On extended symmetries for the Galileon

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We investigate a large class of infinitesimal, but fully nonlinear in the field, transformations of the Galileon and search for extended symmetries. The transformations involve powers of the coordinates x and the field π up to any finite order N. Up to quadratic order the structure of these symmetry transformations is the unique generalization of both the infinitesimal version of the standard Galileon shift symmetry as well as a recently discovered infinitesimal extension of this symmetry. The only higher-order extensions of this symmetry we recover are ("Galileon dual" versions of) symmetries of the standard kinetic term.

I. INTRODUCTION

Galileon field theories are a class of nonlinear field theories with derivative self-interactions. As their name suggests they satisfy an internal shift symmetry $\pi \to \pi + C + B_a x^a$ for any constant C and any constant vector B_a. They can be defined as the maximal class of theories with this shift symmetry which still obeys second-order field equations. Ever since their recent rediscovery [1] they have attracted much attention and these theories have a lot of internal structure yet to be fully explored. Covariant [2] and multifield generalizations [3] have also been considered but in this work we restrict ourselves to single-field Galileon theories in flat space-time.

The main reason for the interest in these theories is that, despite their nonlinear derivative dependence, they still satisfy second-order field equations, thereby avoiding the Ostrogradsky ghost, which generically plagues higher-derivative theories (see also Ref. [4]). They are therefore expected to arise in the *effective field theory* (EFT) description of many physically interesting situations (see e.g. Ref. [5] for recent reviews). In particular, they frequently arise in scaling limits of various modified gravity theories exhibiting Vainshtein screening, e.g. in the decoupling limit of brane-world models *à la* Dvali-Gabadadze-Porrati, [6] or in the decoupling limit of nonlinear massive gravity and its generalizations [7]. For such theories the nonrenormalization theorem of Ref. [8] then ensures that Vainshtein screening can be realized in a controlled fashion within the regime of validity of the EFT, unlike in theories with arbitrary (non-Galileon) irrelevant operators. More generally, Galileons are known to arise in the EFT limit of fluctuating surfaces [9]. All taken in conjunction, they have a potential interest in many concrete applications all across physics.

In this work we do a systematic search for the existence of extended infinitesimal (but fully nonlinear in the coordinates and field) symmetries of Galileon theories. We

find that, up to quadratic order, the symmetry is uniquely fixed to be the standard Galileon shift symmetry plus coordinate/Lorentz transformations together with the quadratic extension recently discovered in Ref. [10]. The only higher-order extensions of this symmetry we find are ("Galileon dual" [11] versions of) symmetries of the standard kinetic term [12].

Conventions: We work in D space-time dimensions and frequently employ Einstein summation. We use the notation $\pi_a \equiv \partial_a \pi$, $\pi_{ab} \equiv \partial_a \partial_b \pi$ for derivatives of the field π.

II. GALILEON FIELD THEORIES

The Galileon field theories are defined via an action constructed out of the following Lagrangian:

$$\mathcal{L} = \sum_{n=1}^{D} \frac{c_n}{(n+1)} \pi \pi_{a_1}^{[a_1} \cdots \pi_{a_n}^{a_n]}. \tag{1}$$

Here and in what follows we have omitted a possible inclusion of a tadpole contribution ($n = 0$), which would only change the background solution (i.e. $c_0 = 0$ here). The c_n are constant parameters of the theory and the $\pi_{a_1}^{[a_1} \cdots \pi_{a_n}^{a_n]}$ are the completely antisymmetric products of $\pi_b^a \equiv \partial^a \partial_b \pi$, normalized with unit weight. More explicitly we have

$$n = 1: \ \pi_a^a,$$
$$n = 2: \ \frac{1}{2}(\pi_a^a \pi_b^b - \pi_b^a \pi_a^b),$$
$$n = 3: \ \frac{1}{6}(\pi_a^a \pi_b^b \pi_c^c - 3\pi_b^a \pi_a^b \pi_c^c + 2\pi_b^a \pi_c^b \pi_a^c),$$
$$\vdots$$
$$n = D: \ \det(\pi_b^a). \tag{2}$$

These are, up to normalization, the unique total derivatives which can be formed out of π_b^a at each order in π. Due to their antisymmetric structure, $\pi_{a_1}^{[a_1} \cdots \pi_{a_n}^{a_n]} = 0$ for any

$n > D$. Note that the $n = 1$ term in Eq. (1) is the standard kinetic term and, with a mostly plus convention for the metric, the value $c_1 = 1$ canonically normalizes this term in the action.

