Numerical simulation of the December 1997 Debris Avalanche in Montserrat, Lesser Antilles

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Abstract. Emplacement of a debris avalanche in the White River valley of southern Montserrat (Lesser Antilles), on 26 December 1997, was caused by sector failure of the south flank of the active Soufriere Hills volcano. Pre- and post-emplacement surveys of the region indicate a debris avalanche deposit volume of about 40-50 x10^6 m^3. This avalanche is modeled as the gravitational flow of a homogeneous continuum governed by a basal friction law. Mass and momentum equations are depth-averaged over the slide thickness. Numerical results show that the observed distribution of debris and duration of emplacement is simulated well for a Coulomb-type friction law with a dynamic friction coefficient dependent upon the thickness and the velocity of the flowing mass.

Introduction

Eruptive activity at the Soufriere Hills volcano, Montserrat, began in 1995 and has led to repeated growth and collapse of a lava dome with associated pyroclastic flows reaching the sea. On 26 December 1997, failure of the southwestern flank of the volcano and adjacent parts of the growing lava dome produced a debris avalanche that flowed in the White River (Fig. 1 and 2), stopping a few tens of meters from the coastline [*Sparks et al.*, 2001; *Voight et al.*, 2001]. This paper is devoted to the simulation of this avalanche, and to the comparison of numerical results with the distribution of observed deposits.

Depending on the concentration of fluid, solid and gas within the flowing material, debris avalanches exhibit two principal types of behaviors, a solid one or a fluid one. The choice of the most appropriate constitutive laws (Bingham, Coulomb-type or Bagnold approaches) is still debated [e.g. *Sousa and Voight*, 1995; *Hutter*, 1996; *Iverson*, 1997; *Laigle and Coussot*, 1997], although many authors have agreed that some form of frictional behavior is appropriate for real avalanches. Following the modeling approach of Savage and Hutter (1989), the avalanche at Montserrat is treated here as the gravitational flow of a dense granular material, governed by Coulomb-type friction laws.

Avalanche Model

The mechanism initiating the debris avalanche is not investigated in this study [*Voight et al.*, 2001] and it is assumed that the whole mass suddenly loses its equilibrium and moves downslope under gravity forces. For simplicity and due to its observed macroscopic fluid-like behavior, the avalanche, composed of volcanic deposits with different sizes of particles, is treated here as an homogeneous and incompressible continuum. The presence of pore fluids, bed erosion and density variations due to dilatance of the material and porosity change are not taken explicitly into account. The model we have developed is based on the one-phase grain-flow model of Savage and Hutter (1989). Following their approach, and considering that the slide thickness is much smaller than the characteristic slide length, mass and momentum conservation equations are depth-averaged over the thickness. This assumption enables us to ignore the variations of the mechanical behavior within the flow. During the flowing process, and especially during early stages of collapse, the slide mass continually breaks into fragments. Most of the fragment collisions and deformations are assumed to be concentrated in the boundary layer near the bed surface [*Kilburn and Sorensen*, 1998]. It is the zone that has been rendered most homogeneous, by breakdown of megaclasts and mixing. Energy dissipation is then neglected within the body of the flow, which is small compared to the energy lost within the boundary layer; thus the profile of the slope-parallel velocity is assumed to be constant over the thickness [*Savage and Hutter*, 1989]. This hypothesis is consistent with the preservation of pre-failure stratigraphy observed in deposits of debris avalanches. It does not prevent lateral and down-slope spreading of the flow, caused by the pressure force due to height gradient. The basal friction law, of Coulomb-type, involves a dynamic friction angle δ between the rough bed and the flowing mass. When the flow is close to rest, the fluid velocity is set to zero as soon as the Coulomb force is larger than the algebraic sum of the forces due to gravity and height gradient. The resulting equations of mass and momentum conservation, written in a coordinate system linked to the topography, read in a conservative form :

$$\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}(hu)+\frac{\partial}{\partial y}(hv)=0 \qquad (1)$$

$$\frac{\partial}{\partial t}(hu)+\frac{\partial}{\partial x}(hu.u)+\frac{\partial}{\partial y}(hu.v)= -\frac{1}{2}\frac{\partial}{\partial x}(gh^2\cos\theta)+gh\sin\theta_x+F_x \qquad (2)$$

