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On the formation of the Kepler-10 planetary system

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ABSTRACT

In this paper, we investigate the conditions required for the 3 and 17 M_{\oplus} solid planets in the Kepler-10 system to have formed through collisions and mergers within an initial population of embryos. By performing a large number of N -body simulations, we show that the total mass of the initial population had to be significantly larger than the masses of the two planets, and that the two planets must have built-up farther away than their present location, at a distance of at least a few au from the central star. The planets had to grow fast enough so that they would detach themselves from the population of remaining, less massive, cores and migrate in to their present location. By the time the other cores migrated in, the disc's inner edge would have moved out so that these cores cannot be detected today. We also compute the critical core mass beyond which a massive gaseous envelope would be accreted and show that it is larger than 17 M_{\oplus} if the planetesimal accretion rate on to the core is larger than $10^{-6} M_{\oplus} \text{ yr}^{-1}$. For a planetesimal accretion rate between 10^{-6} and $10^{-5} M_{\oplus} \text{ yr}^{-1}$, the 17 M_{\oplus} core would not be expected to have accreted more than about 1 M_{\oplus} of gas. The results presented in this paper suggest that a planetary system like Kepler-10 may not be unusual, although it has probably formed in a rather massive disc.

Key words: planets and satellites: atmospheres – planets and satellites: formation – planets and satellites: individual: Kepler-10 – planet – disc interactions – planetary system.

1 INTRODUCTION

Since the detection of the first rocky extrasolar planet (Corot 7b; Léger et al. 2009; Queloz et al. 2009), a large number of similar objects have been observed by *Kepler* (Borucki et al. 2011; Batalha et al. 2013). As most of the planets detected by *Kepler* have not been confirmed by radial velocity measurements, the mass is not in general available and we have to rely on models linking the radius to the mass to classify the planets. Buchhave et al. (2014) and Marcy et al. (2014) have proposed that objects with radii smaller than $\sim 1.5 R_{\oplus}$, between ~ 1.5 and $\sim 4 R_{\oplus}$ and larger than $\sim 4 R_{\oplus}$ are, respectively, terrestrial planets, planets with a rocky core and a hydrogen–helium envelope, and ice or gas giants. According to this classification, the planet Kepler-10c, with a radius of $2.35 R_{\oplus}$, is expected to have a gaseous envelope. Yet, its mass has been determined by radial velocity measurements, and being about 17 M_{\oplus} , it indicates that the planet has a very high density of 7 g cm^{-3} and is likely to be solid (Dumusque et al. 2014).

Solid mass planets are believed to be formed through a process starting with the sedimentation and collisional growth of dust grains in a protostellar disc, followed by solid body accretion of km-sized

objects (Lissauer 1993; Papaloizou & Terquem 2006, and references therein) or cm-sized pebbles (Lambrechts & Johansen 2012). The formation of massive solid cores, which are the nucleus of gas giant planets, is believed to occur through collisions (also called giant impacts) between embryos.

Once the planets reach a mass on the order of a tenth of an Earth mass, they start migrating in the disc on a time-scale comparable to or smaller than the planet formation time-scale (Ward 1997). Recent hydrodynamical simulations (Pierens, Cossou & Raymond 2013) have shown the difficulty of forming very massive cores through giant impacts of terrestrial mass planets. Because of migration, the evolution of a population of such planets tends indeed to result in a resonant chain rather than in a single massive core (see also Terquem & Papaloizou 2007). Very massive cores are found only when starting with a population of planets of at least 2–3 M_{\oplus} . Alternatively, massive cores could form by continuous accretion of planetesimals, but the time-scale for forming a $\sim 10 M_{\oplus}$ core is usually found to be longer than the migration time-scale (see Tanigawa 2008 and references therein).

The planetary system Kepler-10, which comprises at least two planets, harbours the first rocky planet that was discovered by *Kepler*. Radial velocity measurement from Keck–HIRES, made immediately after the detection by *Kepler*, enabled the mass of Kepler-10b to be determined (Batalha et al. 2011). More recent observations

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from HARPS-N have improved the precision on the mass of Kepler-10b, and have allowed the determination of the mass of Kepler-10c: the system has a super Earth of $3.3 M_{\oplus}$ at 0.017 au, and a Neptune-mass planet of $17.2 M_{\oplus}$ at 0.24 au (Dumusque et al. 2014). With a radius of $2.35 R_{\oplus}$, the Neptune-mass planet therefore has a very high density. It is the first known solid planet with a mass above $10 M_{\oplus}$ (Kepler-131b may be similar to Kepler-10c, but its mass has not yet been determined with certainty; Marcy et al. 2014). The fact that Kepler-10c is solid has come as a surprise, as it is commonly believed that the critical core mass, above which accretion of a massive gaseous envelope occurs, is $\sim 10 M_{\oplus}$.

In this paper, we investigate the conditions required for two planets similar to those in the Kepler-10 system to form through collisions and mergers within an initial population of embryos (Section 2). We show that the total mass of the initial population has to be significantly larger than the masses of the two planets, and that the two planets must have built-up farther away than their present location, at a distance of at least a few au from the star. We then compute the critical core mass at the location where the Neptune-mass planet formed (Section 3). We find that it is larger than $17 M_{\oplus}$ if the planetesimal accretion rate on to the core is larger than $10^{-6} M_{\oplus} \text{ yr}^{-1}$. We finally discuss our results in Section 4.

2 FORMATION OF MASSIVE SOLID PLANETS

In this section, we investigate scenarios that could result in a planetary system like Kepler-10, comprising two solid planets of about 3 and $17 M_{\oplus}$ at 0.017 and 0.24 au, respectively.

2.1 In situ formation

Let us first consider whether the planets could have formed in situ. An embryo at 0.017 or 0.24 au from the star could in principle grow through accretion of solid material in the form of either dust, planetesimals or solid cores. However, in situ growth can only happen if the embryo is prevented from migrating on to the central star, i.e. if its orbit is inside the disc's inner edge. Loss of contact with the disc then makes it difficult for the embryo to accrete dust or planetesimals migrating within the disc towards the star. The orbit of more massive cores also migrating in could in principle cross that of the embryo, resulting in collisions and growth. However, as we will see in this section, incoming cores tend to be captured in mean motion resonances rather than collide with cores already within the disc's inner edge. It is therefore unlikely that the planets in the Kepler-10 system have formed in situ.

