of  $10^{-13}$  to  $10^{-14}$  rad.s<sup>-1</sup> vs  $10^{-10}$  to  $10^{-11}$  rad.s<sup>-1</sup> for  
 392 the first three (Lopes et al, 2017; Japaridze et al, 2020). When all components from the trend to the  
 393 Hale cycle are added, they account for 95% of the total variance of the original series. Except for  
 394 the 1.10 and 1.15 yr components, all others are found in the table of planetary interactions (Table  
 395 1).

396

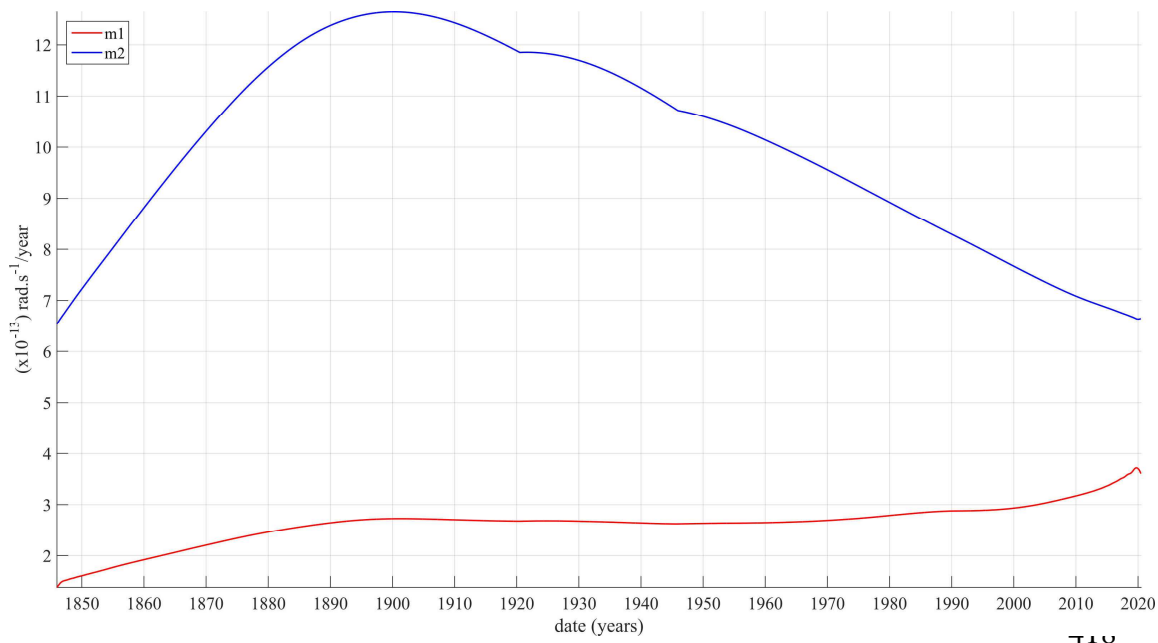
### 397 **5 - On Some Derivatives of SSA Components of Polar Motion**

398 System (D) expresses that there is a link between a force and the derivative of the resulting  
 399 polar motion (Appendix 1, equation 2). In other words Earth acts as a natural integrator (Appendix  
 400 1, equation 2 implies that  $m$  is an integral of  $\xi$ ; see Le Mouél et al, 2010). This leads us to analyze  
 401 the derivatives of the first three (largest) SSA components identified in the previous section.

#### 402 *5.1 Markowitz Drift*

403 We first calculate the derivative of the Markowitz drift (Figure 4), and analyze its major SSA  
 404 components. They are a trend (Figure 8a), a 90 yr pseudo-cycle (Figure 8b), a 40 yr pseudo-cycle  
 405 (Figure 8c), a 22 yr period and an 11 yr period (Figure 8d).

406

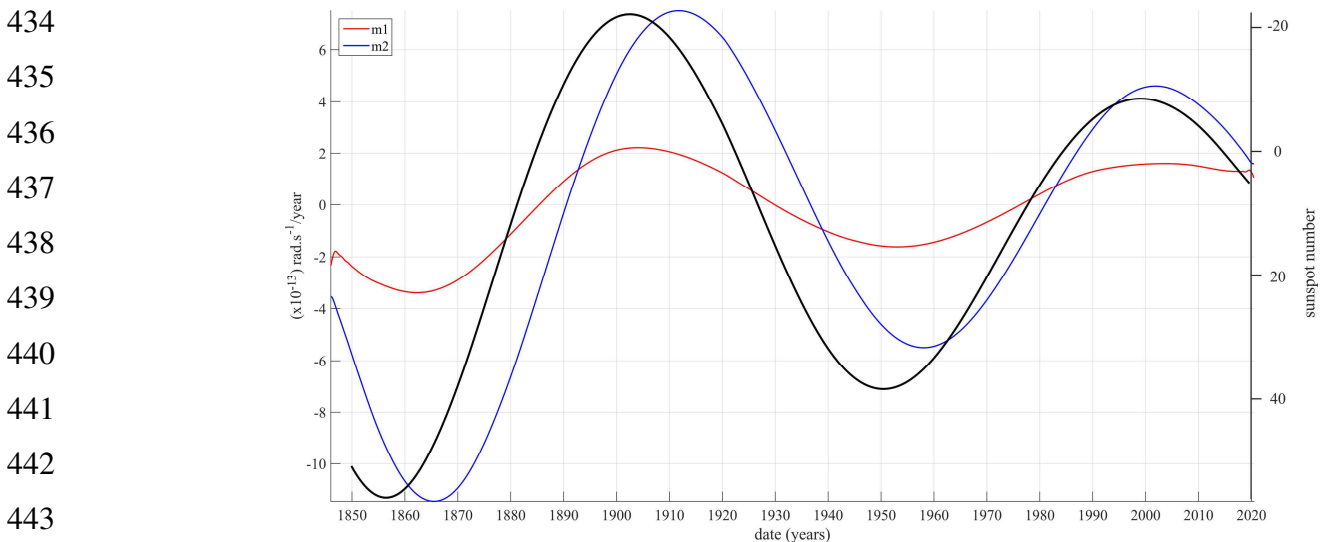


419 **Figure 8a:** First SSA component (trend) of the derivative of the Markowitz drift (first SSA  
 420 component of polar motion). Component  $m_1$  in red and  $m_2$  in blue.

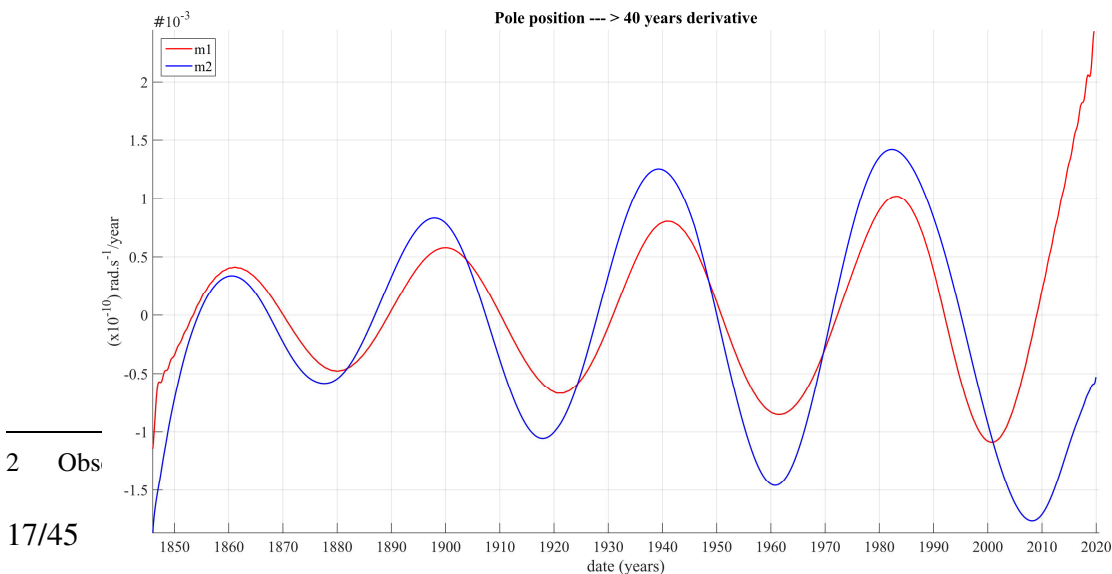
421

422 The 90 yr cycle is strongly correlated with the Gleissberg (1939) solar cycle (Figure 8b). Le

423 [Mouël et al \(2017\)](#) have obtained a period of  $90 \pm 3$  yr from sunspot series `SN_m_tot_V2.0`<sup>2</sup>. It  
 424 corresponds to a characteristic period in the ephemerids of Uranus ([Table 1](#)). There is also a close  
 425 correspondence of periods for the 11 yr oscillation ([Figure 8d](#)). The drift could be linked to the  
 426 modulation and varying “periodicity” of sunspots (8 to 13 yr). This is close to a characteristic  
 427 period of Jupiter’s ephemeris. The trend ([Figure 8a](#)) could be linked to the [Jose \(1965\)](#) 171.5 yr  
 428 cycle, attributed to Neptune ([Table 1](#)) or to the Suess-de Vries ~200 yr cycle ([Stefani et al., 2020](#)).  
 429 Finally, the 40 yr component has been shown by [Mörth and Shlamminger \(1979\)](#); see also [Courtillot](#)  
 430 [et al, 2021](#)) to correspond to a commensurable revolution period of the four Jovian planets. It is  
 431 interesting to point out that in both terrestrial polar motion and solar activity (as studied through the  
 432 proxy of sunspots) the first 3 components that emerge from SSA are a trend, then the Gleissberg  
 433 and Schwabe quasi-cycles.



