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1 **Use of Convolutional Neural Networks with encoder-decoder structure for**  
2 **predicting the inverse operator in hydraulic tomography.**

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11

12 **Abstract:** In this manuscript, we discuss the capabilities of a deep learning algorithm  
13 implemented with the Conventional Neural Network concept to characterize the hydraulic  
14 properties of aquifers. The algorithm called CNN-HT is designed to predict the inverse operator  
15 of hydraulic tomography using a synthetic training dataset in which the hydraulic head data  
16 associated with pumping tests are linked to hydraulic transmissivity field. This approach relies  
17 on an adaptation of the SegNet network that was initially developed to process image  
18 segmentation. The SegNet is composed of encoders and decoders networks. In the encoder,  
19 sequential operations with multiple filters, as convolution, batch normalization, max-pooling  
20 are performed to identify feature maps of the input data. In the decoder, the up-sampling,  
21 convolution, batch normalization and regression operations are used to prepare the output by  
22 recovering the loss of spatial resolution that occurred in the encoder process. In this adaptation,  
23 we used the least-square iterative formulation at the initial iteration with Jacobian matrix to  
24 resize the hydraulic head data to match the size of the output (transmissivity field). This protocol  
25 was applied to the hydraulic head data computed numerically by solving the groundwater flow  
26 equation for a given transmissivity field, generated geostatistically with Gaussian and spherical  
27 variograms. A part of this data was used for training the network and the other part to test its  
28 performance. The test step confirmed the effectiveness of this tool in reconstructing the main  
29 heterogeneities of the hydraulic properties, and its effectiveness is related to the nature and

30 quantity of the training data. Moreover, the CNN-HT method provided inversion results of the  
 31 same quality than those obtained with the Gauss-Newton algorithm using the finite difference  
 32 or adjoint state method in the computation of the Jacobian matrix. However, the computational  
 33 time is longer in CNN-HT but this time can be less or of the same order as that of Gauss-Newton  
 34 using finite difference method.

## 35 1. Introduction

36  
 37 Since the 1980s, the hydraulic tomography has been adopted in hydrology as an effective  
 38 technique for mapping the heterogeneity of the hydraulic properties of aquifers (Neuman, 1987;  
 39 Gottlieb and Dietrich, 1995; Bohling et al., 2002; Yeh and Liu, 2000). Indeed, this approach  
 40 can effectively provide insights into the spatial variability of the hydraulic transmissivity and  
 41 storage coefficient of a porous or fractured aquifer through a combined interpretation of  
 42 hydraulic data obtained from several pumping tests (Bohling et al., 2002; Berg and Illman,  
 43 2011; Cardiff et al., 2013; Fischer et al., 2018). The approach consists of using an inversion  
 44 algorithm, either deterministic or stochastic, to retrieve the best models that could match the  
 45 observed piezometric data (Fu & Gómez-Hernández, 2009a; Kitanidis, 1997). In general, the  
 46 inverse solution is not unique for that reason the inverse formulation usually incorporates a  
 47 regularization term in the optimization of the objective function  $\Psi(\mathbf{m})$  that is defined as the  
 48 sum of data misfit and regularization terms, as below (Tarantola and Valette, 1982):

$$49 \quad \Psi(\mathbf{m}) = \underbrace{\left[ \mathbf{F}(\mathbf{m}) - \mathbf{h}_{\text{obs}} \right]^T \mathbf{C}_d^{-1} \left[ \mathbf{F}(\mathbf{m}) - \mathbf{h}_{\text{obs}} \right]}_{\text{Data misfit term}} + \underbrace{\left[ \mathbf{m} - \mathbf{m}_0 \right]^T \mathbf{C}_m^{-1} \left[ \mathbf{m} - \mathbf{m}_0 \right]}_{\text{constraint term}}, \quad (1)$$

50 where the data misfit term assesses the pertinence of the model in term of matching the data  
 51 and is expressed as a sum of square differences between the observed hydraulic data  $\mathbf{h}_{\text{obs}}$  and  
 52 the numerical hydraulic data derived by solving a forward operator  $\mathbf{F}(\mathbf{m})$ . In hydraulic  
 53 tomography, the forward problem involves a numerical discretization for solving the

54 groundwater flow equation subject to Darcy's law in transient or steady modes.  $m = -\log [T]$   
55 is the spatial distribution of the negative logarithm of the hydraulic transmissivity  $T$ , which is  
56 regarded as an unknown field and is expressed logarithmically to ensure its positivity during  
57 the inverse process.  $C_d$  is a diagonal matrix for incorporating data uncertainty in the  
58 optimization. On the other side, the regularization term is expressed with a prior model  $m_0$  and  
59 its covariance matrix  $C_m$ . This term can be obtained from geological, geophysical and tracer  
60 investigations to constrain the inversion to offer only plausible solutions (Lochbühler et al.,  
61 2013; Zhao et al., 2016; Soueid et al., 2016). In general, the use of geostatistical constraints is  
62 still widely applied in hydraulic tomography by using statistical properties such as mean and  
63 covariance to obtain an imagery of hydraulic properties with smooth features (Kitanidis, 1997;  
64 Yeh and Liu, 2000).

65 Once the parameters of the objective function are formulated, its optimization is processed  
66 iteratively using one of the three categories of optimization algorithms: deterministic (Li et al.,  
67 2005), stochastic (Jimenez et al., 2016) or global (Castagna and Bellin, 2009). These algorithms  
68 involve a repetitive numerical solving of the forward problem by applying finite elements or  
69 finite difference methods to assess the ability of the proposed model to reach the convergence  
70 state. Therefore, the computation time is dependent on the numerical tools, the dimensionality  
71 of the unknown parameters and the nature of the optimization algorithm employed. In general,  
72 deterministic algorithms are the most used in high-dimensional inverse problems due to their  
73 advantage of reaching convergence in a short time compared to stochastic or global algorithms  
74 by exploiting the geometric properties of the objective function (gradient and Hessian)  
75 (Tarantola and Valette, 1982). Nevertheless, their efficiency depends strongly on the choice of  
76 the initial model as they only provide local minima. On the other hand, stochastic methods (such  
77 as Markov chain Monte Carlo MCMC) are based on the concept of sampling where the solution

78 is obtained by exploring the optimization performance of many randomly generated models and  
79 selecting the best of them in terms of minimization of the objective function. Most of the time,  
80 algorithms belonging to this category are easy to implement and insensitive to the initial model.  
81 However, their applicability to inverse problems with a large number of unknown parameters  
82 remains limited due to the high repetitive and heavy computation of the forward problem during  
83 the sampling process (Oliver et al., 1997; Fu and Gómez-Hernández, 2009b; Wang et al. 2017;  
84 Jardani et al., 2012; Elsheikh et al., 2012). Other global optimization algorithms such as particle  
85 swarm optimization, genetic, simulated annealing can also be used to identify the best solution  
86 by iteratively evaluating the objective function until the state of convergence. However,  
87 convergence can be slow, especially in the case of high-dimensional inverse problems (Scales  
88 et al., 1990; Fernández-Martínez et al., 2011).