We have assumed in the above discussion that the embryo would stop migrating after entering the cavity. However, Masset et al. (2006) have suggested that cores would be trapped at the edge of the disc, rather than penetrating inside the cavity, due to the effect of the corotation torque. In this context, the embryo would not lose contact with the disc and could continue to accrete dust and/or planetesimals migrating within the disc. However, it is not clear that trapping of the cores would happen in the presence of MHD turbulence, which is likely to have been present in the disc at the location of the planets in the Kepler-10 system. Whether the disc keeps the planet trapped or not depends strongly on the profile of the surface density at the edge (Masset et al. 2006). Also, recent MHD simulations indicate that planets with masses as small as $\sim 10 M_{\oplus}$ can open up gaps in turbulent regions of discs with net vertical magnetic flux (Zhu, Stone & Rafikov 2013). The corotation torque acting on such planets would be much reduced, so that trapping would not occur.

We now investigate whether the dynamical evolution of a population of cores migrating inwards within the disc can result in the formation of a super Earth (with a mass of a few Earth masses) and a massive solid planet (with a mass similar to that of Neptune) at 0.017 and 0.24 au, respectively.

2.2 Numerical integration

To compute the evolution of a population of cores migrating through a disc, we use the N -body code described in Papaloizou & Terquem (2001) in which we have added the effect of the disc torques (see also Terquem & Papaloizou 2007).

The equations of motion for each core are:

$$\frac{d^2 \mathbf{r}_i}{dt^2} = -\frac{GM_* \mathbf{r}_i}{|\mathbf{r}_i|^3} - \sum_{j=1, j \neq i}^N \frac{GM_j (\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|^3} - \sum_{j=1}^N \frac{GM_j \mathbf{r}_j}{|\mathbf{r}_j|^3} + \mathbf{\Gamma}_i, \quad (1)$$

where G is the gravitational constant and M_* , M_i and \mathbf{r}_i denote the mass of the central star, that of core i and the position vector of core i , respectively. The third term on the right-hand side is the acceleration of the coordinate system based on the central star (indirect term).

Acceleration due to tidal interaction with the disc is dealt with through the addition of extra forces as in Papaloizou & Larwood (2000, see also Terquem & Papaloizou 2007):

$$\mathbf{\Gamma}_i = -\frac{1}{t_{m,i}} \frac{d\mathbf{r}_i}{dt} - \frac{2}{|\mathbf{r}_i|^2 t_{e,i}} \left(\frac{d\mathbf{r}_i}{dt} \cdot \mathbf{r}_i \right) \mathbf{r}_i - \frac{2}{t_{i,i}} \left(\frac{d\mathbf{r}_i}{dt} \cdot \mathbf{e}_z \right) \mathbf{e}_z, \quad (2)$$

where \mathbf{e}_z is the unit vector perpendicular to the disc mid-plane and $t_{m,i}$, $t_{e,i}$ and $t_{i,i}$ are the time-scales over which, respectively, the angular momentum, the eccentricity and the inclination with respect to the disc mid-plane of the orbit of core i change due to tidal interaction with the disc. Note that the time-scale on which the semi-major axis decreases is $t_{m,i}/2$ (e.g. Teysandier & Terquem 2014). As here we are not interested in following the evolution of a core after it gets close to the star, we will not include contribution from the tides raised by the star nor from relativistic effects.

2.3 Type-I migration and collisions

The cores we consider here are small enough that they undergo type I migration. Radiation-hydrodynamical simulations of disc/planet interactions have shown that cores with masses between about 4 and $30 M_{\oplus}$ and eccentricities below ~ 0.015 undergo outward migration, due to the effect of the corotation torque (Paardekooper & Mellema 2006; Kley, Bitsch & Klahr 2009; Bitsch & Kley 2010). Planets more massive than about $30 M_{\oplus}$ open up a gap, which reduces the corotation torque, so that the total torque is negative and migration is inward. However, as mentioned above, recent MHD simulations indicate that planets with masses significantly smaller (by at least a factor of 3) than $30 M_{\oplus}$ can open up gaps in turbulent regions of discs with net vertical magnetic flux (Zhu et al. 2013). Therefore, the range of planet masses for which outward migration occurs may be much smaller than suggested by the hydrodynamical simulations. In this context, we will assume in this paper that type I migration is always inward. Note that our results would not be significantly affected if cores with masses in a narrow range and eccentricities below ~ 0.015 were migrating outward.

In the regime of inward type-I migration, Papaloizou & Larwood (2000) have shown that $t_{m,i}$ and $t_{e,i}$ can be written as:

$$t_{m,i} = 146.0 \left[1 + \left(\frac{e_i}{1.3H/r} \right)^5 \right] \left[1 - \left(\frac{e_i}{1.1H/r} \right)^4 \right]^{-1} \times \left(\frac{H/r}{0.05} \right)^2 \frac{M_\odot}{M_d} \frac{M_\oplus}{M_i} \frac{a_i}{1 \text{ au}} \text{ yr}, \quad (3)$$

$$t_{e,i}^d = 0.362 \left[1 + 0.25 \left(\frac{e_i}{H/r} \right)^3 \right] \times \left(\frac{H/r}{0.05} \right)^4 \frac{M_\odot}{M_d} \frac{M_\oplus}{M_i} \frac{a_i}{1 \text{ au}} \text{ yr}, \quad (4)$$

and $t_{i,i} = t_{e,i}$. Here e_i is the eccentricity of core i , H/r is the disc aspect ratio and M_d is the disc mass contained within 5 au. The equations above assume that the disc surface mass density varies like $r^{-3/2}$.

Collisions between cores are dealt with in the following way: if the distance between cores i and j becomes less than $R_i + R_j$, where R_i and R_j are the radii of the cores, a collision occurs and the cores are assumed to merge. They are subsequently replaced by a single core of mass $M_i + M_j$ with the position and the velocity of the centre of mass of cores i and j .

2.4 Initial set up

We start with a population of N cores on circular orbits in the disc mid-plane spread between an inner radius R_{in} and an outer radius R_{out} . The initial distance between a core and the star is chosen randomly. The disc is assumed to be truncated at an inner radius R_{cav} , which in some simulations will increase with time.

We assume that once a core reaches this radius R_{cav} it loses contact with the disc and stops migrating. As indicated in Section 2.1, it has been suggested that the cores may be trapped at the disc inner edge rather than penetrate inside the cavity. When that happens, if the disc inner edge then expands, the planet may stay coupled to the disc and also move outward (Masset et al. 2006). However, such a shepherding of the planet by the disc requires that the disc can transfer enough angular momentum to the planet so that it can move outward as fast as the disc radius (Lyra, Paardekooper & Mac Low 2010). This cannot be satisfied if X-ray photoevaporation is responsible for the expansion of the disc's inner cavity (Owen, Ercolano & Clarke 2011), as the surface density of gas in the vicinity of the planet decreases to zero. Therefore, in the simulations presented below, a planet reaching the disc inner radius will be assumed to decouple from the disc and will stay at its location when this radius moves out.