444 **Figure 8b:** Second SSA component (90 yr period)) of the derivative of the Markowitz drift (first  
 445 SSA component of polar motion). Component  $m_1$  in red and  $m_2$  in blue. In black: Gleissberg cycle  
 446 extracted from sunspots (sign reversed).



454 **Figure 8c:** Third SSA component (40 yr period)) of the derivative of the Markowitz drift (first  
 455 SSA component of polar motion). Component  $m_1$  in red and  $m_2$  in blue.

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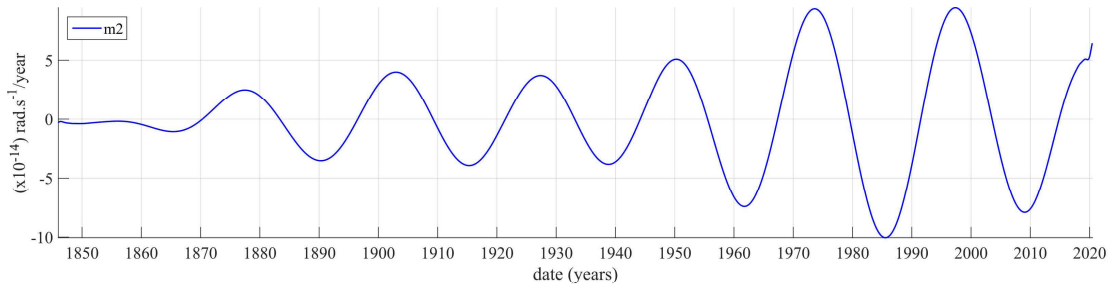
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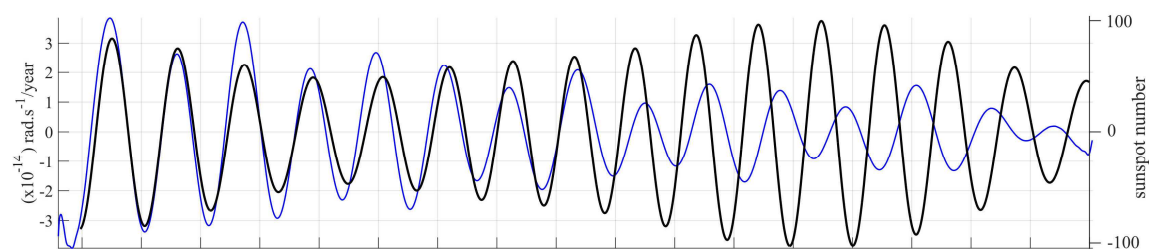
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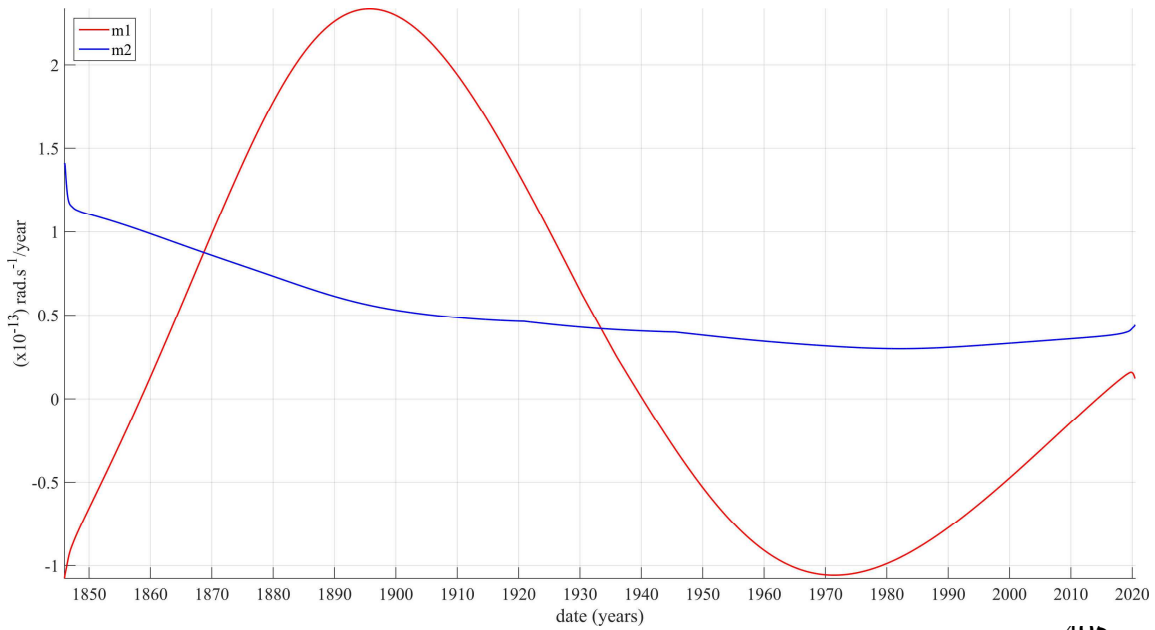
468 **Figure 8d:** 22 yr SSA component (top, component  $m_2$  in blue) and 11 yr SSA component  
 469 (bottom, component  $m_2$  in blue) of the derivative of the Markowitz drift (first SSA component of  
 470 polar motion). Bottom, black curve: the 11 yr Schwabe cycle extracted by SSA from the sunspot  
 471 series (sign reversed).

472

### 473 5.3 Envelope of the Forced Annual Oscillation

474 We next turn to the derivative of the envelope of the forced annual oscillation (**Figure 5**). Its  
 475 first SSA component, the trend, is shown in **Figure 9a**. For  $m_1$  this trend is compatible with a little  
 476 more than one period of a sine curve with a period close to 170 yr, that is the Jose (**1965**) solar  
 477 cycle, corresponding to the ephemeris of Neptune (or again given uncertainties to the Suess-de  
 478 Vries ~200 yr cycle). The next SSA component is a 70 yr cycle for  $m_1$  and a 60 yr cycle for  $m_2$   
 479 (**Figure 9b**). These periods, or pseudo-periods, are among those resulting from combinations of  
 480 ephemerids of the Jovian planets (**Mörth and Schlamming, 1979; Scafetta, 2020; Table 1**). The 60  
 481 yr cycle had already been found in sunspot series by **Scafetta (2010)** and **Le Mouél et al (2020a)**.  
 482 We had also seen it as an important component of series of global temperature and PDO and AMO  
 483 oceanic indices (**Courtilot et al, 2013**).

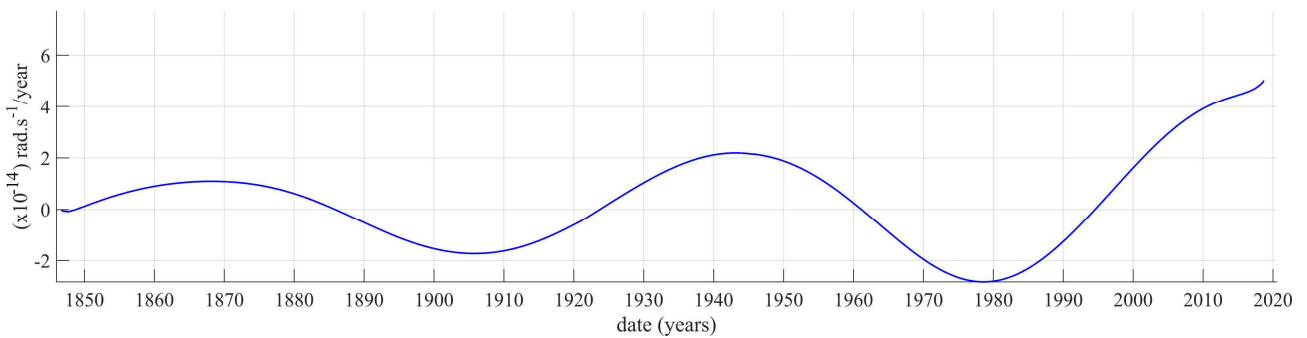
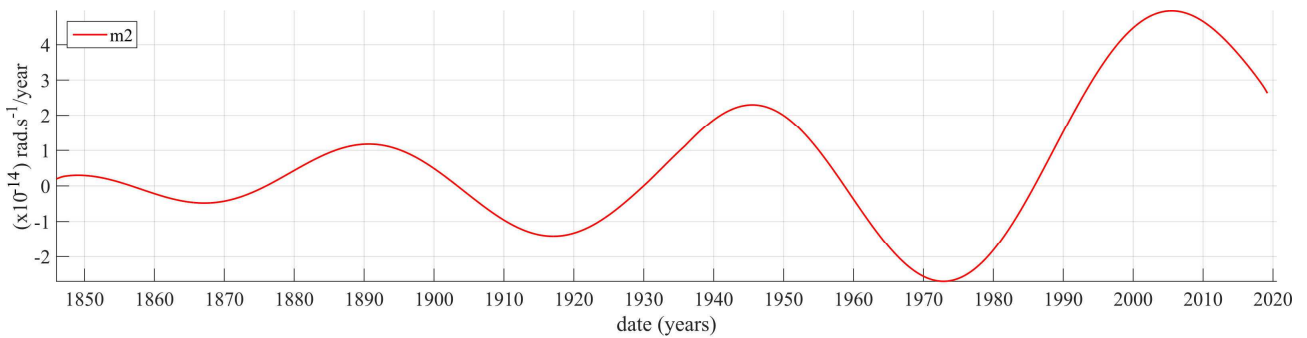
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495

496 **Figure 9a:** First SSA component (trend) of the derivative of the envelope of SSA component 2  
497 (annual oscillation) of polar motion.

