Comment on “Diffusion of epicenters of earthquake aftershocks, Omori’s law, and generalized continuous-time random walk models”

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PHYSICAL REVIEW E **69**, 063101 (2004)

Comment on "Diffusion of epicenters of earthquake aftershocks, Omori's law, and generalized continuous-time random walk models"

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(Received 18 April 2003; revised manuscript received 3 October 2003; published 10 June 2004)

Modeling of earthquake sequences using an epidemic-type aftershock sequence model by Helmstetter and Sornette [Phys. Rev. E **66**, 061104 (2002)] has led these authors to conclude that previous analyses of apparent earthquake diffusions were flawed. We show here that diffusion analyses based on spatiotemporal correlation measures for earthquake populations are an appropriate method for capturing the space-time coupling present in earthquake triggering processes.

DOI: 10.1103/PhysRevE.69.063101 PACS number(s): 05.40.Fb, 91.30.Px

Helmstetter and Sornette [1] (HS) use a space-time point process [epidemic-type aftershock sequence (ETAS)] model to show that the diffusion of aftershocks can be explained in terms of a cascade of earthquakes generating their own aftershocks, which in turn produce their aftershocks and so on. Their model is a particularly interesting tool, well adapted to investigate such a cascading phenomenon, which is expected to either generate diffusion by itself (as shown in HS) or to modulate potential direct diffusion physically originating from a given source. The latter case would correspond, in the framework of a space-time ETAS model, to coupled space-time "bare" kernels.

However, some of the assumptions made in HS are questionable, when it comes to applying their method to real seismicity data. In the real world, no one can arbitrarily "reset the clock" (p. 7 of Ref. [1]), as needed in their treatment; while in HS, all events occurring at $t>0$ are due to the mainshock that took place at $t=0$ (in the sense that, without this event, none of these subsequent events would have existed), this cannot generally be said for real earthquakes. In particular, when observing at long-time scales, the decision of whether a given earthquake has been triggered (or depends on) a previous one cannot be made unambiguously.

A possible solution to this problem was proposed by Marsan and co-workers [2–4] (MC), which consists in recalling that statistical dependence only makes sense when considering distributions. Denoting by ρ and τ the two random variables giving the epicentral/hypocentral distances and time separation between any two earthquakes, it can be shown that, at large τ, the distance ρ tends to a stationary, "background" distribution. Such a distribution corresponds to pairs of earthquakes that are uncorrelated, and only reveals the permanent, quasistationary (at the time scales of years) network of active faults. MC studied how the distribution of ρ relaxes to this background distribution, as τ increases. A mean distance $R(t)$ was defined such that

$$E\{\delta(t-\tau)\}R(t) = E\{\rho\delta(t-\tau)\} - E\{\rho\}E\{\delta(t-\tau)\}, \quad (1)$$

where $E\{\cdot\}$ is the expectation and δ the Dirac generalized function. Since ρ becomes independent of τ at large τ, $R(t)$ tends to 0 at large t. For a finite number of earthquake pairs, as with a real catalog, $R(t)\to 0$ rather takes the form of random fluctuations of R around 0. Prior to this random walk [4] (see also Ref. [5] for a measure of when this transition occurs), R is seen to grow with t following a power law, revealing the existence of a subdiffusive process.

While HS focus on *aftershock* diffusion (hence originating from a well-identified mainshock), MC analyze how an earthquake population is statistically correlated, hence with no assumption on the mainshock/aftershock nature of these earthquakes. The latter analyze probe, in the mean-field sense, on how two fault segments can have synchronous earthquake productions, and how this correlation changes with the distance and the time scale.

Since the two methods have their own assumptions and examine different types of relation (mainshock aftershock for HS, any two earthquakes for MC), it is difficult to link their respective observations. In particular, a temporal decorrelation of the seismicity field with a rate depending on the epicentral distance, as seen by MC, does not necessarily imply a direct diffusion of aftershocks.