All the cores are supposed to have an identical mass density $\rho = 1 \text{ g cm}^{-3}$. Note that this is smaller than the densities in the Kepler-10 system, which are inferred to be 5.8 and 7.1 g cm^{-3} for the 3 and 17 M_\oplus planets, respectively. Therefore, the radii of the cores in our simulations, which are given by $R_i = [3M_i/(4\pi\rho)]^{1/3}$, are almost twice as large as they would be if we adopted those higher values of the density. Thus, collisions between cores are favoured in our model. This, however, does not affect our results, as we will find that collisions are not efficient enough for the evolution of the population of cores to result in a 17 M_\oplus at 0.24 au.

In the simulations presented below, we have adopted $M_* = 1 M_\odot$, $M_d = 10^{-3} M_\odot$ and $H/r = 0.05$. For these values of the parameters,

equations (3) and (4) give $t_{m,i} \simeq 10^5 \text{ yr}$ and $t_{e,i} \simeq 4 \times 10^2 \text{ yr}$, respectively, for a 1 M_\oplus planet on a circular orbit at 1 au.

We now describe the results of our simulations.

2.5 A super Earth at 0.017 au

To investigate whether the dynamical evolution of a population of migrating cores could result in a 3 M_\oplus planet at 0.017 au and a 17 M_\oplus planet at 0.24 au, and nothing else, we have run a series of simulations with a total mass of cores equal to 20 M_\oplus . We have considered cores with initial masses between 1 and 3 M_\oplus , and N in the range 7 to 20. In some simulations, all the cores have the same mass, while in others, there is a mixture of different masses. The inner edge of the disc is taken to be $R_{\text{cav}} = 0.017 \text{ au}$ to start with, and is moved up to 0.24 au after a total mass of cores of a few M_\oplus has reached it. The initial inner and outer radii of the population of cores, R_{in} and R_{out} , are in the range 0.1–3 au and 1–5 au, respectively.

In Fig. 1, we plot the results of a simulation with $N = 14$ cores initially spread between $R_{\text{in}} = 1 \text{ au}$ and $R_{\text{out}} = 3 \text{ au}$ in a disc with an inner cavity below $R_{\text{cav}} = 0.017 \text{ au}$. The six outermost cores have a mass of 2 M_\oplus , while the eight innermost cores have a mass of 1 M_\oplus . Very quickly after the beginning of the simulation, a 5 M_\oplus core builds up through collisions and migrates in. It reaches the disc's inner cavity at around $t = 1.6 \times 10^4 \text{ yr}$, while the other cores are still beyond 0.5 au. After that time, the radius of the inner cavity is moved up to 0.24 au. The other cores continue to migrate in, and at around $t = 2.5 \times 10^4 \text{ yr}$, three cores with masses 4, 4 and 2 M_\oplus reach the new inner cavity's radius $R_{\text{cav}} = 0.24 \text{ au}$. As two last cores reach this radius at around $t = 1.1 \times 10^5 \text{ yr}$, collisions occur, and finally two cores with masses 5 M_\oplus and 10 M_\oplus are left at 0.22 and 0.18 au, respectively, in a 7:5 mean motion resonance. After $t = 1.6 \times 10^5 \text{ yr}$, the disc is removed to make sure the system is stable. The two outer planets, being in a resonance, have rather large eccentricities, on the order of a few hundredth, whereas the innermost planet has an eccentricity below 10^{-3} .

In the simulation described above, the outer edge of the cavity was assumed to move up rather quickly, on a time-scale of $\sim 10^4 \text{ yr}$. However, this time-scale could be made longer by decreasing the mass of the disc, so that migration would be slower, or by starting the cores further away from the central star.

We have run 37 simulations with a total mass of cores of 20 M_\oplus , an initial $R_{\text{cav}} = 0.017 \text{ au}$ and various R_{in} and R_{out} . In six of these simulations, the eccentricity damping time-scale given by equation (4) was increased by a factor of 2 or 5 to allow eccentricities to reach higher values, which would promote collisions. In five of the simulations, the initial masses of the cores were 3 or 4 M_\oplus , while in all the others they were 1 or 2 M_\oplus .

We have obtained a single core close to $R_{\text{cav}} = 0.017 \text{ au}$ in seven of these simulations. The mass of this planet was 1, 5, 5, 10, 6, 4 or 8 M_\oplus , with the three last cases corresponding to simulations with increased eccentricity damping time-scale. An inner core with 1 M_\oplus was obtained when one core in the initial distribution was detached from the rest of the population and closer in than the others. In all of the six other cases, the core that came to a halt close to 0.017 au built up through collisions very early on in the simulations. Being heavier than the others, it then migrated in faster and reached the inner edge of the disc before the other cores had time to join.

In the other 30 simulations, several cores of a few Earth masses ended up in mean motion resonances close to 0.017 au. In most cases, the cores would grow on their way in, at the same time as they were migrating.