$$\frac{\partial}{\partial t}(hv)+\frac{\partial}{\partial x}(hv.u)+\frac{\partial}{\partial y}(hv.v)= -\frac{1}{2}\frac{\partial}{\partial y}(gh^2\cos\theta)+gh\sin\theta_y+F_y \qquad (3)$$

where the z coordinate is normal to each small patch of ground (cell size), (x,y) denote local slope parallel coordinates, $h(x,y,t)$ is the layer thickness perpendicular to the local slope, u and v are the depth-averaged velocities parallel to the local bed in the x and y direction respectively, $\boldsymbol{F}=-gh\cos\theta\ \tan\delta\ \boldsymbol{u}/|\boldsymbol{u}|$ is the friction force with $\boldsymbol{u}=(u,\ v)$,

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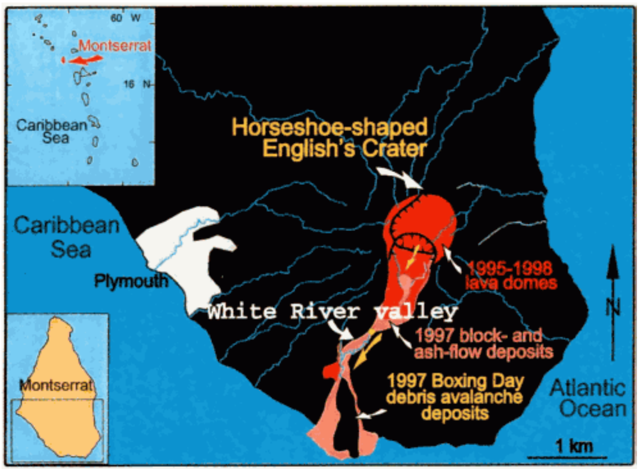

Figure 1. Map of Montserrat, Lesser Antilles.

$\theta(x,y)$ is the local steepest slope angle, and θ_x and θ_y are the slope angles along the x- and y-axes, respectively. The first term on the right-hand side of (2) and (3) represents the force linked to variations in avalanche thickness, the second term is the driving force due to gravity, and the third term is the Coulomb frictional force. The equations are solved by a staggered-grid finite-difference method with a grid step of 10 m. The numerical model uses a Godunov-type scheme, extended to second order by using the concept of Vanleer [*Alcrudo et al.*, 1993; *Mangeney*, 2000]. It is particularly well adapted to deal with significant discontinuities and has been recently used to model submarine landslides [*Assier et al.*, 2000; *Heinrich et al.*, 2001].

Geotechnical testing of the avalanche debris on Montserrat has suggested effective-stress friction angles for sand-textured debris of 25-35° (peak) and 17-25° (residual), with minor cohesive strength [*Voight et al.*, 2001]. The 'apparent' friction values associated with a flowing avalanche are much reduced from these values, due to high pore water pressures in disaggregated fumarolically-altered debris [*Voight and Elsworth*, 1997]. Hence, the test values are consistent with values inferred from the simulations, which are lower than 15°. In addition, some local seams of clay-rich materials have very low frictional strength (3-14°), but these materials are only locally observed and cannot account for bulk avalanche mobility.

The simple Coulomb friction law is based on a constant friction angle which implies a constant ratio of the shear stress to the normal stress at the base. However, it may be criticized, since laboratory experiments on granular flows show a dependence of this angle on the Froude number [*Savage and Hutter*, 1991; *Pouliquen*, 1999]. Similarly, Kilburn and Sorensen (1998) and Dade and Huppert (1998) suggest that deformation of the basal layer obeys a collisional regime, where the shear stress is proportional to the square of the velocity [*Bagnold*, 1954]. More precisely, Pouliquen (1999) has shown that the constant friction assumption fails for granular flows over rough bedrock for a range of inclination angles, for which steady uniform flows can be observed in laboratories. In this range, the frictional force is able to balance the gravity force, indicating a shear rate dependence. Pouliquen (1999) proposes an empirical friction coefficient $\mu=\tan\delta$ as a function of the Froude number and the thickness h of the granular layer