From Eq. (1) the Galileon equations of motion are

$$\mathcal{E} \equiv \sum_{n=1}^{D} c_n \pi_{a_1}^{[a_1} \cdots \pi_{a_n}^{a_n]} = 0 \qquad (3)$$

and we notice that under an infinitesimal transformation, $\pi \to \pi + \epsilon \delta \pi$, at linear order in ϵ (i.e. infinitesimally), the Lagrangian (1) shifts by

$$\Delta \mathcal{L} = \sum_{n=1}^{D} c_n \delta\pi_{a_1}^{[a_1} \cdots \pi_{a_n}^{a_n]}. \qquad (4)$$

III. SYMMETRIES AT QUADRATIC ORDER

Do any Galileon theories exist that are invariant (up to total derivatives) under extensions of the standard Galileon shift symmetry $\pi \to \pi + C + B_a x^a$? The most general transformation for π that is a function of π itself and coordinates x^a (up to second order in x^a and π combined and up to first derivatives acting on π; we will refer to this as *quadratic order*) can be written as

$$\delta\pi = d_{(0,0)} + d_{(1,0)} b_a^{(1)} x^a + d_{(0,1)}^{(2)} \pi + d_{(0,1)} b_a^{(2)} \pi^a$$
$$+ d_{(0,2)}^{(1)} \pi^2 + d_{(0,2)}^{(2)} \pi b_a^{(3)} \pi^a + d_{(1,1)}^{(2)} r_{ab} x^a \pi^b$$
$$+ d_{(2,0)} s_{ab} x^a x^b + d_{(1,1)} p_{ab} x^a \pi^b + d_{(0,2)} q_{ab} \pi^a \pi^b. \qquad (5)$$

Bracketed indices are labels and all scalar, vector and matrix coefficients $d_{(r,m)}, b_a, s_{ab}, \ldots$ are constant. s_{ab}, p_{ab}, q_{ab} are symmetric, whereas r_{ab} is antisymmetric. Plugging the *ansatz* (5) into the variation (4) we compute the contribution to the equations of motion,

$$\Delta\mathcal{E} = \frac{\partial \Delta \mathcal{L}}{\partial \pi} - \partial_a \frac{\partial \Delta \mathcal{L}}{\partial \pi_a} + \partial_a \partial_b \frac{\partial \Delta \mathcal{L}}{\partial \pi_{ab}}. \qquad (6)$$

Forcing this to vanish provides conditions relating the parameters c_n of the Lagrangian and the parameters $d_{(r,m)}$ of the *ansatz* (5) and allows us to efficiently find any symmetries. Doing so (for details see Ref. [13]), the most general infinitesimal symmetry transformation up to this order is

$$\delta\pi = d_{(0,0)} + d_{(1,0)} b_a x^a$$
$$+ s_{ab}(d_{(2,0)} x^a x^b + d_{(1,1)} x^a \pi^b + d_{(0,2)} \pi^a \pi^b)$$
$$+ d_{(0,1)} b_a^{(2)} \pi^a + d_{(1,1)}^{(2)} r_{ab} x^a \pi^b, \qquad (7)$$

where s_{ab} is symmetric *and* traceless. All coefficients are free, except $d_{(2,0)}, d_{(1,1)}, d_{(0,2)}$, which have to satisfy

$$d_{(0,2)} = \frac{c_2^2 - c_1 c_3}{c_1^2} d_{(2,0)}, \qquad d_{(1,1)} = \frac{c_2}{c_1} d_{(2,0)} \qquad (8)$$

and, if any of $d_{(2,0)}, d_{(1,1)}, d_{(0,2)}$ are nonzero, we have to restrict to Galileon Lagrangians satisfying

$$c_4 = \frac{2c_2 c_3}{c_1} - \frac{c_2^3}{c_1^2}. \qquad (9)$$

The first line in Eq. (7) is precisely the standard Galilean shift symmetry, the second line is the nonlinear extension recently found in Ref. [10] and the third line is the unique completion of these other symmetries at quadratic order, which simply consists of a coordinate shift and a Lorentz transformation respectively. In Ref. [13] we extend this argument to higher orders and also consider terms with higher derivatives acting on π.