89 In this paper, we test a new generation of deep learning algorithm that can be considered as a  
90 global optimizer in which prediction does not depend on an initial model and the computation  
91 of the sensitivity matrix as it is the case for gradient-based methods (Zio, 1997). The concept  
92 of this approach is different from previous optimization methods as the process focuses on  
93 approximating the inverse function by finding a universal relationship linking the input and  
94 output data (in the case of hydraulic tomography between hydraulic pressure and hydraulic  
95 transmissivity). Indeed, the deep learning is a powerful generalizer of non-linear and complex  
96 functions by identifying a set of parameters such as weights and biases allocated to the hidden  
97 layer neurons that process the input data and link them to their corresponding output. The first  
98 generation of deep learning algorithms uses a set of hidden layers with a large number of  
99 neurons fully connected via high-dimensional weights, which complicates computation and  
100 requires a lot of time, especially for handling the imaging tasks (Shen, 2018). However, the  
101 emergence of new deep learning architectures with Convolutional Neural Networks (CNN)  
102 concept that have been proven in high-resolution image processing have opened the way for

103 broadening their applicability in the hydraulic characterization (LeCun et al., 1998; Sun, 2018;  
104 Zhu and Zabararas, 2018). Indeed, the CNN concept has succeeded in reducing drastically the  
105 image processing time thanks to the convolution calculation, which allows to retrieve the local  
106 features of the image through small filters (Indolia et al., 2018). These filters cover the whole  
107 input image but their activation is done zone by zone with local connections between pixels.  
108 This local convolution reduces the number of parameters to optimize during the learning  
109 process (LeCun et al., 1998). Among the first attempts at adopting the CNN concept in the  
110 realm of tomography by inversion in geosciences, we cite: Sun (2018) trained the Generative  
111 Adversarial networks for linking the hydraulic head map and the spatial distribution of  
112 hydraulic conductivity. Zhu and Zabararas (2018) proposed a deep convolutional encoder-  
113 decoder network to reconstruct the image of hydraulic conductivity from hydraulic head map.  
114 Laloy et al. (2018) used the Generative Adversarial networks to generate randomly in 2D and  
115 3D binary hydraulic conductivity fields and MCMC to perform the inversion process. In  
116 geophysics realm, Wu and Lin et al. (2019) applied CNN with encoder-decoder networks for  
117 mapping the subsurface velocity from seismogram data. Puzyrev and Swidinsky. (2019) used  
118 the CNN network to determine the vertical subsurface heterogeneity of electrical conductivity  
119 by training electromagnetic data.

120 In this article, we discuss the use of the convolution neural networks formed according to the  
121 SegNet architecture to process hydraulic tomography. This technique was initially designed for  
122 the semantic segmentation of objects learned on the images (Badrinarayanan, et al. 2017). To  
123 illustrate the inversion principle with the CNN-HT structure, we organize this manuscript with  
124 the following outline: the first section will be devoted to the generation of data used in training,  
125 validation and test, and the second to the introduction of concept of the CNN-SegNet structure.  
126 In the application sections, we discuss the relevance of the approach on theoretical cases under  
127 different conditions on training data with high and low resolution and contaminated or not by

128 the noise. We also devote a section to the comparison of the results obtained with CNN-SegNet  
 129 code with those determined with the Gauss-Newton algorithm.

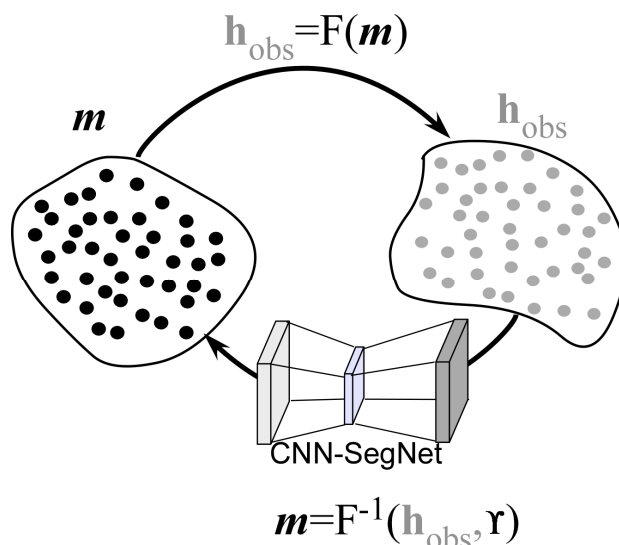
## 130 **2. Theoretical concept:**

### 131 **2.1.Preparation of Training data:**

132 In deep learning algorithm, the first step concerns the construction of training data that  
 133 will condition the accuracy of the result obtained from the predicted inversion operator (see  
 134 figure 1). Given the impossibility of having a real dataset connecting the spatial distribution of  
 135 hydraulic properties of aquifers and the piezometric responses, we use synthetic models in  
 136 which the hydraulic parameter fields are generated geostatistically and their corresponding  
 137 piezometric responses are obtained by solving the numerical groundwater equation in steady  
 138 state (forward problem).

$$139 \quad \mathbf{F}(\mathbf{m}) : \begin{cases} \nabla \cdot (T \nabla h) = q_p \delta(\mathbf{x}_p - \mathbf{x}), \\ h = h_0, \quad \Gamma_d \end{cases}, \quad (2)$$

140 where  $\mathbf{m}$  is the negative of logarithm of transmissivity  $T=10^{-\mathbf{m}}$  randomly generated and  
 141 distributed on the studied domain.  $h$  is the hydraulic head responses due to water extraction  
 142 represented by a point source term  $q_p \delta(\mathbf{x}_p - \mathbf{x})$  defined at pumping well locations  $\mathbf{x}_p$  ( $\delta$  is  
 143 Dirac delta function). The pumping tests will be done sequentially on several wells. The  
 144 forward problem is solved by finite element technique using the Comsol software with a  
 145 constant hydraulic level  $h_0$  imposed as Dirichlet boundary at all limit of the domain  $\Gamma_d$  that are  
 146 positioned far from the studied area for reducing their impact.