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501 **Figure 9b:** Second SSA component (60 years and 70 years) of the derivative of the envelope  
502 of SSA component 2 (annual oscillation) of polar motion.



503 *5.4 Free Chandler Oscillation*

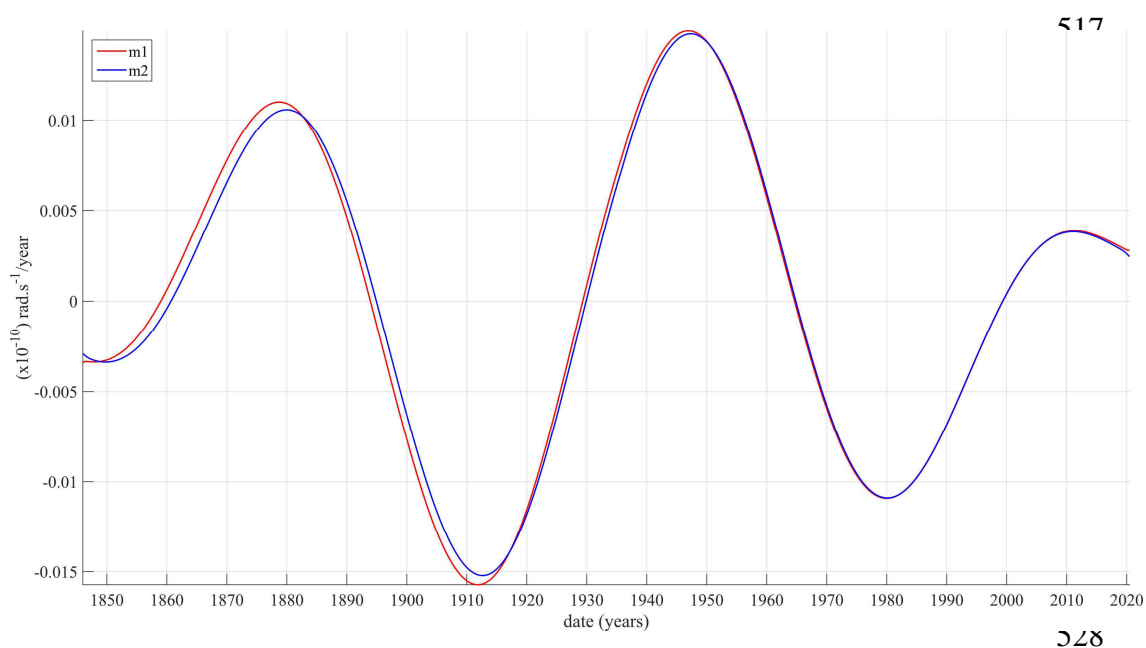
504 We now undertake the SSA analysis of the derivative of the envelope of the Chandler  
 505 oscillation (Figure 6). We find components with periods 70, 40, 30 and 22 yr (Figures 10a to 10d).  
 506 It is remarkable that the components for  $m_1$  and  $m_2$  are quasi-identical and have a very regular  
 507 behavior, close to sine functions but with some slower modulation: they could be described as  
 508 “astronomical” (as opposed to “astrophysical”, as defined by [Mayaud, 1980](#)).

509

510 **6 – Further Examples**

511 We can illustrate further how Jovian planets influence polar motion with the combined effects  
 512 of the pair Uranus (84 yr) – Neptune (165 yr): this pair has revolution periods compatible with the  
 513 envelopes in Figures 5, 6 and 8b. Figure 11a shows the sum of the kinetic moments of these two  
 514 Jovian planets as seen by Earth, which explains the annual oscillation whose envelope is of interest  
 515 to us. Using again SSA, we have extracted the first four components of that signal; we show three

516



529 **Figure 10a:** First SSA component (70 yr quasi-period) of the derivative of the envelope of the  
 530 Chandler oscillation.

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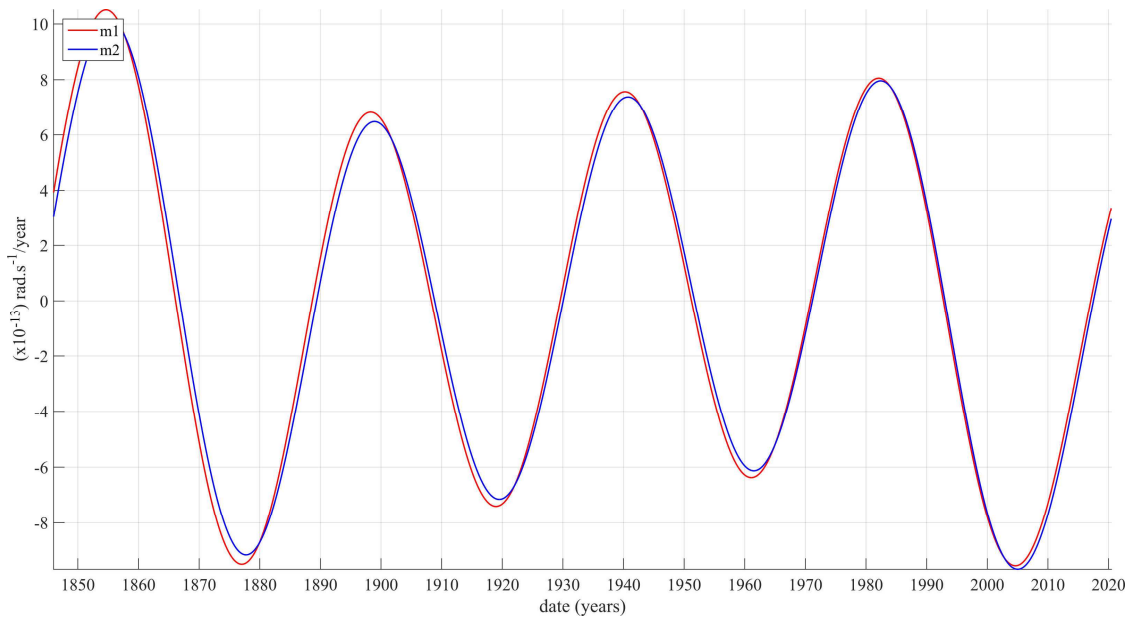
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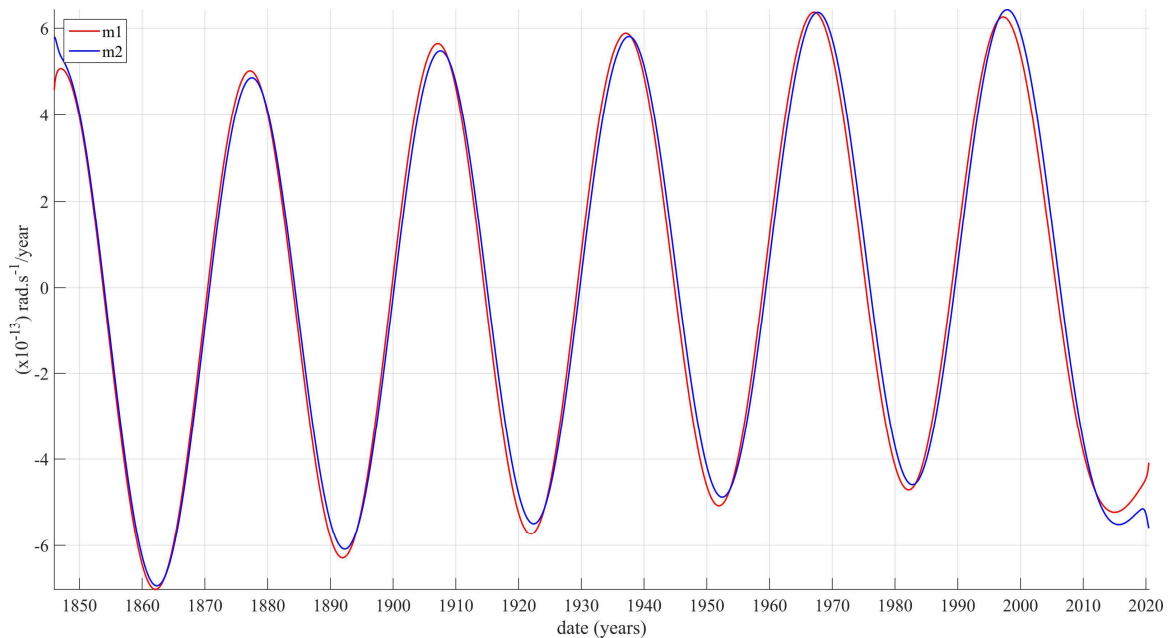
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Figure 10b: Second SSA component (40 yr quasi-period) of the derivative of the envelope of the Chandler oscillation.