IV. HIGHER-ORDER SYMMETRIES

Can this quadratic order symmetry be generalized to higher orders? Extensions involving partially antisymmetric coefficient tensors are somewhat complicated and will be discussed in Ref. [13], but here we conjecture that the general higher-order generalization of the symmetric (s_{ab}-dependent) piece of Eq. (7) is

$$\delta\pi = \sum_{(r,m)_p}^{N} d_{(r,m)} Q_{a_1 \cdots a_r b_1 \cdots b_m} x^{a_1} \cdots x^{a_r} \pi^{b_1} \cdots \pi^{b_m}, \qquad (10)$$

i.e. a power series in the components of x^a and π^a, where the Q's are totally symmetric and traceless constant coefficient tensors. The sum in Eq. (10) runs over all the ordered partitions (r, m) of integers $p = r + m$ (including 0) up to some N and we define the Q's appropriately whenever 0 is part of the partition. In general, for each p there are $p + 1$ such partitions. This means that the *ansatz* contains at most $\sum_{p=0}^{N}(p+1) = \frac{1}{2}N(N+3) + 1$ arbitrary parameters $d_{(r,m)}$. In order to avoid confusion we stress that we label these partitions according to,

$$(r, m) = (\# \text{ of } x^a, \# \text{ of } \pi^a). \qquad (11)$$

For example, considering $N = 2$ we would sum over the values $p = 0, 1, 2$ with (r, m) taking values in the sets $\{(0,0)\}, \{(1,0),(0,1)\}$ and $\{(2,0),(1,1),(0,2)\}$ respectively. Also note that the pattern of generalization (10) is simply to add, at each order in x^a and π^a combined, all possible terms with symmetric and traceless constant coefficient tensors which, at each order, differ at most by an overall constant. We stress that this *ansatz* is motivated by the above explicit calculation of general symmetry transformations at the lowest orders.

We now wish to evaluate Eq. (6) for the conjectured higher-order symmetry (10). For this it is convenient to define the *traceless symmetric* matrices,

$$^{(r,m)}\mathbb{Q}^c{}_d \equiv x^{a_1} \cdots x^{a_r} \pi^{b_1} \cdots \pi^{b_m} Q_{a_1 \dots a_r b_1 \dots b_m}{}^c{}_d. \quad (12)$$

These matrices satisfy the following useful identities (which hold for any traceless matrix $\mathbb{Q}$):

$$n\mathbb{Q}_{a_1 b}\pi^{b[a_1}\pi_{a_2}^{a_2} \cdots \pi_{a_n}^{a_n]} = -(n+1)\mathbb{Q}_{a_1}^{[a_1}\pi_{a_2}^{a_2} \cdots \pi_{a_{n+1}}^{a_{n+1}]}, \quad (13)$$

and,

$$\begin{aligned}
\mathbb{Q}_{bc}\pi^{bc}\pi_{a_1}^{[a_1} \cdots \pi_{a_n}^{a_n]} &= n\mathbb{Q}_{a_1 b}\pi^{bc}\pi_c^{[a_1}\pi_{a_2}^{a_2} \cdots \pi_{a_n}^{a_n]} \\
&\quad - (n+2)\mathbb{Q}_{a_1}^{[a_1}\pi_{a_2}^{a_2} \cdots \pi_{a_{n+2}}^{a_{n+2}]}. \quad (14)
\end{aligned}$$

These identities can be used to convert all terms appearing in the evaluation of Eq. (6) into functions of the following form (now there is no nonderivative π dependence):

$$\mathcal{I}_{(n,r,m)} \equiv n^{(r,m)}\mathbb{Q}_{a_1}^{[a_1}\pi_{a_2}^{a_2} \cdots \pi_{a_n}^{a_n]}. \quad (15)$$

Due to the tracelessness of $\mathbb{Q}$ and the antisymmetric structure we have the very important properties that, independently of the values (r, m),

$$\mathcal{I}_{(1,r,m)} = 0, \qquad \mathcal{I}_{(n,r,m)} = 0 \quad \forall \, n > D. \quad (16)$$

In terms of these functions we find that,

$$\begin{aligned}
\Delta\mathcal{E} = \sum_{n=1}^{D} c_n \sum_{(r,m)_p} d_{(r,m)}[m(m-1)\mathcal{I}_{(n+2,r,m-2)} \\
+ r(r-1)\mathcal{I}_{(n,r-2,m)} - 2rm\mathcal{I}_{(n+1,r-1,m-1)}]. \quad (17)
\end{aligned}$$

It is now straightforward, for any given N, to find the conditions, which need to be satisfied in order to achieve $\Delta\mathcal{E} = 0$. Since all the $\mathcal{I}_{(n,r,m)}$ are independent, we simply collect the coefficients of each one of them and demand that they all vanish. This implies the recurrence relation

$$\begin{aligned}
c_n d_{(r,m+2)}(m+1)(m+2) + c_{n+2}d_{(r+2,m)}(r+1)(r+2) \\
- 2c_{n+1}d_{(r+1,m+1)}(r+1)(m+1) = 0. \quad (18)
\end{aligned}$$

This can be solved for any (r, m) but its form is not very illuminating. Before providing an explicit example, some very general remarks can be made by inspection of Eq. (17) and observing the properties (16).