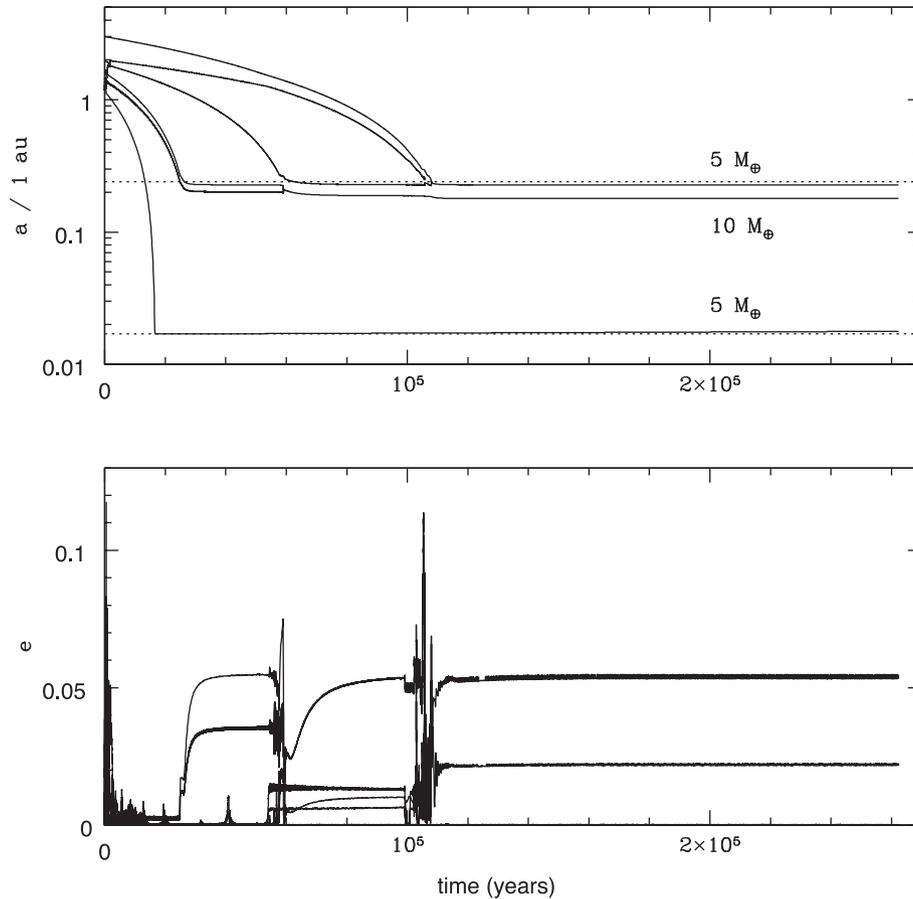


Figure 1. Evolution of the semi-major axes (in units of au and in logarithmic scale; upper plot) and of the eccentricity (lower plot) of the 14 cores in the system versus time (in units of years). Initially, the six outermost cores have a mass of $2 M_{\oplus}$, while the others have a mass of $1 M_{\oplus}$. The solid lines correspond to the different cores. A line terminates just prior to a collision. On the upper plot, the dotted lines indicate the location of the inner cavity ($R_{\text{cav}} = 0.017$ au initially, 0.24 au after 1.6×10^4 yr). The disc is removed after 1.6×10^5 yr. There are three cores left at the end of the simulation. Their masses are indicated on the upper plot.

These simulations therefore indicate that, if a single core of a few M_{\oplus} at 0.017 au has grown by collisions and mergers of smaller cores, most likely it has assembled further away. It grew and detached itself from a population of other smaller cores at a distance of at least a few au from the central star.

2.6 A massive planet at 0.24 au

We now investigate how a massive core which comes to a halt at 0.24 au could have formed.

In the seven simulations described above where a single core ended up close to 0.017 au, the other cores would still be beyond 0.5 au when the inner core reached R_{cav} . We therefore subsequently moved R_{cav} up to 0.24 au to investigate whether a single other core could be obtained at this location. In none of these simulations did we obtain a single other core. At least two cores in mean motion resonances were left close to 0.24 au, as observed in Fig. 1.

To study more generally whether a single core could grow through collisions and mergers within a population of cores with a total mass of $17 M_{\oplus}$, we performed another 29 simulations starting with cores with masses between 1 and $3 M_{\oplus}$, N in the range 6 to 17 and $R_{\text{cav}} = 0.24$ au initially. The initial inner and outer radii of the population of cores, R_{in} and R_{out} , were in the range 1 – 3 au and

2 – 5 au, respectively. In six of the simulations, the initial spacing between two cores was set to be 4 or 4.5 times their mutual Hill radius (as in Pierens et al. 2013). In all the other simulations, the location of the cores was chosen randomly between R_{in} and R_{out} . Migration and eccentricity damping time-scales were computed from equations (3) and (4). In 10 cases, the simulation ended with two cores in mean motion resonance close to the disc inner edge. In the other cases, there were at least three cores left. None of the simulations ended with only one core.

We then performed another 14 simulations with a larger total mass of cores, to study whether a massive core could build-up through collisions and migrate quickly to the inner edge before the others had time to join. In some of the simulations, the edge of the outer cavity was assumed to increase linearly with time so that $R_{\text{cav}} = 1$ au after 10^5 yr. In two of the simulations, we obtained a rather massive core (9 or $10 M_{\oplus}$) at around 0.3 au. In Fig. 2, we plot the results of one of these simulations. It starts with $N = 14$ cores spread between 2 and 4 au. Initially, the five outermost cores have a mass of $3 M_{\oplus}$, the innermost core has a mass of $1 M_{\oplus}$ and the others have a mass of $2 M_{\oplus}$, so that the total mass is $32 M_{\oplus}$. The edge of the inner cavity starts at $R_{\text{cav}} = 0.24$ au and increases to 1 au after 10^5 yr. We terminate the simulation after 5×10^4 yr, when there is a $9 M_{\oplus}$ core at 0.34 au, 2 cores in mean motion resonance close to 0.5 au and still two cores between 1 and 2 au migrating in.

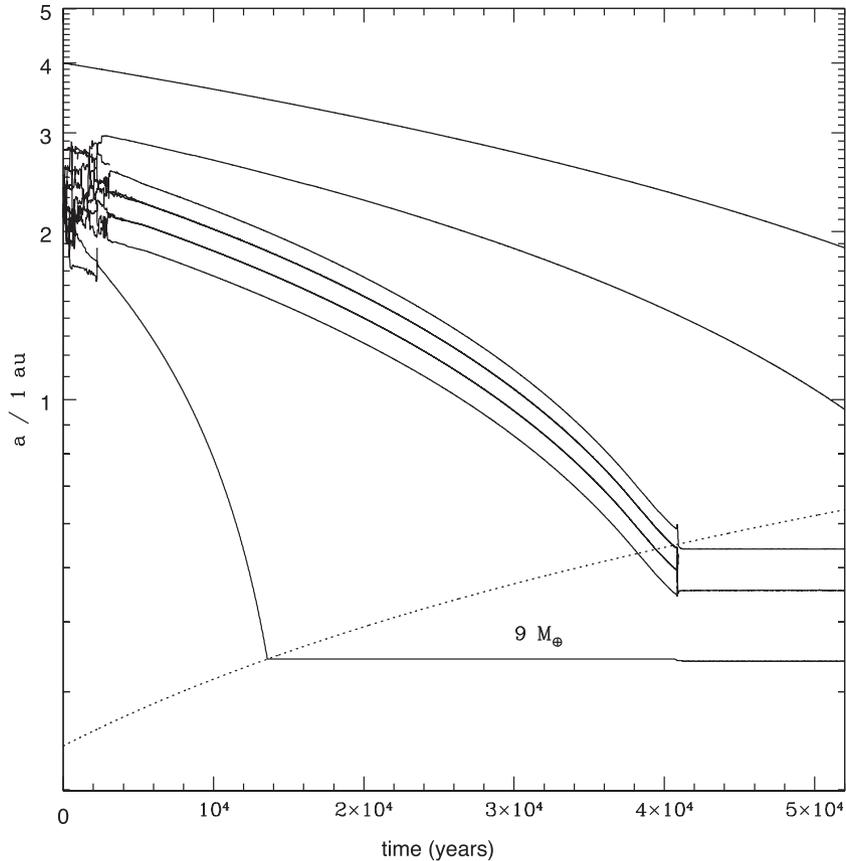


Figure 2. Evolution of the semi-major axes (in units of au and in logarithmic scale) of the 14 cores in the system versus time (in units of years). Initially, the five outermost cores have a mass of $3 M_{\oplus}$, the innermost core has a mass of $1 M_{\oplus}$ and the others have a mass of $2 M_{\oplus}$. The solid lines correspond to the different cores. A line terminates just prior to a collision. The dotted line indicates the location of the inner cavity. At the end of the simulation, there is a $9 M_{\oplus}$ core at 0.34 au, two cores in mean motion resonance close to 0.5 au and still two cores between 1 and 2 au migrating in.