Figure 10c: Third SSA component (30 yr quasi-period) of the derivative of the envelope of the Chandler oscillation

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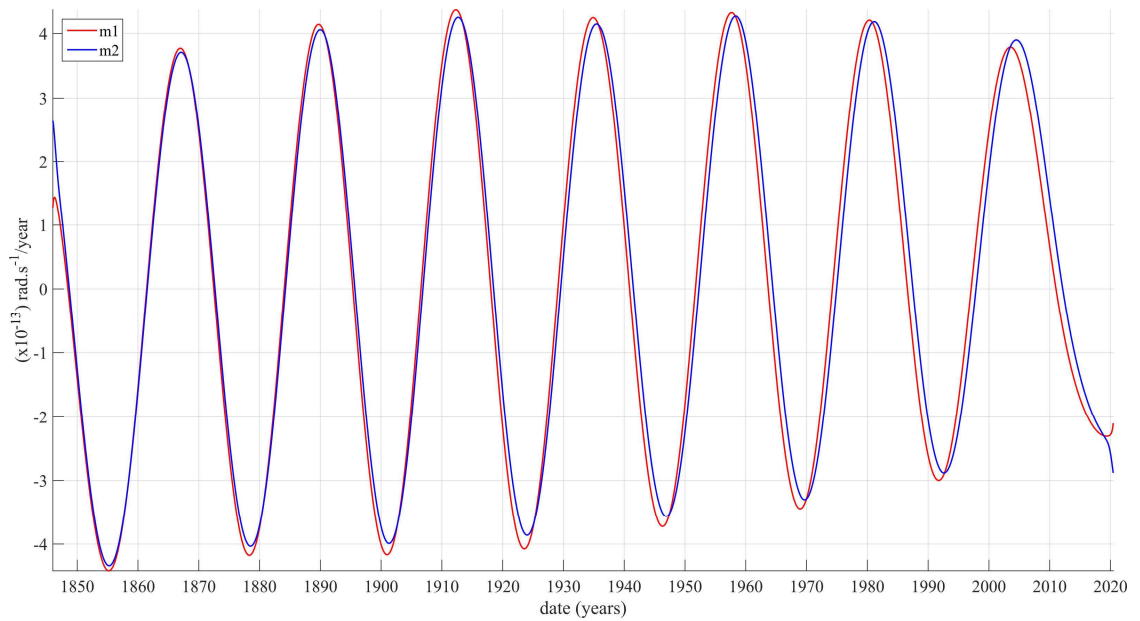
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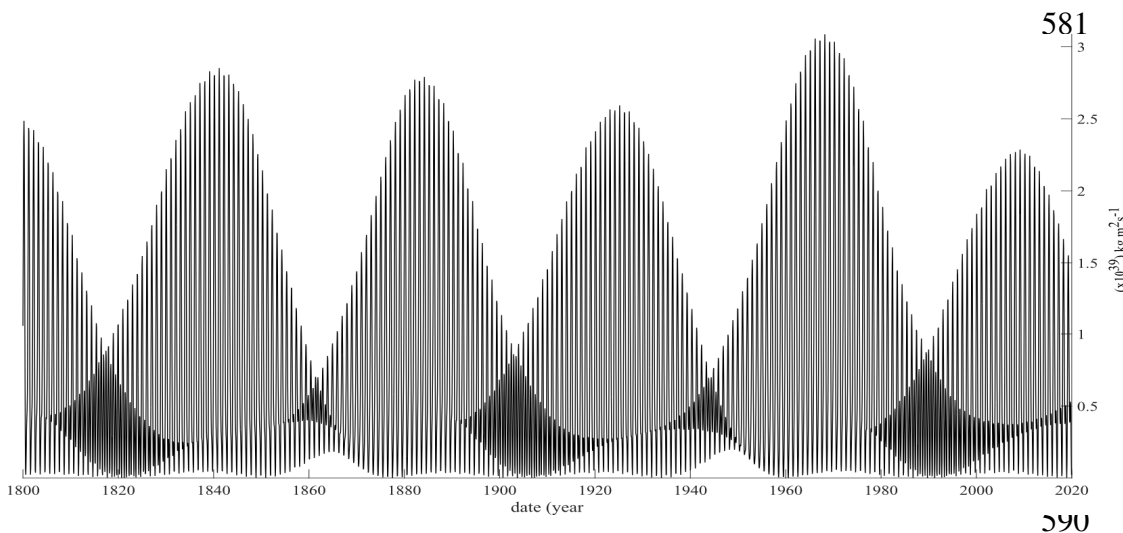


578

578 **Figure 10d:** Fourth SSA component (22 yr quasi-period) of the derivative of the envelope of

579 the Chandler oscillation

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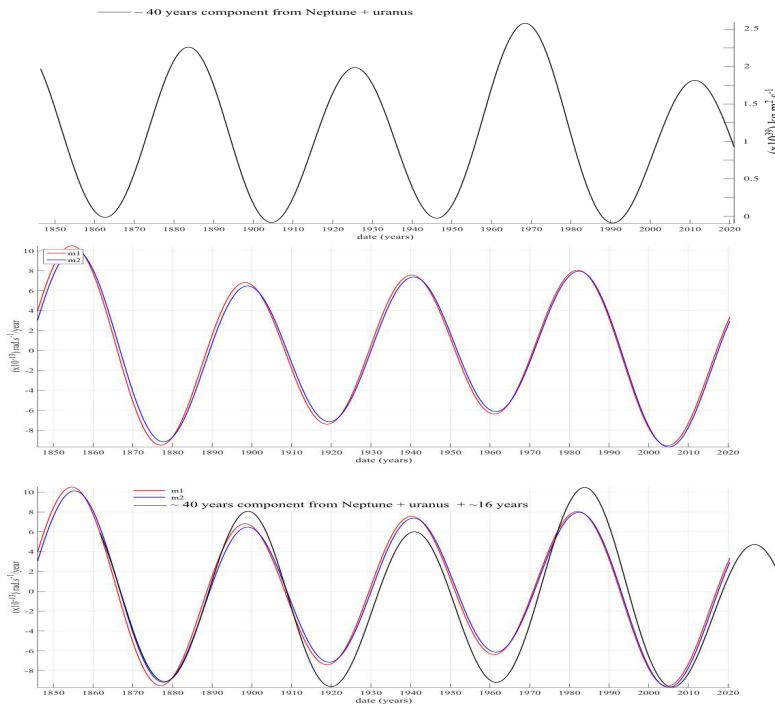


591 **Figure 11a:** The sum of the kinetic moments of Uranus and Neptune in a geocentric reference

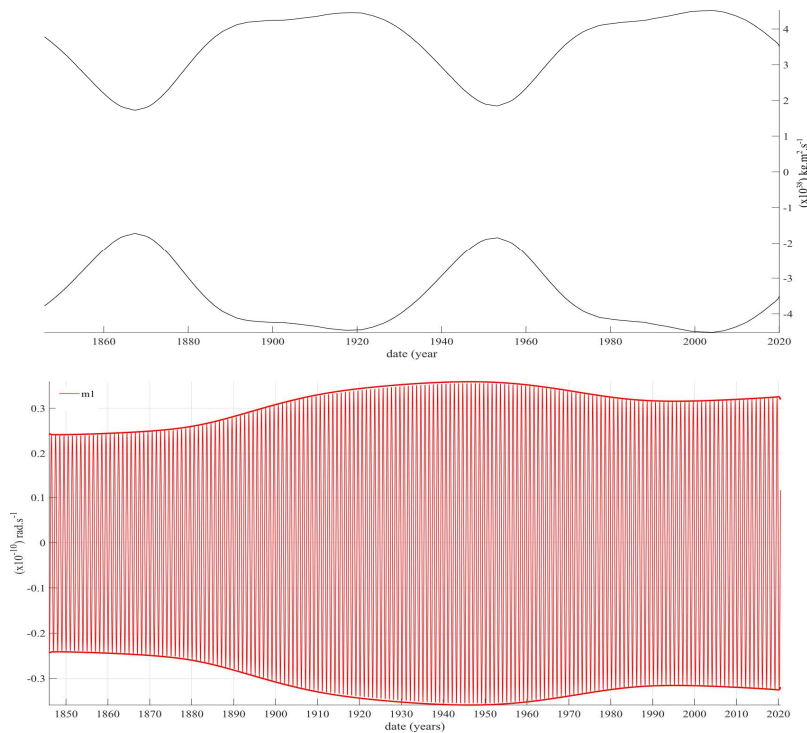
592 frame.

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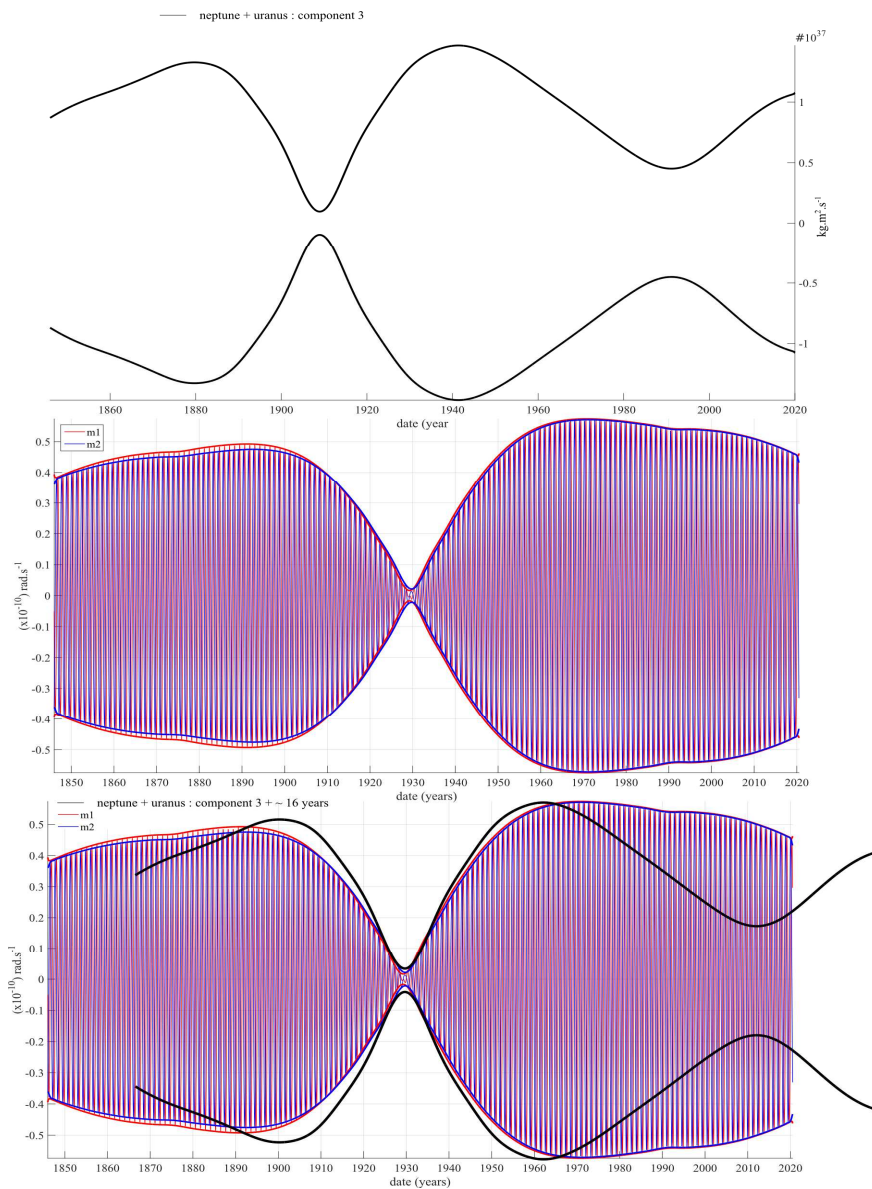