(i) Any terms with $r = 0$ [i.e. a string of π^a in Eq. (10)] leave the c_D and c_{D-1} terms invariant. Similarly, any terms with $r = 1$ [i.e. one x^a together with a string of π^a in Eq. (10)] leave the c_D term invariant.

(ii) Considering the minimum (maximum) value of $2 \leq n \leq D$, the coefficients of $\mathcal{I}_{(n_{\min}, r-2, m)}$ and $\mathcal{I}_{(n_{\max}, r, m-2)}$ in Eq. (17) have to vanish separately. This implies that the presence of a nonzero $d_{(r,m)}$ with $r \geq 2$ $(m \geq 2)$ requires $c_1 \neq 0$ $(c_D \neq 0$ or

$c_{D-1} \neq 0$). Since c_1 parametrizes the standard kinetic term, this is also necessary to avoid infinitely strongly coupled solutions.

V. A CUBIC SYMMETRY IN $D = 4$

In order to elucidate these points we present the conditions that arise for the $p = 2$ and $p = 3$ terms in four dimensions, i.e. considering $N = 3$. The ordered partitions fall into the sets $\{(2,0),(1,1),(0,2)\}$ and $\{(3,0),(2,1),(1,2),(0,3)\}$. Computing $\Delta\mathcal{E}$ we find the following set of nontrivial equations:

$$(c_1 d_{(1,1)} - c_2 d_{(2,0)})\mathcal{I}_{(2,0,0)} = 0,$$
$$(c_1 d_{(0,2)} - c_2 d_{(1,1)} + c_3 d_{(2,0)})\mathcal{I}_{(3,0,0)} = 0,$$
$$(c_2 d_{(0,2)} - c_3 d_{(1,1)} + c_4 d_{(2,0)})\mathcal{I}_{(4,0,0)} = 0,$$
$$(2c_1 d_{(1,2)} - c_2 d_{(2,1)})\mathcal{I}_{(2,0,1)} = 0,$$
$$(2c_1 d_{(2,1)} - 3c_2 d_{(3,0)})\mathcal{I}_{(2,1,0)} = 0,$$
$$(3c_1 d_{(0,3)} - 2c_2 d_{(1,2)} + c_3 d_{(2,1)})\mathcal{I}_{(3,0,1)} = 0,$$
$$(c_1 d_{(1,2)} - 2c_2 d_{(2,1)} + 3c_3 d_{(3,0)})\mathcal{I}_{(3,1,0)} = 0,$$
$$(3c_2 d_{(0,3)} - 2c_3 d_{(1,2)} + c_4 d_{(2,1)})\mathcal{I}_{(4,0,1)} = 0,$$
$$(c_2 d_{(1,2)} - 2c_3 d_{(2,1)} + 3c_4 d_{(3,0)})\mathcal{I}_{(4,1,0)} = 0. \quad (19)$$

It is straightforward to see that the coefficients solving these equations obey the general recurrence relation (18).

A solution to all of the above nine equations (the unique solution for nonzero parameters) is given by,

$$c_3 = \frac{3c_2^2}{4c_1}, \qquad c_4 = \frac{c_2^3}{2c_1^2}, \qquad d_{(1,1)} = \frac{c_2 d_{(2,0)}}{c_1},$$

$$d_{(0,2)} = \frac{c_2^2 d_{(2,0)}}{4c_1^2}, \qquad d_{(2,1)} = \frac{3c_2 d_{(3,0)}}{2c_1},$$

$$d_{(1,2)} = \frac{3c_2^2 d_{(3,0)}}{4c_1^2}, \qquad d_{(0,3)} = \frac{c_2^3 d_{(3,0)}}{8c_1^3}. \quad (20)$$

Note that this leaves two Lagrangian parameters, e.g. c_1 and c_2, as well as two parameters, e.g. $d_{(2,0)}$ and $d_{(3,0)}$, of the transformation undetermined. Furthermore, only the ratio c_2/c_1 appears in the $d_{(r,m)}$.

We define $\alpha \equiv c_2/2c_1$ and set $d_{(2,0)} = d_{(3,0)} = 1$ by absorbing them into the definition of the corresponding Q's. Ignoring any contributions from (partially) antisymmetric coefficient tensors (like r_{ab}), the Galileon theory specified by the above values for the c_n has a symmetry up to cubic order in fields and coordinates given by,

$$\begin{aligned}
\delta\pi = C + B_a x^a + Q_a \pi^a + Q_{ab} x^a x^b + 2\alpha Q_{ab} x^a \pi^b \\
+ \alpha^2 Q_{ab} \pi^a \pi^b + Q_{abc} x^a x^b x^c + 3\alpha Q_{abc} x^a x^b \pi^c \\
+ 3\alpha^2 Q_{abc} x^a \pi^b \pi^c + \alpha^3 Q_{abc} \pi^a \pi^b \pi^c. \quad (21)
\end{aligned}$$

This result generalizes Eq. (7) and earlier results by going one order further, but is in some sense trivial. To see this more clearly we discuss the general version of this symmetry.