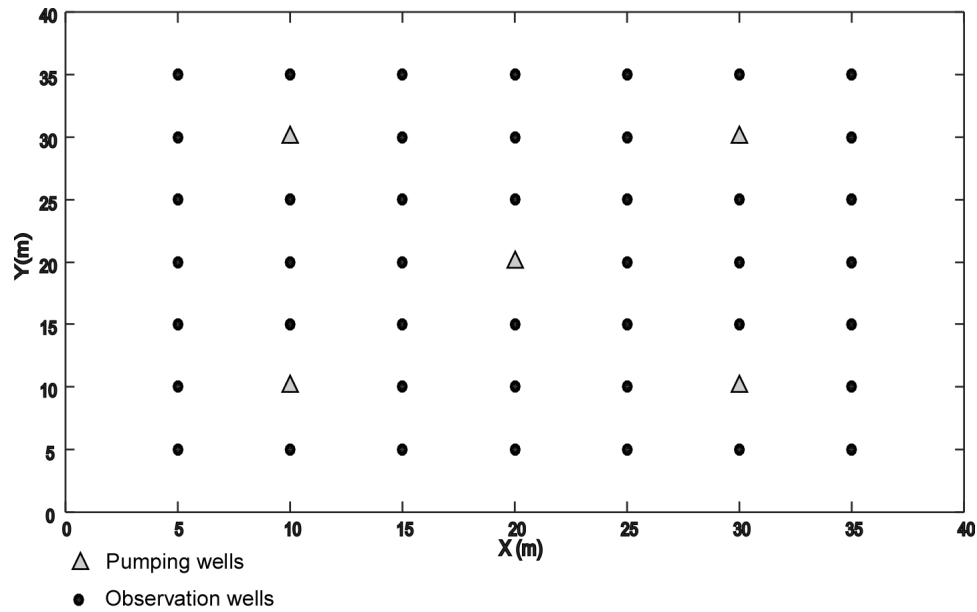


147

148 **Figure 1:** A sketch describing the concept of hydraulic tomography with the CNN-SegNet. This  
 149 concept is based on the prediction of inverse operator  $\hat{\mathbf{F}}^{-1}(\mathbf{h}_{\text{obs}}, \mathbf{Y})$  by the CNN-SegNet  
 150 algorithm using a training dataset in which the hydraulic transmissivity fields are generated  
 151 geostatistically and the hydraulic head responses due to pumping tests are obtained numerically  
 152 by solving the groundwater flow equation (Forward problem  $\mathbf{F}(\mathbf{m})$ ).

153 As the nature of the training models conditions the result of the CNN-HT inversion, we have to  
 154 use the prior information on the studied field to establish a cluster of models for training and  
 155 targeting a type of solution with certain characteristics. In this paper, prior information is  
 156 expressed in statistical terms, with a Gaussian distribution wherein the mean and covariance,  
 157 are held constants during model generation. Once the parameters of the distribution are defined,  
 158 we launch the generation of thousands of hydraulic transmissivity fields using the SGeMS code  
 159 implemented in Matlab (Remy et al., 2009). The generated transmissivity fields are then  
 160 assigned to a domain of [40 m  $\times$  40 m] on which we set up a dense distribution of wells to better  
 161 cover the heterogeneity of the field. On this configuration, we conduct 5 pumping tests in such  
 162 a way to hydraulically disturb the whole area. Each pumping test allows us to retrieve 44  
 163 measurements from the remaining observation wells; in the end, we obtain 220 measurements  
 164 for each field (see figure 2).





165

166 **Figure 2:** Position of the wells used in the generation of the hydraulic head data. 5 pumping  
 167 tests were carried out successively in the wells marked with a triangle and the responses of the  
 168 hydraulic head were collected in 44 wells with the dot symbol. This configuration provides 220  
 169 hydraulic data that are then used in the reconstruction of the hydraulic transmissivity.

170

171 To predict an inversion operator using convolutional neural networks with the encoder-decoder  
 172 architecture, we need to reshape the piezometric data into a field with the same dimension than  
 173 the hydraulic transmissivity field in order to link tow images with the same size. To do this, we  
 174 can use one of two methods:

175 (i) The first consists in interpolating the piezometric data recorded on the observation wells  
 176 for each pumping test in order to form images with the same resolution in the target  
 177 transmissivity field. In this case, the input data are composed of a number of maps equal to the  
 178 number of pumping tests. In our example, we have 5 pumping tests and we will then obtain 5  
 179 maps that will be treated jointly as 5 channels in convolutional neural networks.

180 (ii) This second is more compact than the first in which all piezometric data set recorded  
 181 for different pumping tests are introduced by a single map using a projection operation derived  
 182 from the Gauss-Newton formulation. This formulation results from the minimization of the

183 objective function presented in the introduction section ( $\frac{\partial \Psi(\mathbf{m})}{\partial \mathbf{m}} = 0$ ) (Tarantola & Valette,  
 184 1982):

$$185 \quad \mathbf{m}_{i+1} = \mathbf{m}_i + \left[ \mathbf{J}_i^T \mathbf{C}_d^{-1} \mathbf{J}_i + \mathbf{C}_m^{-1} \right]^{-1} \left[ \mathbf{J}_i^T \mathbf{C}_d^{-1} (\mathbf{h}_{\text{obs}} - \mathbf{F}(\mathbf{m}_i)) + \mathbf{C}_m^{-1} (\mathbf{m}_i - \mathbf{m}_0) \right], \quad (3)$$

186 where  $\mathbf{J}_i$  is Jacobian matrix with a size of  $N \times M$ , ( $N$  and  $M$  are number of the data and the  
 187 unknown parameters respectively). At the first iteration, usually we start with the prior model  
 188 then:

$$189 \quad \mathbf{m}_1 = \mathbf{m}_0 + \left[ \mathbf{J}_0^T \mathbf{C}_d^{-1} \mathbf{J}_0 + \mathbf{C}_m^{-1} \right]^{-1} \mathbf{J}_0^T \mathbf{C}_d^{-1} (\mathbf{h}_{\text{obs}} - \mathbf{F}(\mathbf{m}_0)), \quad (4)$$

190 and this equation can be reformulated under this following form:

$$191 \quad \mathbf{m}_1 = \boldsymbol{\beta} + \boldsymbol{\Lambda} \mathbf{h}_{\text{obs}}, \quad (5)$$

$$192 \quad \text{where} \quad \boldsymbol{\beta} = \mathbf{m}_0 - \left[ \mathbf{J}_0^T \mathbf{C}_d^{-1} \mathbf{J}_0 + \mathbf{C}_m^{-1} \right]^{-1} \mathbf{J}_0^T \mathbf{C}_d^{-1} \mathbf{F}(\mathbf{m}_0), \quad (6)$$

$$193 \quad \text{and} \quad \boldsymbol{\Lambda} = \left[ \mathbf{J}_0^T \mathbf{C}_d^{-1} \mathbf{J}_0 + \mathbf{C}_m^{-1} \right]^{-1} \mathbf{J}_0^T \mathbf{C}_d^{-1}, \quad (7)$$

194  $\mathbf{m}_1$  is the hydraulic transmissivity model resulting from the first iteration; obviously, it does not  
 195 fit the observed data, but it can be applied as a projection term for all piezometric data in order  
 196 to convert them into matrices with same size as the transmissivity fields.  $\mathbf{J}_0$  is the sensitivity  
 197 matrix of the prior homogenous model  $\mathbf{m}_0$  representing the mean of the fields. In this case, the  
 198 Jacobian can be derived analytically using the Thiem's equation for confined aquifers to express  
 199 the radial flow field and its adjoint state operator (for details, see Appendix).