607 **Figure 11b:** Superposition (bottom) of the 40yr SSA (first) component of the curve shown in  
 608 **Figure 11a** (top) and the component of polar motion with similar pseudo period in **Figure 10b**  
 609 (middle). The top curve has been shifted by 16 yr with respect to the middle one in the bottom  
 610 comparison.



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 624 **Figure 11c:** Second component of the Uranus-Neptune pair (top) and forced annual oscillation  
 625 of the polar motion  $m_1$  (bottom).

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649 **Figure 11d:** Third SSA component of the Uranus-Neptune couple (top), Chandler oscillation (third  
650 SSA component of polar motion; middle) and comparison of the two with a 16 yr offset.

651

652 of them, from which the annual and semi-annual oscillations are absent. The first one (**Figure 11b**)  
653 has a pseudo period of about 40yr, similar to the envelope of the component of polar motion shown  
654 in **Figure 10b**. When a 16yr phase shift is taken into account, the correlation is remarkable (**Fig 11b**,  
655 bottom frame). The second component has nodes and bellies that are reminiscent of the envelope of  
656 the annual forced  $m_1$  component of polar motion (**Figure 11c**). The third component of the Uranus-  
657 Neptune pair compares well with the Chandler free oscillation, particularly so when a 16 yr offset is

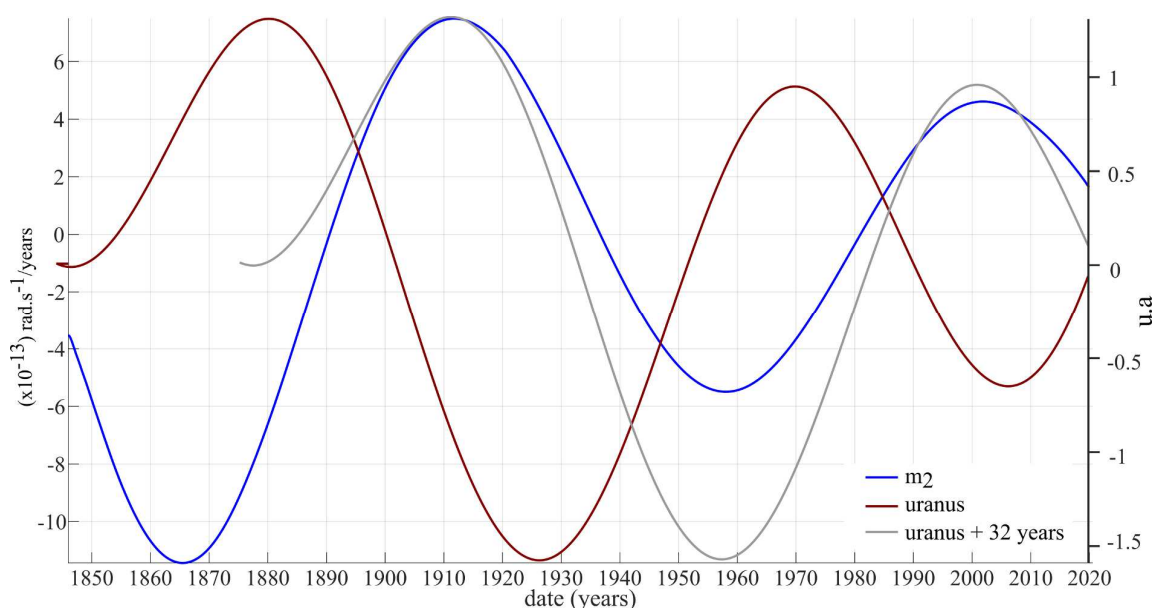
658 taken into account (Figure 11d).

659 Finally, in Figures 12a to 12c, we superimpose the signatures (components) of the ephemeris  
 660 of Jovian planets on the components of polar motion. In Figure 12a, the 90 yr component of the  
 661 envelope of  $m_2$  matches the ephemeris of Uranus offset by 32 years. In Figure 12b, the 165 yr  
 662 component of the envelope of  $m_1$  matches the ephemeris of Neptune, also offset by 32 years. In  
 663 Figure 12c, the 30 yr component of the envelope of  $m_1$  of the Chandler oscillation matches the  
 664 ephemeris of Saturn offset by 15 years. The 11 yr component detected in the  $m_2$  component of the  
 665 derivative of the Markowitz drift (Figure 8d) has a variable phase drift with respect to the  
 666 ephemeris. But, whereas “solar” components (periodicities) do appear at 22 and 11 yr (and 5.5 yr?)  
 667 in polar motions, they are 3 to 4 orders of magnitude smaller than the leading components we  
 668 discuss here.

669 We have seen that the sum of the Markowitz drift, annual oscillation and Chandler oscillation  
 670 explain some 70% of polar motion. The same is true for the leading components of sunspots, i.e. the  
 671 sum of the trend (Jose ~171.5 yr cycle), Schwabe cycle (~11 yr) and Gleissberg cycle (~90 yr) (on  
 672 the same time range). These periods correspond to those of Neptune (~165 yr), Uranus (84 yr ) and  
 673 Jupiter (11.8 yr ).

674 Many if not most of the (quasi-)periods found in the SSA components of polar motion, of  
 675 their modulations, of their derivatives can be associated with the Jovian planets. Only one, the 432-  
 676 434 day period is due to the Earth’s mass and moments of inertia and not to the Jovian planets, as  
 677 predicted by Laplace (1799).

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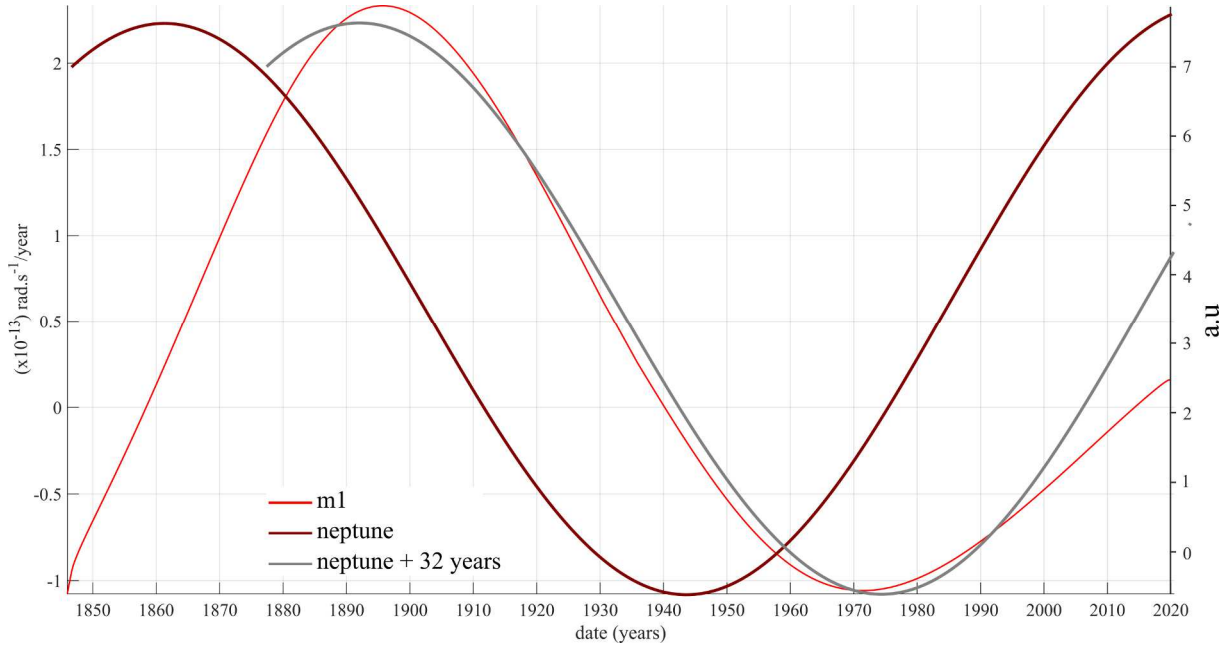


679  
 680 Figure 12a: Superimposition of the ~90 yr SSA component of the envelope of  $m_2$  (blue curve; see



681 **Figure 8b)** with the ephemerids of Uranus (red curve). Grey curve: Uranus curve delayed by 32  
 682 years.