The simulation described above results in a core less massive than the one detected in the Kepler-10 system at 0.24 au, and there are two other massive cores rather near by. However, it does illustrate that it is possible to get a massive core at a few tenths of an au starting with a massive population of cores further away. The mass of the core reaching the inner edge could be increased by increasing the total mass of the population of cores. Also, if it grew further away from the central star and detached itself from the rest of the population, it would reach the inner edge while the other cores would still be far away, so that at the end of the evolution no other core would be found near by. Note that the time-scale over which the edge of the cavity is moved is rather fast, so that we could perform a large number of simulations, but again this time-scale could be made longer by starting the cores further away.

Here again, we note that the core that comes to a halt at around 0.3 au has assembled very early on in the simulation, at a distance of ~ 1 au from the central star.

3 CRITICAL CORE MASS

The results presented in the previous section indicate that the planets have formed at a distance of at least a few au from the central star before migrating in. We therefore calculate what the critical core mass is at this location and all the way down to 0.24 au. Because the planets in the Kepler-10 system are very dense, they have not

accreted much gas, and therefore should not have attained the critical core mass (see the discussion at the end of Section 3.4). In the section below, we study the conditions which are required for the critical core mass to be above $17 M_{\oplus}$ within a distance of a few au from the central star.

3.1 Structure of the protoplanet atmosphere

Because the critical core mass corresponds to the mass of the core above which no atmosphere can exist at equilibrium around it, we solve the equations describing an atmosphere at equilibrium as a function of the core mass. The critical core mass is reached when these equations no longer have a solution.

The equations governing the structure of the protoplanet atmosphere at hydrostatic and thermal equilibrium have been presented in Papaloizou & Terquem (1999) and we recall them below.

We assume that the protoplanet is spherically symmetric and non-rotating. We denote ϖ the radius in spherical coordinates in a frame with origin at the centre of the protoplanet. The equation of hydrostatic equilibrium is:

$$\frac{dP}{d\varpi} = -g\rho. \quad (5)$$

Here, P is the pressure, $g = GM(\varpi)/\varpi^2$ is the acceleration due to gravity, with $M(\varpi)$ being the mass interior to radius ϖ (this includes the core mass if ϖ is larger than the core radius) and G

is the gravitational constant. The mass $M(\varpi)$ is related to the mass density per unit volume ρ through:

$$\frac{dM}{d\varpi} = 4\pi\varpi^2\rho. \quad (6)$$

We use the equation of state for a hydrogen and helium mixture given by Chabrier et al. (1992) for mass fractions of hydrogen and helium of 0.7 and 0.28, respectively. The luminosity L_{rad} that is transported by radiation through the atmosphere is related to the temperature gradient $dT/d\varpi$ through the standard equation of radiative transport:

$$\frac{dT}{d\varpi} = \frac{-3\kappa\rho}{16\sigma T^3} \frac{L_{\text{rad}}}{4\pi\varpi^2}, \quad (7)$$

where κ is the opacity, which in general depends on both ρ and T , and σ is the Stefan–Boltzmann constant.

The total luminosity is transported by both radiation (in the outer parts of the atmosphere) and convection (in the inner parts). Here, the only energy source for the atmosphere that we consider comes from the planetesimals that are accreted by the protoplanet and release their gravitational energy as they collide with the surface of the core. The corresponding total core luminosity L_c is:

$$L_c = \frac{GM_c\dot{M}_c}{r_c}, \quad (8)$$

where M_c and r_c are, respectively, the mass and the radius of the core, and \dot{M}_c is the planetesimal accretion rate.

The radiative and adiabatic temperature gradients, ∇_{rad} and ∇_{ad} , are given by:

$$\nabla_{\text{rad}} = \left(\frac{\partial \ln T}{\partial \ln P} \right)_{\text{rad}} = \frac{3\kappa L_c P}{64\pi\sigma G M T^4}, \quad (9)$$

and

$$\nabla_{\text{ad}} = \left(\frac{\partial \ln T}{\partial \ln P} \right)_s, \quad (10)$$

where the subscript s indicates that the derivative has to be evaluated at constant entropy.

When $\nabla_{\text{rad}} < \nabla_{\text{ad}}$, there is stability to convection and therefore all the energy is transported by radiation, i.e. $L_{\text{rad}} = L_c$. In the regions where $\nabla_{\text{rad}} > \nabla_{\text{ad}}$, there is instability to convection and therefore part of the energy is transported by convection, i.e. $L_c = L_{\text{rad}} + L_{\text{conv}}$, where L_{conv} is the luminosity associated with convection. Using the mixing length theory (Cox & Giuli 1968), we obtain:

$$L_{\text{conv}} = \pi\varpi^2 C_p \Lambda_{\text{ml}}^2 \left[\left(\frac{\partial T}{\partial \varpi} \right)_s - \left(\frac{\partial T}{\partial \varpi} \right) \right]^{3/2} \times \sqrt{\frac{1}{2} \rho g \left| \left(\frac{\partial \rho}{\partial T} \right)_P \right|}, \quad (11)$$

where $\Lambda_{\text{ml}} = |\alpha_{\text{ml}} P / (dP/d\varpi)|$ is the mixing length, α_{ml} being a constant of order unity, $(\partial T/\partial \varpi)_s = \nabla_{\text{ad}} T (d \ln P/d\varpi)$, and the subscript P denotes evaluation at constant pressure. The different thermodynamic parameters needed in the above equation are given by Chabrier et al. (1992), and we fix $\alpha_{\text{ml}} = 1$.

3.2 Boundary conditions

As we solve the above equations for the three variables P , M and T as a function of ϖ , we need three boundary conditions.

We take for the mass density of the core $\rho_c = 7 \text{ g cm}^{-3}$, which is approximately the value inferred for the $17 M_{\oplus}$ planet in the Kepler-10 system (Dumusque et al. 2014).

We can then calculate the inner boundary of the atmosphere, which is equal to the core radius r_c , given by:

$$r_c = \left(\frac{3M_c}{4\pi\rho_c} \right)^{1/3}. \quad (12)$$

The first boundary condition is that $M(r_c) = M_c$.