200 To further simplify this projection term so that it is less dependent on prior information, we  
 201 remove the bias term  $\beta$ , replace the covariance matrices  $\mathbf{C}_m$  and  $\mathbf{C}_d$  by simple identity  
 202 matrices, now the new projection term takes this form:

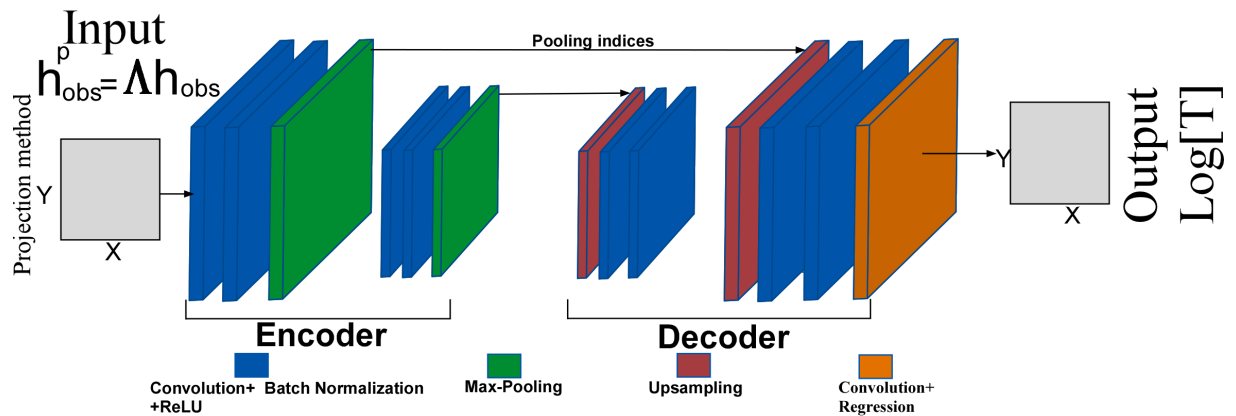
$$203 \quad \mathbf{h}_{\text{obs}}^p = \Lambda \mathbf{h}_{\text{obs}} \quad \text{With } \Lambda = \left[ \mathbf{J}_0^T \mathbf{J}_0 + \mathbf{I}_m \right]^{-1} \mathbf{J}_0^T. \quad (8)$$

204 Thanks to this simple multiplication, the hydraulic data  $\mathbf{h}_{\text{obs}}$  will be transformed into a vector  
 205  $\mathbf{h}_{\text{obs}}^p$  of the same size as the output hydraulic transmissivity field and used in matrix form as the  
 206 input field in the network. This formulation allows to link linearly  $\mathbf{h}_{\text{obs}}$  and  $\mathbf{h}_{\text{obs}}^p$  and preserves  
 207 all the hydraulic information to establish a relationship between  $\mathbf{h}_{\text{obs}}^p$  and transmissivity field.

208 The comparison of the results of the two methods (linear interpolation and projection)  
 209 in terms of inverse operator prediction is presented in the last section. However, the first  
 210 sections all applications are done with the projection method.

## 211 **2.2. Inversion by Convolutional Neural Networks with encoder-decoder structure**

212 In this article, we will explore for the first time the efficiency of SegNet architecture to  
 213 process the inversion in hydraulic tomography. The approach uses the Conventional Neural  
 214 Networks concept built with an encoder-decoder architecture (see figure 3). This approach was  
 215 originally conceived to process semantic segmentation of images by delineating the shapes of  
 216 learned objects (Badrinarayanan, et al., 2017).



217

218 **Figure 3:** SegNet Convolutional Neural Networks with encoder-decoder structure. *Modified*  
 219 *from* (Badrinarayanan, et al., 2017). Here, the piezometric data can be resized to the same  
 220 dimension of the output field using the projection method

221

222 In the encoder network, the input image is processed sequentially by a set of encoders. In  
 223 our case, we use only 2 encoders chosen after an analysis of the inversion results obtained with  
 224 networks having 1, 2 and 3 encoders. The network with 2 encoders gave the best result in terms  
 225 of training and generalization with a low sensitivity to overfitting. Each encoder starts with a  
 226 convolutional process in which multiple filters with the small size ( $3 \times 3 \times 64$ ) are convoluted on  
 227 the input layer zone by zone to identify their features (Badrinarayanan, et al., 2017). This  
 228 calculus permits to establish a local connectivity between pixels with these shared small filters,  
 229 which remain simple to predict in the training process. The convolutional operations are  
 230 followed by Batch normalization which is a linear operation that plays the role of a regularizer  
 231 in scaling the convolution outputs in order facilitate and accelerate the training (Ioffe and  
 232 Szegedy, 2015). This step is followed by the application of the ReLU function, which  
 233 introduces the nonlinearity in the process. Then, the dimensionality of the result of the previous  
 234 sequence will be considerably reduced in the max-pooling layer with stride of 2 and  $2 \times 2$   
 235 windows to keep only the main features. Therefore, the construction of feature maps in the  
 236 encoder network leads to a decrease in the spatial resolution that will be restored in the decoder  
 237 network. Indeed, the decoder network is configured in a symmetrical form with 2 decoders to

238 prepare the output by recovering the resolution lost in the decoder with up-sampling operations  
 239 from the max-pooling indices obtained in the previous max-pooling layers carried out in each  
 240 encoder (Badrinarayanan, et al., 2017). These results of up-sampling will be convoluted with  
 241 other filters to build feature maps with more details. Then the Batch normalization and ReLU  
 242 are applied successively on the convolutional outputs as in the encoder network. At the end of  
 243 the encoder network, we add a convolution layer having the same size of the output and a  
 244 regression layer to assess the performance of the training. This last layer replaces the Softmax  
 245 layer used in the original version of SegNet network for the segmentation task. The set of  
 246 weights and biases  $\mathbf{Y}$  used in the various encoder-decoder operations are chosen in way to  
 247 match the training data  $\left\{ \mathbf{m}_i, \mathbf{h}_{\text{obs},i}^p \right\}$  by minimizing this objective function:

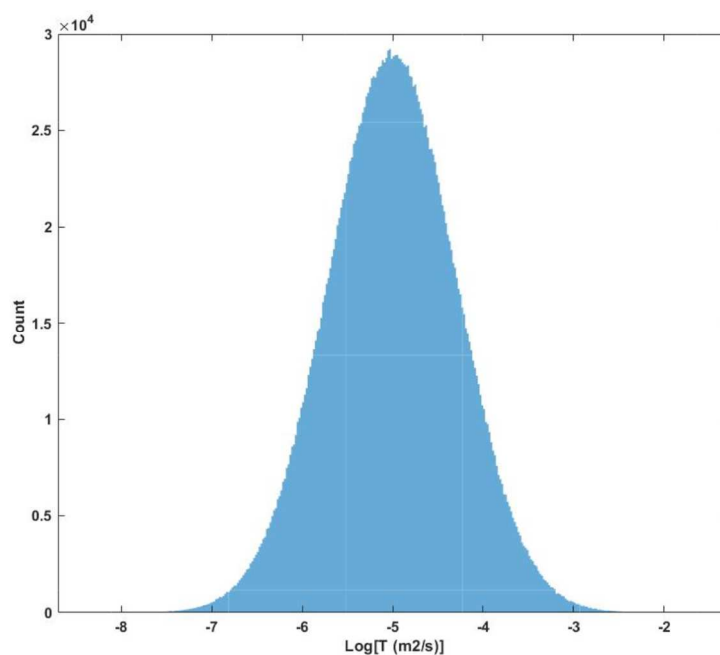
$$248 \quad \mathbf{Y} = \arg \min \left\{ \sum_{i=1}^{Nt} \left\| \mathbf{m}_i - \hat{\mathbf{F}}^{-1}(\mathbf{h}_{\text{obs},i}^p, \mathbf{Y}) \right\| \right\}, \quad (9)$$

249 where  $i$  is the sample index of the training data set with  $Nt$  as size of the training data ,  
 250  $\hat{\mathbf{F}}^{-1}(\mathbf{h}_{\text{obs},i}^p, \mathbf{Y})$  denotes the inverse function to identify by using the training data in the prediction  
 251 of the networks parameters  $\mathbf{Y}$  with ADAM optimization algorithm (Kingma and Ba, 2014).  
 252 This optimizer was run with a batch size of 70 and a learning rate starting at 0.01 and decreasing  
 253 by 0.1 every 50 epochs. The computation was performed on a workstation with (*Intel(R)*  
 254 *Xeon(R) Silver 4110 CPU @ 2.10GHz*) and 128 G of RAM with a single GPU. After training  
 255 is complete, we use a sample of generated data not used during training phase to check the  
 256 quality of the inversion results by the CNN-HT code.