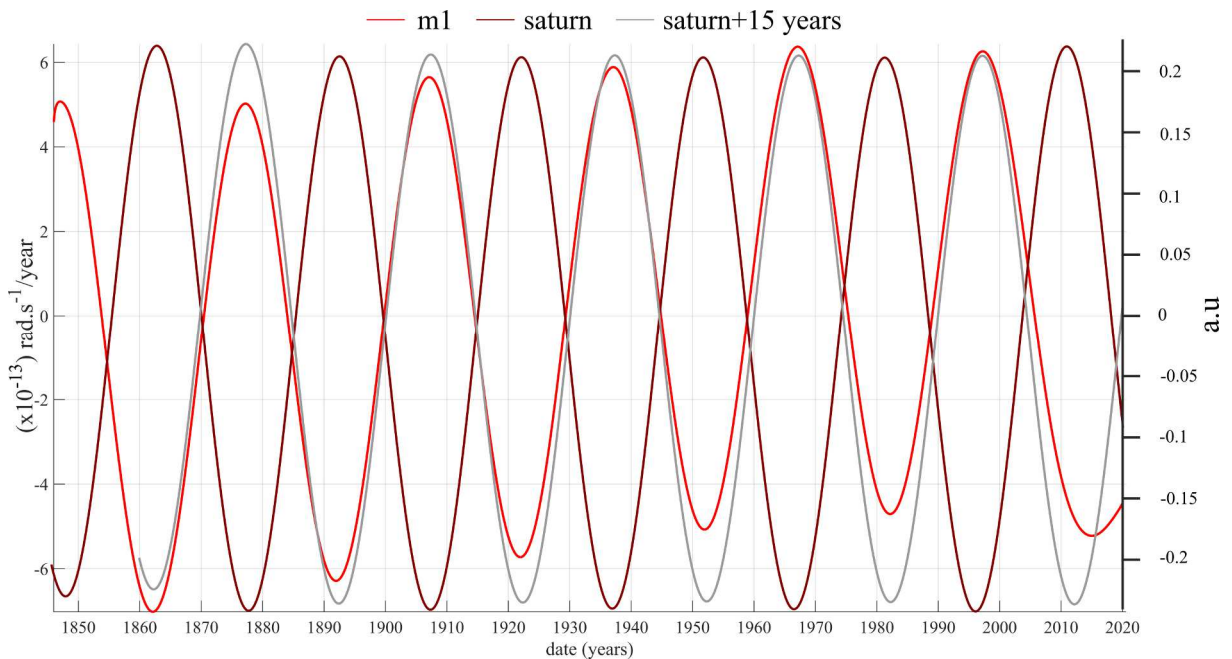
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684

685 **Figure 12b:** Superimposition of the ~165 yr SSA component of the envelope of  $m_1$  (red curve;  
 686 see **Figure 10a)** with the ephemerids of Neptune (dark red curve). Grey curve: Neptune curve  
 687 delayed by 32 years.

688



689

690 **Figure 12c:** Superimposition of the ~30 yr SSA component of the envelope of  $m_1$  (red curve; see  
 691 **Figure 10c)** with the ephemerids of Saturn (dark red curve). Grey curve: Saturn curve delayed by 15

692 years.

693

694

## 695 **7 – Summary, Discussion and Conclusion**

696

697 The general laws that govern the motions of celestial bodies have been derived and discussed  
698 by Laplace (1799) in his remarkable *Traité de Mécanique Céleste*. Laplace established the system  
699 of linear differential equations now known as the Liouville-Euler equations. He provided the full set  
700 of equations for the three Euler angles that specify the motions of a body’s axis of rotation. Laplace  
701 differs from most later authors in the way he uses the Liouville-Euler system. Laplace makes full  
702 use of the system (D) for a rotating body that undergoes both rotation and translation, and solves the  
703 algebraic transcendent equations of Appendix 3, given all astronomical parameters. Most others use  
704 a simplified version with the formalism of excitation functions (Appendix 1, equation 2; a second  
705 order system) in which the possibility of a translation of the body’s rotation axis is denied.

706 When Laplace obtains system (D) on page 74 of Chapter 7 of Book I, after 7 chapters that led  
707 him to these equations, he recognizes the fact that the system accounts for rotation as well as  
708 translation of a rotating body’s polar axis. When Lambeck (for instance) follows the same route, his  
709 Chapter 3 (entitled «*Rotational Dynamic*») on page 30 begins with the following sentence : «*The*  
710 *fundamental equations governing the rotation of a body are Euler’s dynamical equations*».  
711 Lambeck links the angular momentum to the torque that generated it. One means only rotation: that  
712 would be valid if the Earth’s inclination were zero or a constant. The equations are the same, but  
713 one soon forgets that the momentum that is the source of the torque (Lambeck’s system (3.1), page  
714 30) is a 3D vector (with no reason to be restricted to 2D, since the Earth is neither flat, nor is its  
715 inclination constant; its rotation axis revolves about the Sun and is therefore subjected at least to our  
716 star’s kinetic momentum). This oversight has some consequences. Since one only considers  
717 rotations, not translations, then the (Chandler) free rotation is obtained by zero-ing all torques and  
718 disregarding the third equation for the  $m_3$  polar coordinate (that is assumed constant). Then, the  
719 forced annual oscillation cannot be due to the revolution of Earth about the Sun and one must find  
720 causes for these forced oscillations (the excitation functions). Laplace of course knew that polar  
721 coordinates  $m_1$  and  $m_2$  were connected to  $m_3$ . Therefore, Laplace did not constrain polar motion to  
722 the two surface components ( $m_1, m_2$ ) but represented it by two meaningful components, the axis’  
723 inclination  $\theta$  and the time derivative of its declination  $\psi$  depends on the inclination



724 (previously calculated as a solution of the first Liouville-Euler equation). Laplace showed that there  
725 existed a free oscillation that would drift with a period between 306 (conjunction nodes) and 578  
726 days (conjunction bellies), fully determined by the Earth’s moments of inertia. This free oscillation,  
727 the Chandler oscillation, has a current value of 432-434 days. We now have long time series, up to a  
728 couple of centuries long, available and we use series of coordinates of the rotation pole  $m_1$  and  $m_2$   
729 (Figure 1) to extend some of Laplace’s (1799) results. A simple Fourier transform (Figure 2) shows  
730 the dominant spectral lines at 1 yr (forced annual oscillation) and 1.19 yr (free Chandler  
731 oscillation).

732 Singular spectral analysis (SSA) allows to better characterize the three leading components,  
733 the trend (~13cm/yr) called the (free) Markowitz drift (Figure 4), then the (forced) annual  
734 oscillation (showing different modulations for  $m_1$  and  $m_2$ , Figure 5) and the Chandler oscillation  
735 (with a very large modulation and a phase change in 1930, similar for  $m_1$  and  $m_2$ , Figure 6). Under  
736 the current theory, modulation is thought to be a response to reorganization of oceanic and  
737 atmospheric masses (e.g. Lambeck, 2005, chapter 7). Taken together, the first three SSA  
738 components explain 73% of the signal’s total variance (Figure 7). The smaller components that  
739 follow have (pseudo-) periods of 1.22 (with an 18.6 yr modulation), 1.15 and 1.10 yr. Some of these  
740 periods have been encountered in sunspot series and in the ephemerids of Jovian planets (Le Mouél  
741 et al., 2020a; Courtillot et al., 2021).

742 We have next analyzed in the same way the envelopes of the derivatives of the first three SSA  
743 components of polar motion (Figure 8). We find a trend in the derivative of the Markowitz drift,  
744 that could also correspond to the 171.5 yr Jose cycle (associated with the ephemeris of Neptune) or  
745 the ~200 yr Suess-de Vries cycle). Next, a 90 yr component, reminiscent of the Gleissberg solar  
746 cycle (associated with the ephemeris of Uranus), a 40 yr component, corresponding to a  
747 commensurable revolution period of the four Jovian planets, a 22 yr and an 11 yr component, that  
748 can be associated with Jupiter and/or the Sun. For the modulation of the annual component of polar  
749 motion, SSA finds periods of 165, 70 and 60 years (Figure 9). The 60 yr component has been found  
750 in sunspots, global temperature of Earth’s surface, and the oceanic oscillation patterns PDO and  
751 AMO (and Saturn). Finally, for the Chandler component, excellent matches are found for  $m_1$  and  $m_2$   
752 with periods of 70, 40, 30 and 22 years (Figure 10).

753 One can think in Laplace’s terms that the kinetic moments of planets act directly on Earth, or  
754 that these moments act on the external layers of the Sun (which is a fluid mass) and perturb its  
755 rotation, hence its revolution and kinetic moment  $M$  (Appendix 1), eventually affecting the Earth’s

756 axis of rotation: classical mechanics allows the hypothesis that planets influence the Earth’s rotation  
757 axis (hence climate and other global phenomena). Laplace (1799) has shown that one should  
758 consider the orbital kinetic moments of all planets and that the Earth’s rotation axis should undergo  
759 motions with the signatures/periods of the Sun and planets: the moments of the Jovian planets range  
760 from 1.7 to 19.3  $10^{42}$  kg.m<sup>2</sup>.s<sup>-1</sup>, and for the Sun, an equivalent is 1.7  $10^{41}$  kg.m<sup>2</sup>.s<sup>-1</sup>. To first order,  
761 the total kinetic moment applied to the Earth’s rotation axis is simply the sum of individual (Jovian)  
762 planetary kinetic moments plus the Solar kinetic moment.

763 We have shown (Figure 3) that the  $m_1$  component of polar motion reconstructed with SSA,  
764 with the Markowitz trend removed, matches remarkably well the sum of kinetic moments of the  
765 four Jovian planets. We have also computed these kinetic moments from the planetary ephemerids  
766 of Uranus and Neptune (Figure 11a). They “predict” remarkably well (Figure 11b) the 40 yr SSA  
767 component of the derivative of the envelope of the Chandler oscillation (Figure 11d).