The outer boundary of the atmosphere is taken to be at the Roche lobe radius r_L of the protoplanet, which is given by:

$$r_L = \frac{2}{3} \left(\frac{M_p}{3M_{\star}} \right)^{1/3} r, \quad (13)$$

where $M_p = M_c + M_{\text{atm}}$ is the planet mass, M_{atm} being the mass of the atmosphere, and r is the orbital radius of the protoplanet in the disc.

We denote the disc mid-plane temperature, pressure and mass density at the distance r from the central star by T_m , P_m and ρ_m , respectively.

At $\varpi = r_L$, we have $M(r_L) = M_p$ and the two boundary conditions $P = P_m$ and T given by:

$$T = \left(T_m^4 + \frac{3\tau_L L_c}{16\pi\sigma r_L^2} \right)^{1/4}. \quad (14)$$

This equation expresses the fact that the radiative flux at the surface of the protoplanet, σT^4 , is the sum of the radiative flux coming from the disc above the protoplanet, σT_m^4 , and the radiative flux coming from inside the protoplanet, $3\tau_L L_c / (16\pi r_L^2)$. This latter term takes into account the fact that the luminosity escaping from the surface of the protoplanet, L_c , is radiated after passing through an additional optical depth τ_L above the protoplanet atmosphere. In other words, T must be larger than T_m at $\varpi = r_L$ for the luminosity to be radiated away from the protoplanet into the surrounding disc. We approximate τ_L by:

$$\tau_L = \kappa(\rho_m, T_m) \rho_m r_L. \quad (15)$$

As pointed out by Papaloizou & Terquem (1999), the structure of the atmosphere is sensitive to the value of T at $\varpi = r_L$ only when a significant part of the envelope is convective. This occurs in the hot inner parts of the disc, below $\sim 0.1 \text{ au}$. Therefore, at the location of the $17 M_{\oplus}$ in the Kepler-10 system and beyond, the critical core mass is not sensitive to the boundary condition given by equation (14).

3.3 Kelvin–Helmholtz time-scale

For a fixed \dot{M}_c and at a given radius r , there is a critical core mass M_{crit} above which no solution to the above equations can be found. As long as $M_c < M_{\text{crit}}$, the energy lost by the envelope through radiation is compensated for by the gravitational energy which the planetesimals entering the atmosphere release when they collide with the surface of the core. The atmosphere is then in quasi-static and thermal equilibrium. However, when $M_c > M_{\text{crit}}$, the atmosphere can no longer be supported at equilibrium. It has to contract gravitationally to supply part of the energy which is radiated away. Rapid accretion of the gas in the surrounding nebula then occurs.

How fast this accretion process is depends on how fast the envelope can radiate away the energy which is produced by its gravitational collapse. This is given by the Kelvin–Helmholtz time-scale, which can be estimated as:

$$t_{\text{KH}} = \frac{|E_{\text{T}}|}{L_c}, \quad (16)$$

where E_T is the total internal and gravitational energy of the gas in the atmosphere when the core reaches the critical mass. The luminosity that appears in equation (16) is L_c as this is roughly the luminosity of the core when it becomes critical.

3.4 Calculations

We compute the disc mid-plane temperature T_m , pressure P_m and mass density ρ_m assuming a standard steady-state α disc model (see Papaloizou & Terquem 1999 for the details of the computation). Such a model is completely characterized by two parameters, which we take to be α and the gas accretion rate \dot{M}_{gas} through the disc.

For a particular disc model, at a fixed radius r in the disc, for a given core mass M_c and planetesimal accretion rate \dot{M}_c , we solve equations (5), (6) and (7) with the boundary conditions described above to get the structure of the envelope. The opacity is taken from Bell & Lin (1994) and has contributions from dust grains, molecules, atoms and ions. The value of M_c above which the equations have no solution is the critical core mass M_{crit} .

In Table 1, we give the values of M_{crit} , $M_p = M_{\text{crit}} + M_{\text{atm}}$ and of the Kelvin–Helmholtz time-scale t_{KH} for disc models with $\dot{M}_{\text{gas}} = 10^{-8} M_{\odot} \text{ yr}^{-1}$ and $\alpha = 10^{-3}$ or 10^{-2} , at the radii $r = 0.24$ and 1 au in the disc, and for a planetesimal accretion rate $\dot{M}_c = 10^{-7}$, 10^{-6} or $10^{-5} M_{\oplus} \text{ yr}^{-1}$. By comparing M_p and M_{crit} we see that, when the core reaches the critical mass, $M_p \simeq 1.5M_{\text{crit}}$ (in agreement with Bodenheimer & Pollack 1986).

Fig. 3 shows M_{crit} and t_{KH} as a function of \dot{M}_c in the range 10^{-6} – $10^{-5} M_{\oplus} \text{ yr}^{-1}$ at $r = 0.24$ and 1 au and for disc models with $\dot{M}_{\text{gas}} = 10^{-8} M_{\odot} \text{ yr}^{-1}$ and $\alpha = 10^{-3}$ or 10^{-2} .

As was already noted by Papaloizou & Terquem (1999), M_{crit} is essentially independent of r for r larger than about 0.1 au. This is because M_{crit} depends on the boundary conditions only when a large part of the envelope is convectively unstable, which happens only for the highest values of T_m and P_m , i.e. in the disc’s inner parts. The values of M_{crit} beyond 1 au can therefore be taken as being roughly the same as at 1 au.

From Table 1 and Fig. 3, we see that \dot{M}_c has to be larger than $10^{-6} M_{\oplus} \text{ yr}^{-1}$ for M_{crit} to be larger than $17 M_{\oplus}$ beyond 0.24 au.

For such values of \dot{M}_c , a $17 M_{\oplus}$ core forming at a few au from the star and migrating in would not be expected to accrete a massive atmosphere of gas. However, the core could still accrete an envelope that would stay at equilibrium at its surface. The mass of an envelope at equilibrium on to a $17 M_{\oplus}$ core depends on \dot{M}_c . The largest value is attained when the core is very close to being critical, and in that situation $M_p = M_{\text{crit}} + M_{\text{atm}} \simeq 1.5M_{\text{crit}}$, which gives $M_{\text{atm}} = 8.5 M_{\oplus}$. From Table 1, we see that a $17 M_{\oplus}$ core is close to being critical if $\dot{M}_c = 10^{-6} M_{\oplus} \text{ yr}^{-1}$, and the corresponding Kelvin–Helmholtz time-scale is $t_{\text{KH}} \simeq 10^6$ yr at $r \geq 0.24$ au. If $\dot{M}_c = 10^{-5} M_{\oplus} \text{ yr}^{-1}$, we calculate that the mass of the atmosphere at equilibrium on to a $17 M_{\oplus}$ core is much smaller, being $M_{\text{atm}} \simeq 1 M_{\oplus}$, and for such an atmosphere $t_{\text{KH}} \simeq 2 \times 10^4$ yr at $r \geq 0.24$ au in a disc with either $\alpha = 10^{-2}$ or $\alpha = 10^{-3}$.