### 257 **3. Applications to synthetic cases**

258 In this section, we apply the inversion concept with CNN encoder-decoder networks on  
 259 theoretical cases where the log of hydraulic transmissivity fields are generated randomly with

260 a Gaussian variogram and their hydraulic head responses by the groundwater equation as  
 261 explained in the data preparation section. This distribution has  $\mu = 10^{-5}$  (m<sup>2</sup>/s) as mean and an  
 262 isotropic Gaussian variogram with 0.5 and 10 m as variance and range respectively. With this  
 263 variogram model, we performed 11000 realizations in which the transmissivity is ranging over  
 264  $[10^{-8.3}, 10^{-1.6}]$  (m<sup>2</sup>/s); however 95% of these models have transmissivities between  $10^{-6.4}$  and  $10^{-$   
 265  $3.5$  (m<sup>2</sup>/s) (see figure 4). Then, these models are used in a forward operator that takes 2s per  
 266 simulation to get the hydraulic head responses. These realizations aim to evaluate the  
 267 effectiveness of the CNN-SegNet approach in the reconstruction of hydraulic transmissivity  
 268 field and to analyze the sensitivity of this reconstruction to the size of the training dataset and  
 269 the amount and uncertainties of the hydraulic head data collected during the pumping tests.



270

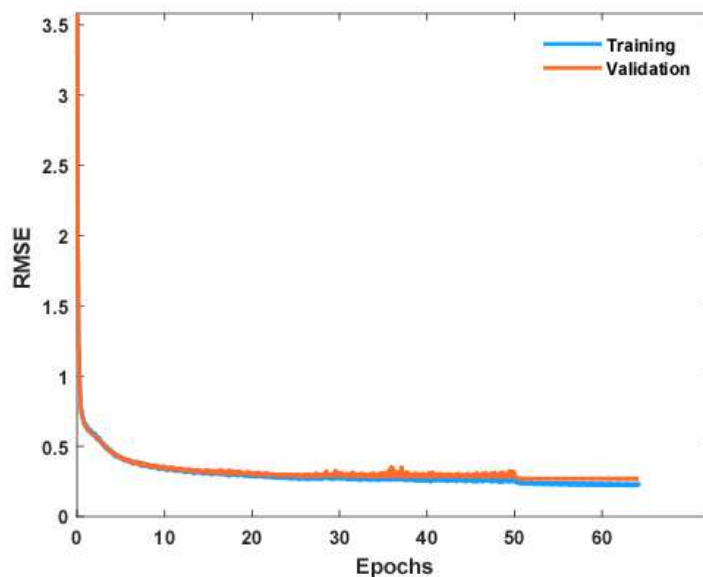
271 **Figure 4:** Gaussian distribution of the log of transmissivity fields used in the training of  
 272 CNN-SegNet network.

273

274 **3.1. Effect of training data size**

275 In the first analysis, we examine the impact of training data size on the reliability of the  
276 inverse operator prediction and determine the amount of data required to obtain a satisfactory  
277 prediction. For this purpose, we performed three assessments with different training data sizes:  
278  $T_1=1000$ ,  $T_2=3000$ ,  $T_3=10000$ , the validation was done with 1000 models and the test with  
279 500 models different from those used in the learning phase (training and validation). During the  
280 optimization process, we followed the quality of the minimization of loss functions with the  
281 training and validation data until they reached optimal values to stop the process manually or  
282 wait until the maximum number of epochs was met (see figure 5). In this way, we avoid the  
283 problem of overfitting which appears especially when we tried to build the network with a  
284 limited number of training data ( $T_1=1000$ ).

285 The results obtained with these different sizes of training data are presented in the Table 1a,  
286 in which we can easily identify through the analysis of the correlation coefficient between the  
287 real and predicted models of test sample that the accuracy of the prediction improved with  
288 increasing training data size. Indeed, the reliability of the predictive model derived from a deep  
289 learning algorithm is highly dependent on the amount of data used in the training, in which a  
290 large amount of data guarantees a good prediction. This implies a long computational time in  
291 the generation of training data, in any case the degree of accuracy desired remains linked to the  
292 nature of the problem studied.