More technically, the rate $N(r,t)$ of HS ("number of events of any possible magnitude at r and t," the origin $r=0$ and $t=0$ being the mainshock that initiated the sequence) bears some analogy with the $N(r,t)-\overline{N}(r)$ of MC [Eq. (1) and (2) of Ref. [3]], except that the latter is computed over all earthquakes taken as "mainshocks." In MC, the subtraction of $\overline{N}(r)$ from $N(r,t)$ is done to ensure that the pairs are temporally correlated in the statistical sense, i.e., in terms of distributions rather than individual pairs. This quantity is then normalized to yield the probability $G(r,t)$ that, knowing an earthquake that occurs at t after an initial earthquake and is temporally correlated with it (in the sense of distributions), it occurs at an epicentral distance r:

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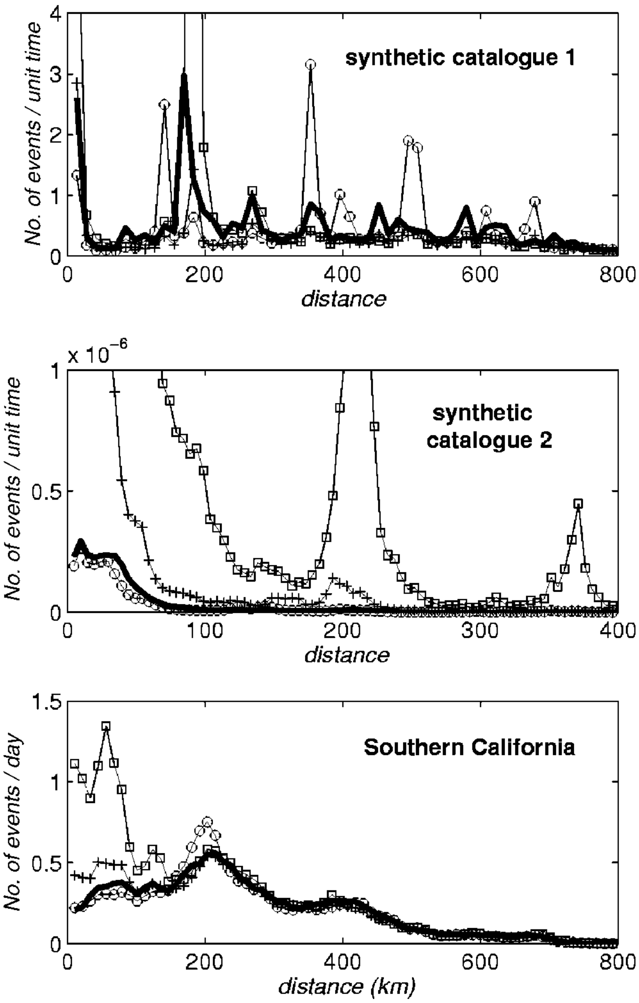

FIG. 1. Average number of earthquakes per unit time vs distance r following any previous earthquake with a varying delay for the two synthetic catalogs of HS (top and center) and for the Southern California data analyzed in Ref. [3] (bottom). The delays t separating the two earthquakes are (top) (□) 12.5–25, (+) 25–50, (◦) 50–100 unit times, (center) (□) 7.9×10^5–7.7×10^6, (+) 7.7×10^6–7.4×10^7, (◦) 7.4×10^7–7.2×10^8 unit times, (bottom) (□) 51–125, (+) 125–305, (◦) 305–743 days. These intervals are chosen so that the last interval stops at a tenth of the total duration of the catalog. The spatial distribution of the temporally uncorrelated pairs is shown by the thick line. The distributions $E\{\delta(r-\rho)\delta(t-\tau)\}$ are seen to relax to this temporally uncorrelated ("background") distribution $E\{\delta(r-\rho)\}E\{\delta(t-\tau)\}$, in the case of the second synthetic catalog and for the Southern California data, but not for the first synthetic catalog.