Therefore, if $\dot{M}_c = 10^{-6} M_{\oplus} \text{ yr}^{-1}$, as the Kelvin–Helmholtz time-scale is much longer than the migration time-scale, the core may not have had time to accrete the $8.5 M_{\oplus}$ of gas that could be supported at equilibrium before it reached the disc’s inner cavity. In contrast, if $\dot{M}_c = 10^{-5} M_{\oplus} \text{ yr}^{-1}$, the Kelvin–Helmholtz time-scale is much shorter than the migration time-scale, so the core can accrete the whole atmosphere that can be supported at equilibrium, but that would only be about $1 M_{\oplus}$. Therefore, in both cases, we may expect an atmosphere at most on the order of an Earth mass on top of the core.

As this atmosphere is not detected today, it has been stripped away. Let us first show that Jean’s escape at 0.24 au from the central star cannot account for the disappearance of the atmosphere. The escape velocity from a core with mass M_c and radius r_c is $v_{\text{esc}} = (2GM_c/r_c)^{1/2}$. With $M_c = 17 M_{\oplus}$ and r_c given by equation (12), in which we take $\rho_c = 7 \text{ g cm}^{-3}$, we obtain $v_{\text{esc}} \simeq 3 \times 10^4 \text{ m s}^{-1}$. As the luminosity of the star in the Kepler-10 system is similar to that of the Sun, the temperature of the planet atmosphere due to stellar irradiation, after the disc has disappeared, is $T = [L_{\odot}/(4\pi\sigma r^2)]^{1/4}$, where r is the distance between the star and the planet. As this assumes that the atmosphere behaves like a blackbody, the derived temperature is only a crude estimate. At $r = 0.24$ au, we obtain $T \simeq 574 \text{ K}$. This gives the thermal velocity of a hydrogen molecule, $v_{\text{th}} = (kT/m_p)^{1/2} \simeq 2 \times 10^3 \text{ m s}^{-1}$, where k is

Table 1. Critical core mass and Kelvin–Helmholtz time-scale.

α	r (au)	T_m (K)	P_m (erg cm $^{-3}$)	\dot{M}_c ($M_{\oplus} \text{ yr}^{-1}$)	M_{crit} (M_{\oplus})	M_p (M_{\oplus})	t_{KH} (10^6 yr)
10^{-2}	0.24	1001.1	41.0	10^{-5}	25.8	38.9	0.16
–	–	–	–	10^{-6}	18.4	27.4	1.3
–	–	–	–	10^{-7}	13.1	19.5	11.0
–	1	273.0	1.1	10^{-5}	24.3	36.3	0.13
–	–	–	–	10^{-6}	16.7	24.6	0.96
–	–	–	–	10^{-7}	11.2	16.6	7.1
10^{-3}	0.24	1180.8	359.8	10^{-5}	24.6	36.2	0.14
–	–	–	–	10^{-6}	17.5	26.4	1.2
–	–	–	–	10^{-7}	12.5	18.7	9.9
–	1	480.7	8.5	10^{-5}	23.9	35.6	0.12
–	–	–	–	10^{-6}	16.6	24.6	0.95
–	–	–	–	10^{-7}	11.4	16.5	7.3

Listed are the parameter α used in the disc models (Column 1), the orbital radius r of the core in au (Column 2), the disc mid-plane temperature in K (Column 3) and pressure in erg cm $^{-3}$ (Column 4) at this radius, the planetesimal accretion rate on to the core \dot{M}_c in $M_{\oplus} \text{ yr}^{-1}$ (Column 5), the critical core mass M_{crit} in M_{\oplus} (Column 6), the total mass of the planet $M_p = M_{\text{crit}} + M_{\text{atm}}$ in M_{\oplus} (Column 7) and the Kelvin–Helmholtz time-scale t_{KH} in Myr for a core with the critical mass (Column 8).

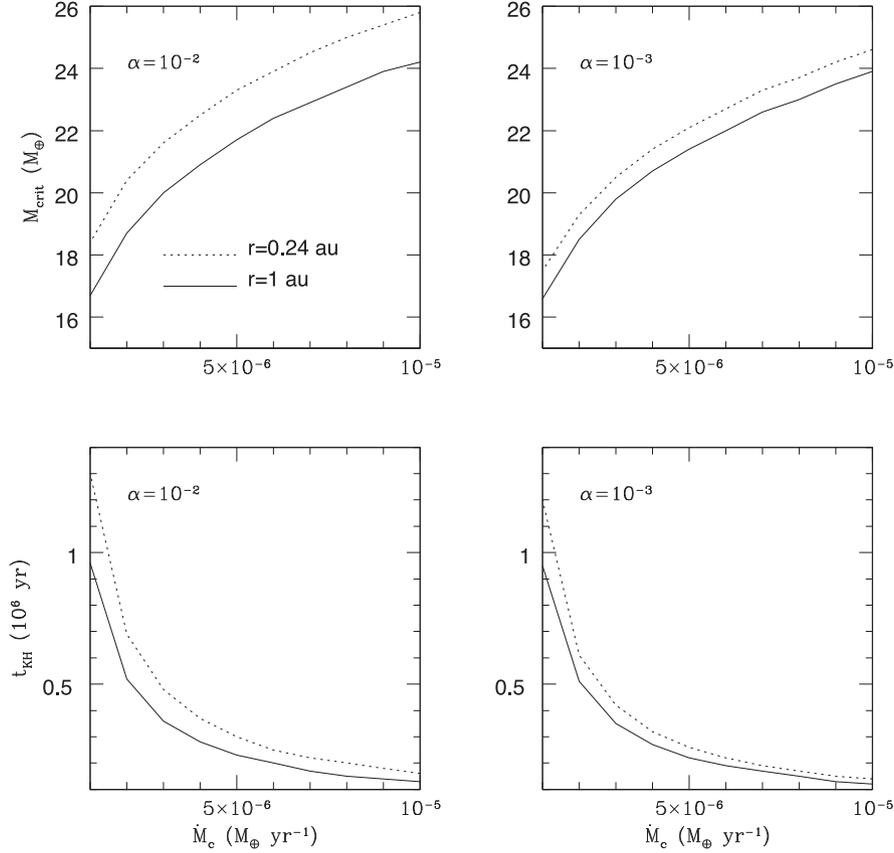


Figure 3. Critical core mass M_{crit} in units of M_{\oplus} (upper panels) and Kelvin–Helmholtz time-scale t_{KH} in units of 10^6 yr for a core with the critical mass (lower panels) as a function of the planetesimal accretion rate on to the core \dot{M}_c in $M_{\oplus} \text{ yr}^{-1}$ at $r = 1 \text{ au}$ (solid lines) and $r = 0.24 \text{ au}$ (dotted lines) for a disc model with $\alpha = 10^{-2}$ (left panels) and 10^{-3} (right panels). The values of M_{crit} beyond 1 au are roughly the same as at 1 au.

the Boltzmann constant and m_p is the mass of the proton. As v_{th} is an order of magnitude smaller than v_{esc} , Jean’s escape cannot have operated for the $17 M_{\oplus}$ core at 0.24 au. An alternative for stripping away the atmosphere would be stellar wind (as has been proposed for Mars), giant impacts or planetesimal accretion (see Schlichting, Sari & Yalinewich 2014, and references therein) or mass loss due to the stellar XUV flux (Rogers et al. 2011).