$$\mu(u,h) = \tan\delta_1 + (\tan\delta_2 - \tan\delta_1)\exp\left(-\frac{h}{D}\frac{\sqrt{gh}}{u}\right) \qquad (4)$$

where δ_1, δ_2 and D are characteristics of the material that can be measured from deposits, D being of the order of ten times the mean diameter of particles. Equation (4) provides a friction angle, ranging between two values δ_1 and δ_2. The higher the velocity, the higher is the friction angle. Small thicknesses, corresponding to high shear rates, are slowed down by high friction coefficients in equation (4). Inversely, large thicknesses are subject to smaller friction angles, which helps to account for the large observed run-out of large volume avalanches. Contrary to the simple Coulomb friction law, the particles diameter is required in (4). For complex geophysical problems, this diameter has to be considered as an empirical parameter, which is one way of making the rheology more flexible. What this empirical law means in terms of microscopic forces, is still an open question.

Three-dimensional simulation of the Boxing Day debris avalanche

Description of the avalanche emplacement

Survey of the debris avalanche deposit emplaced within the White River [*Sparks et al.*, 2001] indicates a volume of about $40\text{-}50\times10^6$ m^3. A large spoon-shaped scar formed as a result of the failure. The debris avalanche deposit itself was largely confined to the White River valley, being emplaced at its far end onto a new fan of pyroclastic flow deposits which had extended the coastline by as much as 500 m. However, some of the debris avalanche spilled out of the White River valley onto the slopes above the village of Morris'. The debris avalanche surface displays typical hummocky topography. Avalanche emplacement was immediately followed by a high-energy pyroclastic density current which scoured the surface, smoothed the hummocks, and locally infilled topographic lows on the debris avalanche. The debris avalanche deposit is composed largely of a wide variety of volcanic breccias with lithologic domains typically several meters in size. The

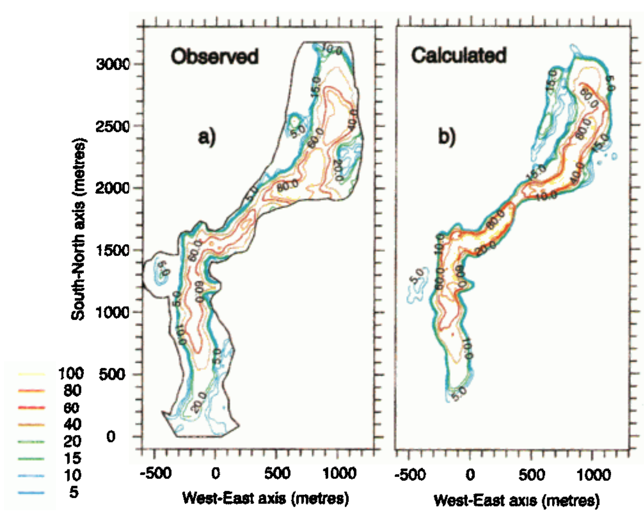

Figure 2. (a) Map of deposit thickness of the debris avalanche in the White River valley, estimated from pre- and post-avalanche topographic surveys (b) Deposit thickness calculated by Pouliquen's law, $\delta_1=11°$, $\delta_2=25°$, and D=15m.

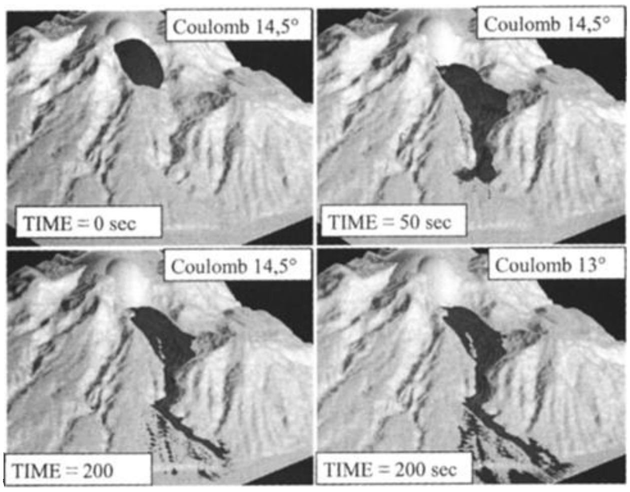

Figure 3. (a) Initialization of the debris avalanche by a parabolic shape volume of 50×10^6 of m^3. (b-c) Flow evolution for a simple-Coulomb law with a friction angle δ= 14,5°, 50 and 200 seconds after the release time. (d) Flow height at t=200s for a smaller friction angle δ= 13°.

material is heterogeneous and varies from clay particles to megablocks, a few 100 m wide.