768 We have previously determined the characteristic SSA components of solar activity, using  
769 sunspot numbers as a proxy (Le Mouél et al., 2019b). The sum of the Markowitz drift, annual  
770 oscillation and Chandler oscillation explain over 70% of polar motion. The same is true for  
771 sunspots, on the same time range, regarding the sum of the trend (Jose ~171.5 yr), Schwabe (~11  
772 yr) and Gleissberg (~90 yr) cycles. These periods correspond to those of Neptune (~165 yr), Uranus  
773 (~90 yr) and Jupiter (~11 yr). We have superimposed the signatures (components) of the  
774 ephemerids of Jovian planets on the components of polar motion. The 90 yr component of the  
775 envelope of  $m_2$  matches the ephemerids of Uranus, offset by 32 years (Figure 12a). The 165 yr  
776 component of the envelope of  $m_1$  matches the ephemerids of Neptune, also offset by 32 years  
777 (Figure 12b). And the 30 yr component of the envelope of  $m_1$  of the Chandler oscillation matches  
778 the ephemerids of Saturn, offset by 15 years (Figure 12c).

779 We have followed Mörth and Schlamming (1979), who determined the commensurable  
780 periods of pairs and pairs of pairs of Jovian planets (Table 1): we find that 8 of them, ranging from  
781 1.2 to 165 years, correspond to the SSA components of polar motion identified in sections 4 to 6 of  
782 this paper. Taken together, the curves shown in Figures 3 to 12 strongly argue that many  
783 components of Earth’s polar motion can be explained by Laplace’s classical celestial mechanics,  
784 with the combined instantaneous action of gravitational forces, and the longer, time-integrated  
785 action of kinetic moments. All celestial bodies act on the ( $m_1, m_2$ ) pair of polar coordinates, a list  
786 that starts with the Sun and Moon, and continues with the Jovian planets. It would be satisfying to  
787 undertake a rigorous demonstration of the influence of all planets on the Sun and on the Earth’s

788 rotation axis, that is to calculate the inclination  $\theta$  and declination  $\psi$  following Laplace’s (1799) full  
789 treatment of the equations (Appendix 2). Since the Liouville-Euler equations are linear differential  
790 equations of first order, we have been able to use the frame of small perturbations and we have  
791 considered that the influence of planets can be taken as the sum of individual influences.

792 We have recalled that Lambeck (2005) writes: ‘*The seasonal oscillation in the wobble is the*  
793 *annual term which has generally been attributed to a geographical distribution of mass associated*  
794 *with meteorological causes [...], a conclusion that is still valid today.*’ When one works within this  
795 theoretical frame, there remain unexplained observations such as the 434 day value of the current  
796 period of the Chandler wobble or the 6 month component of oceanic indices (Le Mouël et al.,  
797 2019d). This has led to attempts to increase the complexity of the model, such as the forcing by  
798 climate or the visco-elastic response to glacial isostatic rebound. We have seen that this theory uses  
799 only 2 of the 3 Euler angles. By using the full system of equations in the Liouville-Euler system (D  
800 for Laplace), Laplace (1799) was able to go beyond the synthetic treatments of (for instance) Guinot  
801 (1977) or Lambeck (2005). We have seen in this paper numerous applications of this theory that  
802 explain many pseudo-periodic components of a number of geophysical (and solar) phenomena,  
803 making the leading role of planetary ephemeris clear.

804 The shorter periods (months to a few decades) often show as modulations of even shorter  
805 variations. And trends, with about 200 years of data, are possibly due to periods in the ephemeris  
806 comparable to or longer than the range of available observations. Still, these 200 years allow us to  
807 test Laplace’s work further than he himself could. We have for instance been able to use this  
808 formalism to predict the future evolution of solar Cycle 25 (Courtillot et al, 2021).

809 It is widely assumed that both forced and free oscillations of Earth can, at least in part, be  
810 associated with climate forcings. Such has been the case from Jeffreys (1916) to Lambeck (2005),  
811 and recently to Zotov and Bizouard (2012) and Zotov et al. (2016). In all these works, causality is  
812 absent, be it from a time perspective or based on the orders of magnitude of the forces required to  
813 perturb the Earth’s rotation. The periods that for instance Zotov et al. (2016) associate with an  
814 interaction between Earth’s fluid and rigid envelopes are found in other geophysical phenomena  
815 such as the Earth’s magnetic field or sunspots (Le Mouël et al., 2019a,b,c,d ; Le Mouël et al.,  
816 2020a,b ; Le Mouël et al., 2021; Courtillot et al., 2021 ; and references therein). We have come to  
817 the same conclusion regarding many climatic indices (Le Mouël et al., 2019d). If there is a good  
818 correlation of many characteristic periods, pseudo-periods and components extracted with SSA, for  
819 instance between Earth’s rotation and many features of climate, it is reasonable to assume that this

820 is because they are subject to some common forcings. This is not an overly speculative hypothesis:  
821 with the views of Laplace on tides, we know that the fluid envelopes react on short time scales (to  
822 changes in the Moon’s declination for 2/3rds and the Sun for 1/3<sup>rd</sup>). On longer time scales, the  
823 whole (including solid) Earth responds (e.g. [Dehant et Mathieus, 2015](#)), all being governed by the  
824 Liouville-Euler equations.

825         In the present study, we have been able to find planetary signatures in polar motions, strictly  
826 based on observational data and using only classical mechanics. A possible causal chain thus  
827 emerges that has gravity potential and kinetic moments of planets acting directly or modulating  
828 motions of the fluid parts of celestial bodies, i.e. the Sun’s outer layers (sunspots) and the Earth’s  
829 atmosphere and ocean. These effects are in general not yet modeled: this is a domain where climate  
830 modeling warrants significant research advances.

831         In summary and conclusion of this work, two different mechanisms (causal chains) are likely  
832 at work. One is illustrated by the spectacular and direct effect of the kinetic moments of the (Jovian)  
833 planets on the Chandler wobble, whose intrinsic period (somewhere between 306 and 578 days) is  
834 synchronized to 433 days (a value that depends on Earth properties). The causal chain is directly  
835 from the Jovian Planets to Earth. Another causal chain would be an effect of planetary motions on  
836 the solar dynamo; variations in solar activity would in turn influence meteorological and climatic  
837 phenomena, such as mass transport between the equator and the poles, length of day, sea-level,...  
838 Given the remarkable coincidence between the quasi-periods of many of these phenomena, it is  
839 reasonable to assume that both causal chains are simultaneously at work. In that sense, it is not  
840 surprising to find the signatures of the Schwabe, Hale and Gleissberg cycles in many terrestrial  
841 phenomena, reflecting the characteristic periods of the combined motions of the Jovian planets.

842

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849

Appendix 1: Polar Coordinates and Excitation Functions

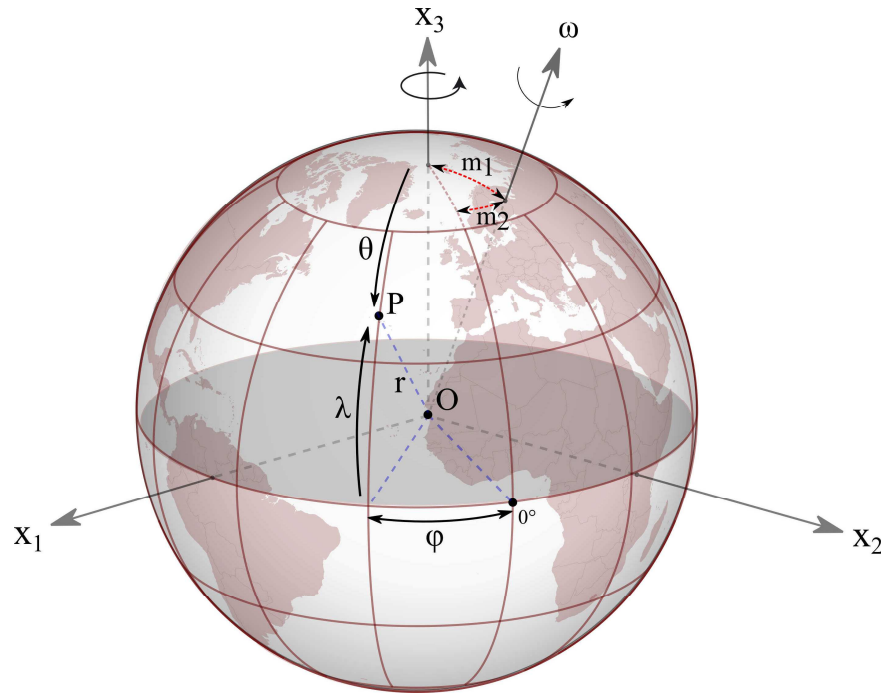


Figure A1 The reference system for the pole ( $m_1$  and  $m_2$ ).

Figure A1 gives the notations for the reference system that we use. The rotation pole is defined by its components  $m_1$  and  $m_2$ , respectively on the Greenwich ( $0^\circ$ ) and  $90^\circ\text{E}$  meridians. We follow Lambeck’s (2005, chapter 3) formalism. The rotation of the pole  $\omega$  can be decomposed into three Euler angles ( $\omega_1, \omega_2, \omega_3$ ) associated with the axes ( $X_1, X_2, X_3$ ) of the fixed reference frame. These Euler angles are a function of the Earth’s mean angular velocity  $\Omega$  and of the apparent position of the pole at the Earth’s surface  $m_1, m_2, m_3$  :

$$\begin{aligned} \omega_1 &= \Omega m_1 \\ \omega_2 &= \Omega m_2 \\ \omega_3 &= \Omega(1 + m_3) \end{aligned} \tag{1}$$

Noting that the Earth rotates about its axis and that its radius is constant, the Liouville-Euler system of equations (D for Laplace, 1799) becomes ( $1/\sigma$  is the Euler period):

$$\begin{aligned} j\left(\frac{\dot{\mathbf{m}}}{\sigma}\right)\Omega + \mathbf{m} &= \boldsymbol{\xi} \\ \dot{m}_3 &= \xi_3 \end{aligned} \tag{2}$$

880 where  $\mathbf{m} = m_1 + jm_2$  and  $\boldsymbol{\xi} = \xi_1 + j\xi_2$ . ( $\xi_1, \xi_2, \xi_3$ ) are the excitation functions (forces and  
881 moments).