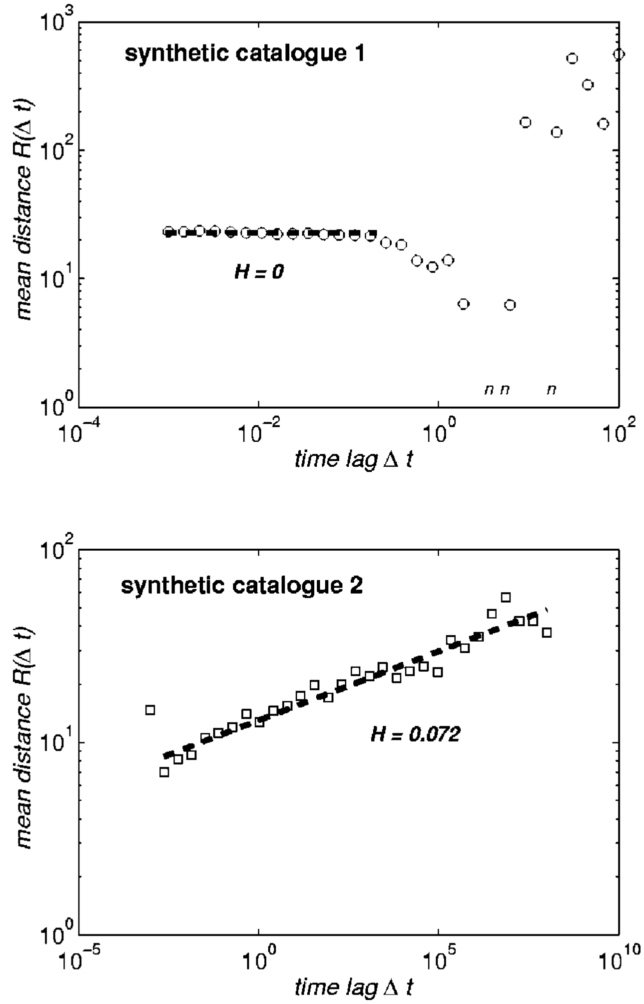

FIG. 2. Mean distance $R(t)$ function of time lag t for the two synthetic catalogs of HS, as obtained with the analysis of Ref. [3,4]. The Green's function G becomes very noisy in the case of the first catalog (for $t\simeq1$), leading to negative values of $R(t)$ (shown with the "n" letters); see the Appendix of Ref. [4] for a discussion on this issue. For shorter time scales, no diffusion ($H=0$) is observed; for longer time scales, the analysis is not robust given the noise in G. The second synthetic catalog is characterized by a diffusion exponent $H=0.072$ (after discarding the first point).

$$E\{\delta(t-\tau)\}G(r,t) = E\{\delta(r-\rho)\delta(t-\tau)\} - E\{\delta(r-\rho)\}E\{\delta(t-\tau)\}. \qquad (2)$$

Taking the integral $\int dr\, r\, G(r,t)$ of this expression leads to Eq. (1). $G(r,t)$ is also equivalent to a propagator, or Green's function. The function $N(r,t)=E\{\delta(r-\rho)\delta(t-\tau)\}$ of MC is the two-point moment function of Kagan and Knopoff [6] and Reasenberg [7], while $N(r,t)-\overline{N}(r)=E\{\delta(r-\rho)\delta(t-\tau)\}-E\{\delta(r-\rho)\}E\{\delta(t-\tau)\}$ is the associated covariance.

HS used the method described above to conduct some tests and concluded that this method is flawed (see also Ref. [8] for a more recent, longer, although still partly misleading, discussion of the method). However, the results of their tests can be shown to be misleading (Fig. 1). These tests were conducted with a flawed software on two synthetic catalogs. Reanalyzing these catalogs with the method of MC, we find, for the first catalog, that there is no effective relaxation of the distribution of ρ to the "background" distribution $E\{\delta(r-\rho)\}E\{\delta(t-\tau)\}$, and this type of analysis is therefore not appropriate. Neglecting this issue, HS went on to report an $H=0.5$ exponent (i.e., a normal diffusion) as resulting from this analysis, even though their catalog was generated from a model with no space-time coupling. It can, however, be shown (see Fig. 2) that the analysis of MC does indeed lead to an $H=0$ value for this catalog, although, as stated above, the lack of proper relaxation to the background structure renders this type of analysis not very robust.

The second synthetic catalog does experience the same type of relaxation as observed for the real data analyzed in MC (e.g., the Southern California data as shown in Fig. 1; see also Fig. 5 of Ref. [3] for a similar graph in the case of a mining-induced seismicity catalog, and Fig. 5 of Ref. [4] in the case of shallow world-wide seismicity). The diffusion exponent for this second synthetic catalog is found to be $H \simeq 0.072$ (Fig. 2), a value that, again, as eventually commented in HS, cannot be compared to the expected theoretical value $H=0.2$ of the ETAS model since these exponents do not measure the same phenomenon.

Finally, and quite contrary to the belief expressed in HS, observation of seismicity diffusion for the space-time ETAS model of HS (1) does imply anomalous stress diffusion, since the stress generated by the subsequent earthquakes diffuses with these earthquakes, and (2) does not save us from investigating what is the physics at work in this process. The "physical ingredients" of the ETAS model still need to be explained in terms of actual physical phenomena. Such ingredients are merely *ad hoc* kernels introduced in order to reproduce basic features of earthquake populations (e.g., Gutenberg-Richter and Omori's laws). More particularly, HS introduce an arbitrary law with algebraic decay for drawing the distance between the trigger and the aftershock, which would need to be substantiated by observations and/or actual crustal processes.

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