The thickness data have been contoured and are displayed in Fig. 2a. In the proximal region, the avalanche was confined in the valley, and the maximum deposit thicknesses range from 60 to 100 m. A part of the avalanche material surmounted the west side of the bend, above Morris', and was deposited further down on a sloping flat area 300 m long by 200 m wide, the thickness ranging from 5 to about 20 m. In the main valley, frontal snout heights of about 20 m were observed at a distance of 500 m from the shoreline.

The White River valley is oriented to the southwest for 2 km then bends to the south at about 1.5 km from the shoreline. The altitude decreases from about 900 m near the scarp of the landslide to 150 m at the bend with slope angles varying from 35° to 20°. Beyond this bend, the altitude decreases slowly with a slope inclination varying from 15° at the bend to 5° at the shore. This flat topography in the distal zone led to rapid loss of kinetic energy and controlled the avalanche run-out.

Simulations using simple-Coulomb friction laws

The landslide is initialized in the simulation by a parabolic-shaped volume of 50×10^6 of m^3, released from rest (Fig. 3a). A first series of numerical simulations has been performed using the simple Coulomb friction law for different values of the basal friction angle. Preliminary tests show that the best agreement is obtained for $13° < \delta < 15°$, which, surprisingly, corresponds approximately to the average angle of the topography, calculated from the observed runout length (about 3500 m) and the assumed elevation difference (about 850 m) between the landslide top and toe. This result suggests that most of the initial potential energy is consumed by basal friction [*Hutter*, 1996] and that local topography variations do not significantly influence the total runout length. The low values of δ are an example of the widely observed ability of large avalanches to travel distances much larger than expected from classical models of slope failure [e.g. *Kilburn and Sorensen*, 1998 ; *Dade and Huppert*, 1998].

Fig. 3b-c shows the flow evolution for a friction angle δ=14,5° , 50 and 200 seconds after the release time. At 200 seconds, the flow is at rest and flow heights may be considered as deposit thicknesses. Fig. 3b at t=50 seconds shows that the flow surmounts the bend on the west side with maximum velocities of about 40m/s and maximum thickness of 5 m during overflowing. The deposit area southwest of the bend is dispersed with maximum heights of 2 to 3 m. Most of the overflowing material does not stop and reaches the sea, flowing down small gulleys parallel to the White River valley. The model avalanche in the main valley does not reach the sea and the maximum deposit heights range only from 1 to 5 m, smaller than the observed ones, at distances from the coast 300 m to 600 m. The same phenomena are observed and accentuated for smaller values of δ, as shown in Fig. 3d for δ= 13°. In this latter case, the avalanche reaches the sea with heights of about 1 m. Deposit heights between 300 and 600 m from the sea range from 5 to 15 m. Deposit areas located west of the main valley do not correspond to the observations.

Simulations using Pouliquen's friction law

A second series of simulations, using Pouliquen's friction law, show that the front evolution and its stopping is governed essentially by δ_1 and that the observed phenomena are approximately reproduced for δ_1 around 10°, δ_2 around 20° and mean particle diameters of the order of 1 m (Fig. 4). By trial and error, the angles δ_1 and δ_2 have been chosen respectively at 11° and 25°, D=15 m in (4). No direct estimates of the velocity are available; however, numerical values are comparable to the local estimate of 35 $m.s^{-1}$ based on height climbed data and a minimum average velocity of 27 $m.s^{-1}$ based on seismic data [*Voight et al.*, 2001].