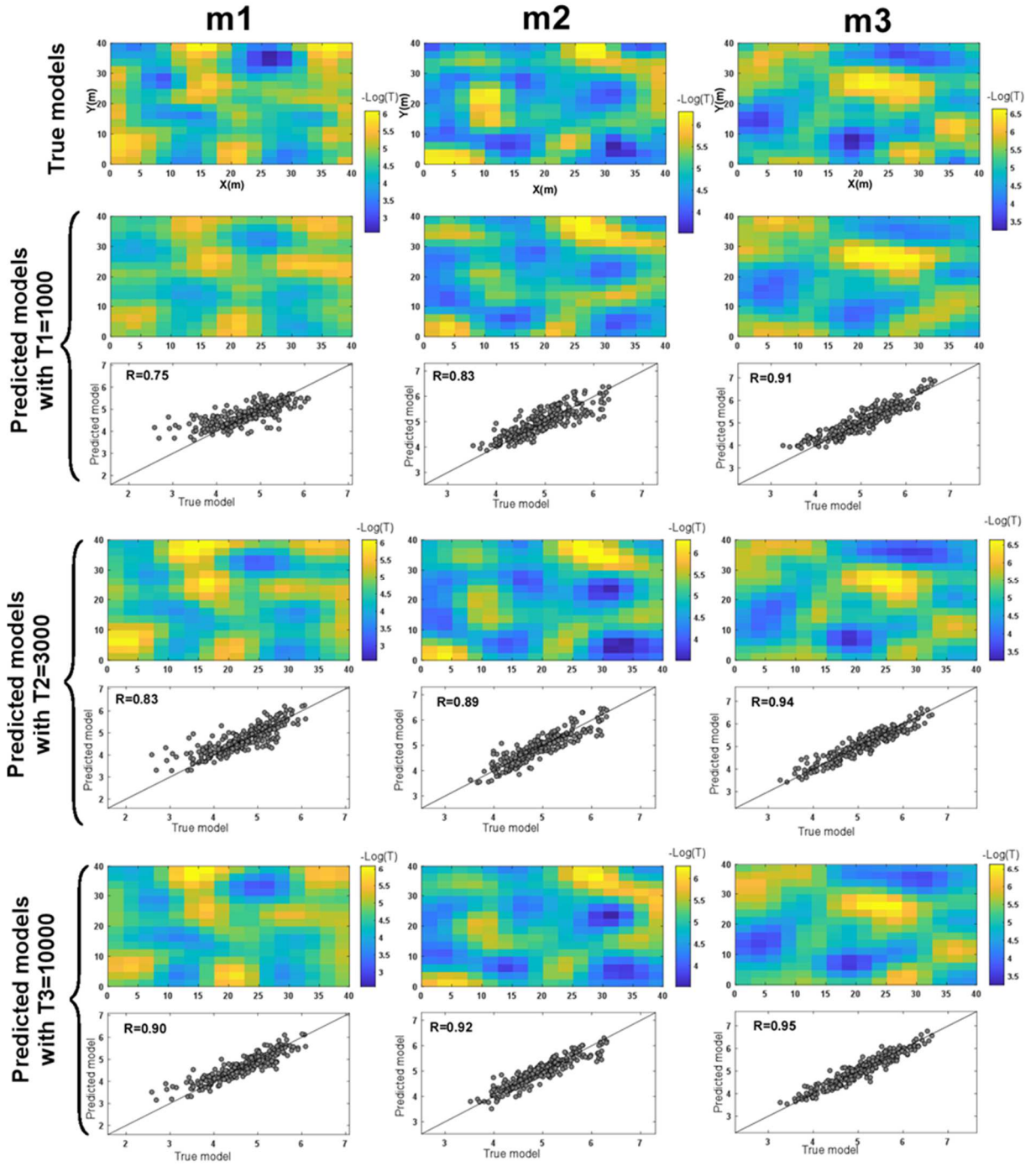
293

294 **Figure 5:** Both curves represent the quality of predictions of the network on validation and  
 295 training data (10000 models for training, 1000 models for validation) during the optimization  
 296 process.(RMSE: Root Mean Square Error).

297 In our case, training with  $T_1=1000$  data allowed us to predict 500 test models with a  
 298 correlation coefficient  $R$  between real and predicted models ranging from 0.74 to 0.93 and a  
 299 mean of 0.86 (see Table 1a). From these models, we choose 3 models: the first one (**m1**) with  
 300 a relative poor prediction ( $R=0.75$ ), a second model (**m2**) ( $R=0.83$ ) close to the mean  
 301 representing quality of the majority of predicted models, and the last one (**m3**) with a good  
 302 reconstruction ( $R=0.91$ ) (see figure 6). These models will be used to analyze the impact of  
 303 training data size on the quality of their estimations (see figure 6). The first model **m1** is  
 304 characterized by a high degree of variability with the small heterogeneities at the boundary of  
 305 the domain and a logarithm of the hydraulic transmissivity that varies between  $[-6, -2.5]$ . Over  
 306 this range, there are outliers in the Gaussian distribution of training data in particular which are  
 307 around  $-2.5$ . Thus, in this case only a few models have been seen in the training with this feature,  
 308 which explains the poor reconstruction (see figure 6). The same problem occurs in the  
 309 prediction of models with low outliers  $< -6.5$ . However, the models having variabilities well  
 310 covered by training models such as **m2** and **m3** with transmissivities values between  $[-6.5, -$



311 3.5], the reconstructions are more accurate. However, if the number of training models  
 312 increases, the models with outliers will be more frequent and will be reconsidered in the  
 313 generalization as shown in the table, where the reconstructions of these models have been well  
 314 improved.



315

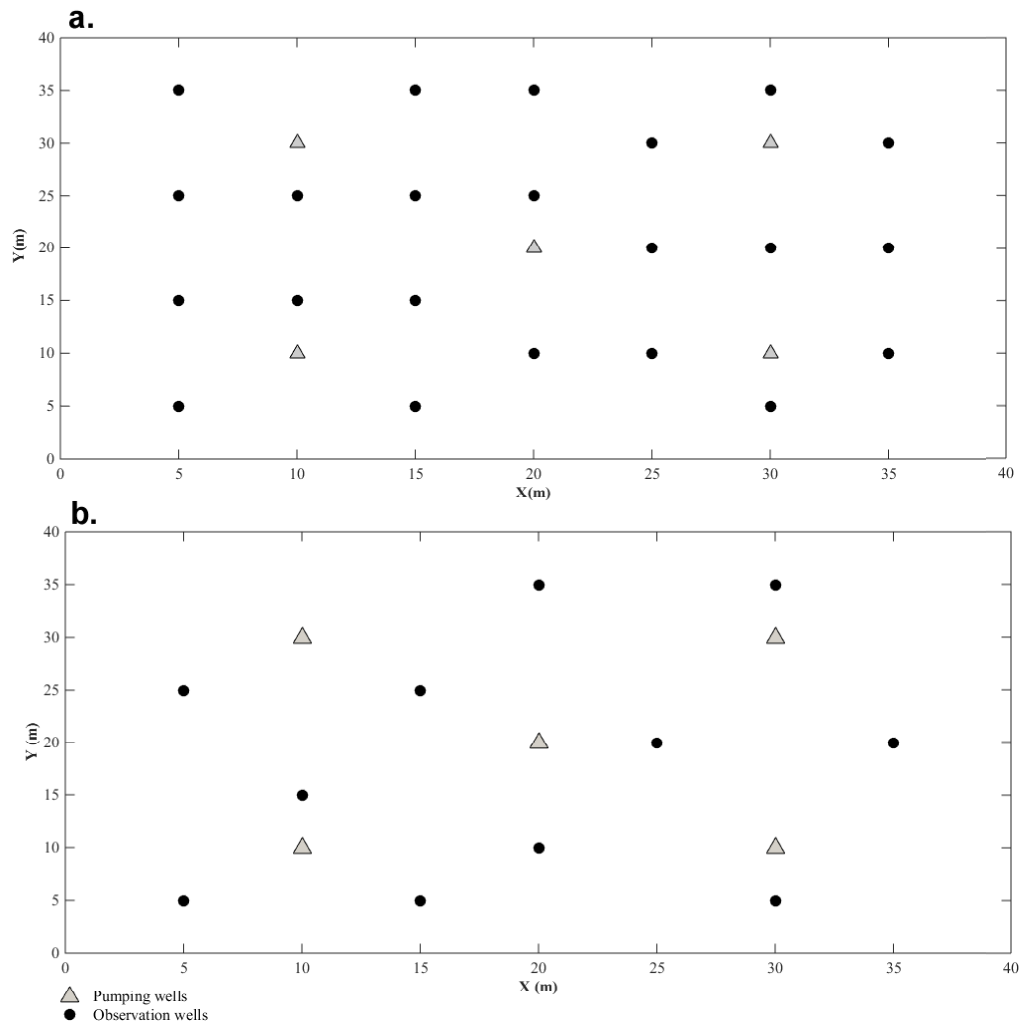
316 **Figure 6:** Three models selected from test models to illustrate the prediction quality of the  
 317 CNN-HT tool by comparing their true and predicted transmissivities. In this figure, the

318 predictions of these models were made with three different amounts of training data, in which  
319 a significant improvement in the reconstructions was obtained when the amount of training data  
320 becomes important.