In the above discussion, we have assumed that the mass of the planet had to be smaller than the critical core mass for a large quantity of gas not to be accreted. In principle though, the planet could be more massive than the critical mass if the Kelvin–Helmholtz time-scale were longer than the migration time-scale. The planet would then reach the disc inner edge and lose contact with the disc before a significant amount of gas could be accreted. We now briefly show that this actually cannot be achieved. If the planetesimal accretion rate were $\dot{M}_c = 10^{-7} M_{\oplus} \text{ yr}^{-1}$, the critical core mass at 1 au would be about $11 M_{\oplus}$. The Kelvin–Helmholtz time-scale on to a core reaching that mass being $\sim 7 \times 10^6 \text{ yr}$, such a core would enter the disc inner cavity without having accreted a significant amount of gas. However, in the case of the Kepler-10 system, the core would have to grow up to $17 M_{\oplus}$ before reaching the disc inner edge. A core of that mass embedded in a disc with $\dot{M}_c = 10^{-7} M_{\oplus} \text{ yr}^{-1}$ has an atmosphere which cannot be at equilibrium, and which therefore is detached from the Roche lobe. Papaloizou & Nelson (2005) have computed the evolution of a core embedded in a disc and which atmosphere is detached from the Roche lobe. They found that such a protoplanet can accrete gas at any rate that may be supplied by the disc without expansion. Therefore, for typical gas accretion rates,

a significant atmosphere would be accreted on to the core before it entered the disc inner cavity.

4 SUMMARY AND DISCUSSION

The simulations we have performed indicate that the planets in a system like Kepler-10 have formed much further away from the central star than the location at which they are detected today. They cannot have assembled through collisions and mergers of a population of low-mass cores with a total mass of $20 M_{\oplus}$ migrating in. This is because the eccentricity damping time-scale is much shorter than the migration time-scale, so that the cores in such a population end up in a resonant chain rather than collide which each other until only two cores are left at 0.017 and 0.24 au.

Either (i) the planets grew all the way up by accreting planetesimals, or (ii) they grew through collisions among a population of cores. In the first case, they had to gain their mass on a time-scale shorter than the migration time-scale. In the second case, they had to grow fast enough that they would detach themselves from the population of remaining cores (which total mass had to be significantly larger than the mass of the two planets) and migrate in to the disc’s inner edge faster than the other, less massive cores. By the time the other cores migrate in significantly, the inner edge of the disc has moved out, so that these cores are further away and cannot be detected. In this situation, the $3 M_{\oplus}$ core would have formed earlier on and/or closer to the central star than the $17 M_{\oplus}$, so that the inner edge of the disc would have had time to move from 0.017 au to 0.24 au out in between their respective arrival in the disc’s cavity.

In both cases, the planets have essentially acquired their mass at a distance of at least a few au from the central star. The physical conditions at this location are then relevant to study the accretion of an atmosphere on to the cores.

As pointed out in Section 2.3, we have assumed that the cores, starting from the initial population, always migrated inward. More specifically, to form a planetary system like Kepler-10, we need the 3 and 17 M_{\oplus} cores to migrate inward starting at a distance of at least a few au. According to the radiation–hydrodynamical simulations of disc/planet interactions (Bitsch & Kley 2010), the 17 M_{\oplus} core would be expected to migrate outward, as its eccentricity is damped below ~ 0.015 by the interaction with the disc. Our results therefore give support to the MHD simulations (Zhu et al. 2013) which show that a 17 M_{\oplus} may open up a gap in a turbulent disc with a net vertical magnetic flux, thus reducing the contribution of the corotation torque and enabling inward migration.

As the 17 M_{\oplus} planet in the Kepler-10 system is very dense and probably does not have an atmosphere (Dumusque et al. 2014), it has not reached the critical mass. We have found that this requires the planetesimal accretion rate on to the core to be larger than $10^{-6} M_{\oplus} \text{ yr}^{-1}$. This value, although in the upper range, is not unphysical and has commonly been used in studies of planet formation (Tanaka & Ida 1999; Ikoma, Nakazawa & Emori 2000, and references therein). A rather high value of the planetesimal accretion rate during the planet formation phase is also consistent with the existence of two rather massive solid planets in the Kepler-10 system, and suggests that this system has formed in a somewhat massive disc. If a core builds up at a few au from the central star and migrates in on a time-scale of $\sim 10^5$ yr, it would accrete only about 0.1 M_{\oplus} of solid material on its way in if the planetesimal accretion rate is uniform and equal to $10^{-6} M_{\oplus} \text{ yr}^{-1}$. As the critical core mass does not depend much on the distance from the central star beyond ~ 0.1 au, the core would therefore remain subcritical. If the planetesimal accretion rate were $10^{-5} M_{\oplus} \text{ yr}^{-1}$ instead, the core would have built-up to about 16 M_{\oplus} at a few au from the central star and grown to its present mass on its way in. In that case, its mass would be much smaller than the critical mass.

Even a subcritical core can accrete a gaseous envelope, which stays at quasi-equilibrium around it. We have found that, for a planetesimal accretion rate between 10^{-6} and $10^{-5} M_{\oplus} \text{ yr}^{-1}$, the core would have accreted an envelope of at most $\sim 1 M_{\oplus}$. This envelope must have been stripped away as it is probably not present today.

The results presented in this paper indicate that a planetary system like Kepler-10 may not be unusual, although it has probably formed in a rather massive disc. It is interesting to note that the observations of both gas giant planets and massive solid planets are consistent with the initial disc mass being a key parameter in determining the final outcome of planetary systems. Massive discs favour the formation of massive planets which migrate in fast and end up on short orbits (as seen in the simulations by Thommes, Matsumura &

Rasio 2008). However, gas giant planets may not necessarily form in those discs if the planetesimal accretion rate is high enough that even rather massive cores remain subcritical.

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