VI. THE GENERAL FORM OF THE SYMMETRY

An attentive reader may have recognised the binomial coefficients appearing in Eq. (21). This is no accident and, for arbitrary D, the higher-order generalization of Eq. (21) can be written

$$\delta \pi = C + B_a x^a + Q_a \pi^a + \sum_{m=2}^{N} Q_{a_1 \dots a_m} \prod_{k=1}^{m} (x^{a_k} + \alpha \pi^{a_k}),$$

$$(22)$$

with the c_n obeying $c_n = K n \alpha^n$ and where K, C, B, Q are constants as well as $Q_{a_1 \dots a_m}$ being symmetric and traceless. To prove this, define $Y^a \equiv x^a + \alpha \pi^a$ and note that under $\pi \to \pi + Q_{a_1 \dots a_m} \prod_{k=1}^{m} Y^{a_k}$, for any $m \geq 2$, we have

$$\Delta \mathcal{E} = \sum_{n=1}^{D} c_n [\alpha^2 \mathcal{J}_m(n+2) - 2\alpha \mathcal{J}_m(n+1) + \mathcal{J}_m(n)],$$

$$(23)$$

where we have defined,

$$\mathcal{J}_m(n) \equiv n m (m-1) Q_{b_1 \dots b_{m-2} a_1}^{[a_1} \pi_{a_2}^{a_2} \dots \pi_{a_n}^{a_n]} \prod_{k=1}^{m-2} Y^{b_k}. \quad (24)$$

The $\mathcal{J}_m(n)$ vanish for $n = 1$ and $n > D$. We then find that $\Delta \mathcal{E} = 0$ provided that (with $n \geq 0$ and $c_0 = 0$),

$$c_n \alpha^2 - 2\alpha c_{n+1} + c_{n+2} = 0 \Rightarrow c_n \propto n \alpha^n. \quad (25)$$

This is exactly satisfied by the above values for the c_n and the constant of proportionality can be fixed by normalizing c_1. This shows that Eq. (22) is indeed an infinitesimal symmetry for arbitrary D.

What is the nature of this symmetry? Due to the existence of Galileon "duality" transformations [11], there is a one-parameter ambiguity. Galileon theories which share this symmetry are therefore unique modulo "duality" transformations. This can be used to fix the value $c_2 = 0$, which transforms the Lagrangian with parameters constrained by Eq. (25) into a free (noninteracting) Lagrangian with only $c_1 \neq 0$. Furthermore the symmetry (22) in this

case transforms into a string of x^a, since the duality transformation is essentially a coordinate transformation $x^a \to x^a + \lambda \pi^a$ for a free parameter λ [14]. This confirms that Eq. (22) is indeed a symmetry of the standard kinetic term [12].

VII. CONCLUSIONS

We have performed a systematic search for extended infinitesimal symmetries of the Galileon. At quadratic order we found that the most general such symmetry is given by Eq. (7), which establishes the result found in Ref. [10] as the unique quadratic-order extension of the standard Galileon symmetries modulo coordinate and Lorentz transformations. Based on this we conjectured a general form for extensions up to any order N in coordinates and field. The parameters c_n of the Lagrangian and the parameters $d_{(r,m)}$ of any such symmetry must obey the recurrence relation (18). Although special cases may be found by studying this recurrence relation, the generic solution for nonzero parameters is uniquely given by Eq. (22). This symmetry is however rather trivial since the parameters of the Lagrangian are constrained such that we are dealing with a "dual" version of the free theory and the symmetry reduces to just a string of x^a, i.e. Eq. (22) is the dual version of solely coordinate-dependent symmetries of the kinetic term. Together with the results of Ref. [15], who found that only one particular Galileon theory has an enhanced soft limit [the quartic Galileon associated with the symmetry found in Ref. [10]—essentially our Eq. (7)], this suggests that no "nontrivial" extension of Eq. (7) to higher orders exists.

Note added.—The main conclusions of this work changed after private communication with K. Hinterbichler and A. Joyce, who pointed out the observation that the higher-order symmetry (22) was the "dual" version of the string of x^a symmetry of the free theory.

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