882

883 **Appendix 2: Orientation of the Classical Orbit of a Planet Moving in the Field of a**  
884 **Central Body**

885 The orientation of the classical orbit of a particle of mass  $\mu$  (the Earth) around a large object  
886 of mass  $m$  (the Sun;  $m \gg \mu$ ) is determined by two conservative vectors, the kinetic moment :

887

$$\vec{M} = \vec{r} \times \vec{p} \tag{B1}$$

889

890 and the vector:

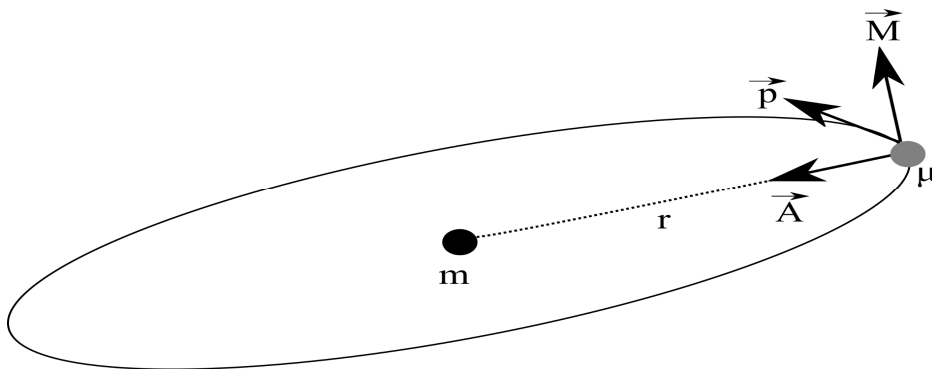
$$\vec{A} = \frac{\vec{p}}{\mu} \times \vec{M} - k \frac{m \vec{r}}{\mu r} \tag{B2}$$

893

894 whose conservation characterizes the Newtonian field (potential  $\phi = -km/r$ ;  $k$  being the constant of  
895 universal gravitation). Vector  $\vec{M}$  is perpendicular to the orbital plane and vector  $\vec{A}$  is oriented along  
896 the major semi-axis towards perihelion (Figure A2); the modulus of  $\vec{A}$  is  $km\mu e$ , where  $e$  is the  
897 orbit’s eccentricity. If  $m$  (the Sun) is rotating, then  $\vec{M}$  is perturbed following the equation:

$$\frac{d\vec{M}}{dt} = \vec{\Omega} \times \vec{M}$$

900  $\vec{\Omega}$  is the angular velocity of vector  $\vec{A}$ , thus the revolution velocity of the perihelion.



901

902

903 **Figure B1:** Particle  $\mu$  (the Earth) rotates about the Sun ( $m \gg \mu$ ).  $\vec{M}$  is its kinetic moment and  $\vec{A}$   
904 derives from the Newtonian potential.

905

906 **Appendix 3: Equations 5 and 6 from Laplace (1799), Book 5, Chapter 1, Pages 317-319**

907 [Laplace \(1799\)](#) establishes the two equations that determine the joint influence of the Moon  
908 and the Sun on the inclination  $\theta$  of the rotation (polar) axis ([Figure A1](#)) as a function of time and  
909 the time derivative of its declination  $\psi$ , that is of its rotation (chapter 1 of book 5 of his treatise, in  
910 sections 5 and 6 to which the interested reader is invited to [turn](#)).

911 The first equation reads (in Laplace’s terms):

$$\theta = h + \frac{3m}{4n} \cdot \left( \frac{2C - A - B}{C} \right) \cdot \left\{ \begin{array}{l} \frac{1}{2} \cdot \sin(\theta) \cdot \left\{ \cos(2\nu) + \frac{\lambda m}{m'} \cdot \cos(2\nu') \right\} \\ -(1 + \lambda) \cdot m \cdot \cos(\theta) \cdot \sum \cdot \frac{c}{f} \cdot \cos(ft + \varsigma) \\ + \frac{\lambda c'}{f'} \cdot \cos(\theta) \cdot \cos(f't + \varsigma') \end{array} \right.$$

912  
913  $h$  corresponds to the inclination of the rotation axis, that was  $26.0796^\circ$  ([Laplace, 1799](#), V (1), page  
914 349), and is now  $23.4333^\circ$ .  $m$  is the Sun’s mass and  $m'$  is the Moon’s mass.  $n$  is the mean angular  
915 rotation velocity of Earth about its third principal axis, that is the length of day.  $A, B, C$  are the  
916 Earth’s principal moments of inertia ( $A=B$ ). Parameter  $\lambda$  is an integration constant obtained for the  
917 free oscillation of the Liouville-Euler system of partial differential equations, equal to:

$$n \sqrt{\frac{C - A}{C - B} AB} = \frac{C - A}{A} = \frac{\sigma}{\Omega}$$

918  
919  $1/\sigma$  is the Euler period mentioned in the paper.  $\nu$  is the Sun’s and  $\nu'$  the Moon’s angular motion,  
920 that is their longitudes measured from the Spring mobile equinox or their right ascensions relative  
921 to Earth. One must also take into account the inclinations of Earth and Moon with respect to the  
922 Sun, represented by  $c$  and  $c'$ . They are associated to the longitudes of ascending nodes of Earth and  
923 Moon with respect to the Sun, also measured from the Spring mobile equinox and noted  $(ft + \zeta)$  and  
924  $(f't + \zeta')$ .

925 The second equation expresses the time variations of the rotation and uses the same notations  
926 and parameters:

$$\frac{d\psi}{dt} = \frac{3m}{4n} \cdot \left( \frac{2C - A - B}{C} \right) \cdot \left\{ \begin{array}{l} (1 + \lambda) \cdot m \cdot \cos(\theta) - \frac{\cos(\theta)}{2dt} \cdot \left\{ d \cdot \sin(2\nu) + \frac{\lambda m}{m'} \cdot d \cdot \sin(2\nu') \right\} \\ + (1 + \lambda) \cdot m \cdot \frac{\cos^2(\theta) - \sin^2(\theta)}{\sin(\theta)} \cdot \sum \cdot c \cdot \cos(ft + \varsigma) \\ + \lambda \cdot m \cdot \frac{\cos^2(\theta) - \sin^2(\theta)}{\sin(\theta)} \cdot c' \cdot \cos(f't + \varsigma') \end{array} \right.$$

927



928 One sees the rotation as the sum of two oscillations, one intrinsic to the Earth linked to the  
929 constant  $3m/4n$ , that varies like  $(1+\lambda).m.\cos \theta$  for all the nodes of luni-solar orbits, and is therefore  
930 a function of inclination  $\theta$ , and another one forced by the Moon and Sun, linked to longitudes  $(ft+\zeta)$   
931 and  $(f't+\zeta')$  and to right ascensions  $\nu$  and  $\nu'$  of these two orbs. Laplace therefore can estimate that  
932 the rotation period varies from 306 and 578 days.

933

934 **Appendix 4: Original Text in French of the Quotation from Laplace**

935 (*Traité de Mécanique Céleste*, vol. 5, cap. 1, page 347, 1799)

936 “Nous avons fait voir (n°8), que le moyen mouvement de rotation de la Terre est uniforme,  
937 dans la supposition que cette planète est entièrement solide, et l'on vient de voir que la fluidité de la  
938 mer et de l'atmosphère ne doit point altérer ce résultat. Les mouvements que la chaleur du Soleil  
939 excite dans l'atmosphère, et d'où naissent les vents alizés semblent devoir diminuer la rotation de  
940 la Terre: ces vents soufflent entre les tropiques, d'occident en orient, et leur action continuelle sur  
941 la mer, sur les continents et les montagnes qu'ils rencontrent, paraît devoir affaiblir insensiblement  
942 ce mouvement de rotation. Mais le principe de conservation des aires, nous montre que l'effet total  
943 de l'atmosphère sur ce mouvement doit être insensible; car la chaleur solaire dilatant également  
944 l'air dans tous les sens, elle ne doit point altérer la somme des aires décrites par les rayons  
945 vecteurs de chaque molécule de la Terre et de l'atmosphère, et multipliées respectivement par leur  
946 molécules correspondantes; ce qui exige que le mouvement de rotation ne soit point diminué. Nous  
947 sommes donc assurés qu'en même temps que les vents analysés diminuent ce mouvement, les autres  
948 mouvements de l'atmosphère qui ont lieu au-delà des tropiques, l'accélèrent de la même quantité.  
949 On peut appliquer le même raisonnement aux tremblements de Terre, et en général, à tous ce qui  
950 peut agiter la Terre dans son intérieur et à sa surface. Le déplacement de ces parties peut seul  
951 altérer ce mouvement; si, par exemple un corps placé au pôle, était transporté à l'équateur ; la  
952 somme des aires devant toujours rester la même, le mouvement de la Terre en serait un peu  
953 diminué; mais pour que cela fut sensible, il faudrait supposer de grands changements dans la  
954 constitution de la Terre”.



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