Calculated deposit heights (Fig. 2b) are in agreement with the observed data (Fig. 2a) up to a distance of 500 m from the sea. The maximum deposit heights southwest of the bend (above Morris's village) are about 10 m, which is close to the observed values. Deposit heights range from 7 to 40 m for distances of 300 to 600 m from the sea, which gives a much more pronounced front than those calculated by simple-Coulomb friction laws. The simulated avalanche stops 200 m before the shoreline, showing that the basal friction angle is probably smaller on this flat area.

Comparison between simple-Coulomb and Pouliquen's friction laws

Compared with the simple-Coulomb law, Pouliquen's law yields (1) greater confinement within the valley, (2) larger deposit thicknesses in the center of the valley before and after the bend, (3) deposit areas closer to the observed ones in the

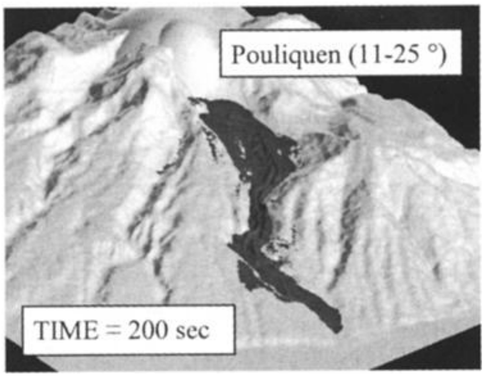

Figure 4. Flow height for Pouliquen's friction law with the same parameters as those defined in Fig. 2b.

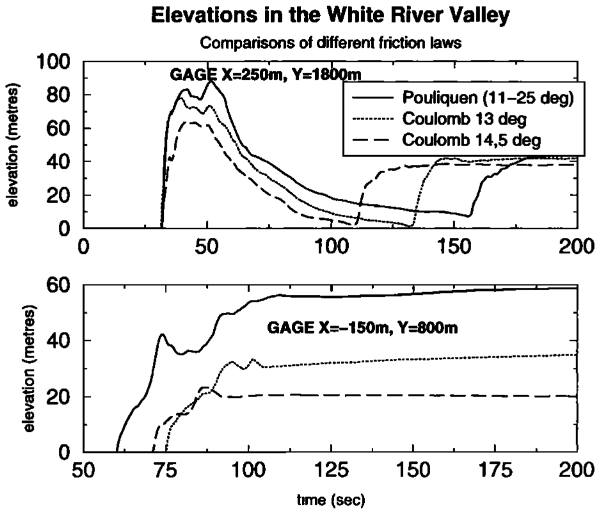

Figure 5. Comparisons between flow heights calculated by simple-Coulomb and Pouliquen's friction laws at two locations, 300 m upstream of the bend (y=1800m) and 700 m downstream of the bend (y=800m). The friction angles are δ= 13° and δ=14,5° for Coulomb laws ; $11<\delta<25°$ for Pouliquen's law.

distal region. Fig. 5 shows comparisons of time series between flow elevations calculated using simple-Coulomb and Pouliquen's laws. Comparisons on the first gage, located upstream 300 m of the bend, show a similar behavior for the flow front between simple-Coulomb laws (δ=13° and δ=14,5°) and Pouliquen's law ($11<\delta<25°$); in the first part of the valley, Pouliquen's law corresponds to a simple-Coulomb law with a basal friction of 11°. Further down, the second gage located 700 m downstream of the bend, shows large height differences between Pouliquen's law and simple-Coulomb laws. The main reason is that a larger volume flows through the bend in the Pouliquen's simulation due to the valley confinement, which itself is the result of the slowing down of small flow thicknesses by Pouliquen's law. Downstream of y=800m, the flow height progressively decreases; δ_2 is then activated in Pouliquen's law, slowing down the flow as it approaches the sea. At a distance of 300 m from the sea, arrival times differ from 30 seconds approximately.

Conclusion

The Boxing Day debris avalanche has been simulated by considering the avalanche as the flow of a homogeneous incompressible continuum subjected to gravity and Coulomb-type friction laws. Comparisons with observed deposits of suggest a dependence of the friction angle on the Froude number and the flow height. Numerical results illustrate the potential of such an empirical approach to estimate the capacity of a debris avalanche to surmount relief, thus affecting a larger area away from the confined channel and valley walls. Such modeling could be extremely useful to assess the hazard linked to these large volume debris avalanches in future volcanic crises.

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