321

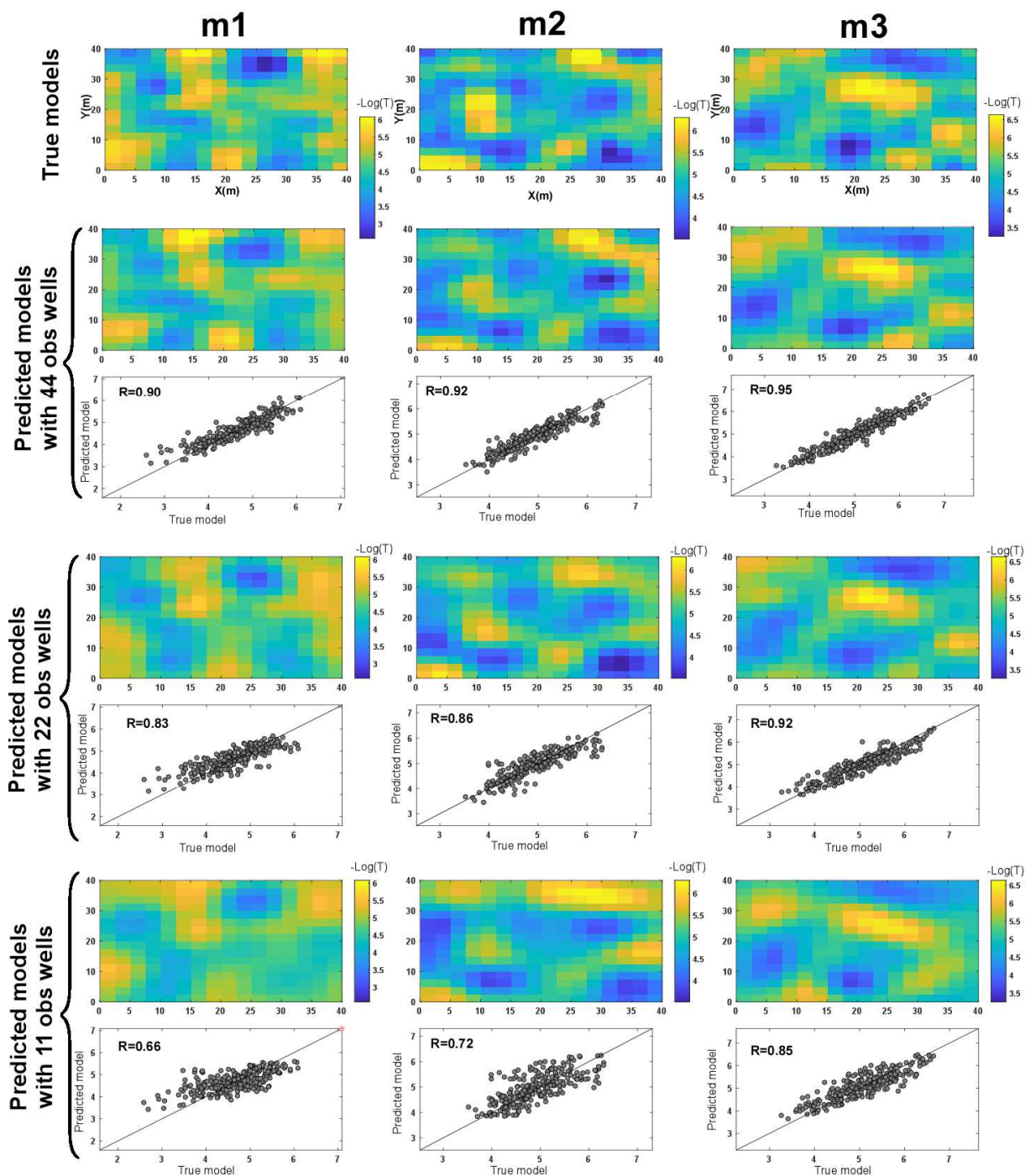
### 322           **3.2 Effect of amount of observation wells**

323           In the second analysis, we study the influence of the number of hydraulic data collected  
324 during the pumping tests on the efficiency of the network to approximate the inverse operator.  
325 In this analysis, we first reduce the number of observation wells to half (22 observation wells)  
326 and in the second step to one quarter (11 observations wells). However, we maintain the number  
327 of pumping tests and the size of the spatial discretization of the transmissivity fields ( $16 \times 16$ ),  
328 as used in previous test. These new well configurations are illustrated in the figure 7. In practice,  
329 only the hydraulic responses of abandoned wells are removed from previous training data. Thus,  
330 these data acquired with the two new observation well configurations are used as training input  
331 to determine the corresponding inverse operators, and the results obtained were compared to  
332 the previous configuration with 44 wells. The training operations were performed with 10,000  
333 models, and the tests were also done on the previous unseen samples. The reconstruction results  
334 of the test models with different series of piezometric data expressed in terms of correlation  
335 coefficients reveal a degradation of the accuracy of prediction with the decrease in the number  
336 of hydraulic data (see Table 1b). Because the number of piezometers becomes insufficient to  
337 cover all the heterogeneities and the complexities of the medium, and this also occurs with  
338 classical inversion methods. This is further confirmed by analyzing the spatial aspect of the  
339 reconstructions of models (**m1**, **m2** and **m3**), in which we observe a loss of resolution and an  
340 increase in smoothness degrees with the reduction of hydraulic data (figure 8). However, the  
341 main heterogeneities can still be identified with less data.



342

343 **Figure 7:** Two new observation well configurations were retained to study the effect of the  
 344 reduction in hydraulic data on the quality of the prediction. In the first configuration, the number  
 345 of observation wells was reduced by half (22 observation wells) and in the second, by a quarter  
 346 (11 observation wells) from the initial configuration shown in Figure 2.



347

348 **Figure 8:** Predictions of the three models are provided with a set of training data  
 349 ( $T_3=10000$ ) in which piezometric responses are collected with 3 different well configurations  
 350 (44, 22 and 11 wells). These predictions showed a degradation in the reconstructions of  
 351 hydraulic transmissivity with a decrease in the number of wells used in the survey.

352

353

354

355 **Table 1** Summarizing metric evaluations of the predictions quality (in term of correlation  
 356 coefficient) of 500 test models with correlation coefficient. **a.** Evaluation of the impact of the  
 357 size of training data (T) on predictions. **b.** Evaluation of the impact of the number of observation  
 358 wells on predictions. **c.** Evaluation of the effect of uncertainties in the piezometric data on  
 359 predictions with incorporation or not of these uncertainties in the learning process.

360

<b>a. Size of training data</b>	<b>Correlation coefficient</b>			
	<b>R</b>	<b>min</b>	<b>mean</b>	<b>max</b>
T1=1000	0.74	0.86	0.93	
T2=3000	0.77	0.90	0.95	
T3=10000	0.84	0.92	0.96	
<b>b. Number of observation data</b>	<b>Correlation coefficient</b>			
	<b>R</b>	<b>min</b>	<b>mean</b>	<b>max</b>
T3=10000, N° data=220	0.84	0.92	0.96	
T3=10000 N° data=110	0.73	0.85	0.93	
T3=10000, N° data=55	0.57	0.77	0.91	
<b>c. Degree of noise on the data</b>	<b>Correlation coefficient</b>			
	<b>R</b>	<b>min</b>	<b>mean</b>	<b>max</b>
T3=10000, N° data=220, std=0 free noise	0.84	0.92	0.96	
T3=10000, N° data=220, std=0.5 on Training and Test data	0.55	0.84	0.93	
T3=10000, N° data=220, std=0.5 only on Test data	0.49	0.79	0.92	

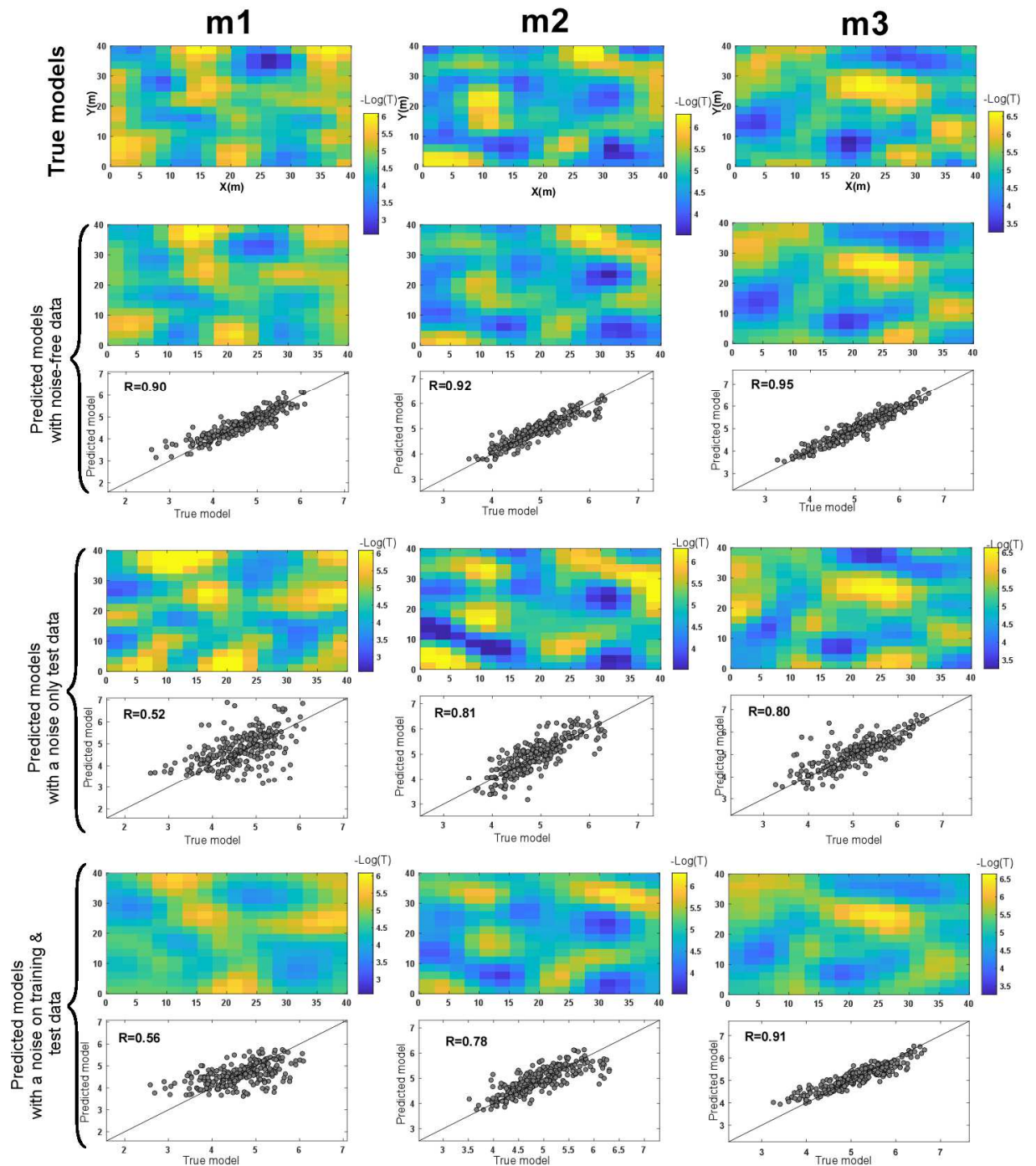
361

### 362 3.3. Effect of observation uncertainties

363 In this section, we analyze the effect of uncertainties that may be associated with  
 364 hydraulic pressure measurements collected during pumping tests. This involves contaminating  
 365 the numerical piezometric data with a random Gaussian noise of 0.5 as the standard deviation,  
 366 and verifying its impact on the quality of hydraulic field reconstructions of test models.

367           As a first analysis, we use an inverse operator established on the basis of uncontaminated  
368 training data to interpret hydraulic head data from a test set that are contaminated by noise.  
369 Here, the training operation did not take into account the noise. The result of this test is  
370 presented by the benchmark models in figure 9 and table 1c of correlation coefficients, which  
371 reveal a sharp degradation in the quality of the predictions compared to the noise-free test  
372 models, with a decrease in the correlation coefficient (from 0.92 to 0.79 mean for all test  
373 models). In the second strategy, noise is taken into account in the estimation of the inversion  
374 function by adding noise to the hydraulic data used in the learning process. This method has led  
375 to a significant improvement in the prediction quality of the test models, as shown by the three  
376 models (**m1,m2 & m3**) and the correlation coefficient noted in table 1c with the average R of  
377 the set of models increasing from 0.79 to 0.84. However, this prediction is less accurate when  
378 compared to the noise-free training and test data, as shown by the R-values. We can, therefore,  
379 conclude that uncertainties in the measured data may have an impact on the results of inversion,  
380 as is the case with classical optimization techniques, but to reduce their impacts, it is necessary  
381 to incorporate these uncertainties in the construction of the inversion model, i.e. in the learning  
382 process.





383

384 **Figure 9:** The purpose of these reconstructions is to analyze the effect of noisy data on  
 385 the efficiency of the inversion with CNN-HT by comparing 3 types of predictions: 1) with the  
 386 training and test data without noise; 2) with the training data without noise but the test models  
 387 are noisy; 3) both the training and the test models are noisy. These comparisons show that the  
 388 uncertainties in the measured data are affecting the quality of the reconstructions; however,  
 389 their impacts could be attenuated by taking into account this noise in the training process.

390

391



#### 392 **4. Comparison of CNN-HT and Gauss-Newton algorithms**

393 In this section, we compare the inversion results obtained with the CNN-HT algorithm  
394 with those determined by the traditional Gauss-Newton (GN) method. To this end, we apply  
395 both algorithms to the reconstruction of a transmissivity field with a mean of  $10^{-5} \text{ m}^2 \text{ s}^{-1}$  and  
396 generated with a spherical covariance having 0.5 and 10 m as variance and range respectively.  
397 However, this time, the models are highly discretized ( $32 \times 32$ ) compared to those used in the  
398 previous sections. The Gauss-Newton algorithm, as described in the introductory section, is a  
399 deterministic approach that relies on the minimization of an objective function (Eq. 1) by  
400 iteratively computing the Jacobian matrix until obtaining a local minimum. The calculation of  
401 the Jacobian matrix remains the most demanding part of this process, especially when the  
402 number of unknowns is important. This calculation is usually done either by the finite difference  
403 or the adjoint state technique. The finite difference method is simple to implement, but its  
404 computation is time consuming because it involves to solve the forward problem for each  
405 unknown parameters (Cardiff & Kitanidis, 2008; Fischer et al., 2017). In our case study, where  
406 the model is discretized into  $32 \times 32$  cells, the construction of the Jacobian will therefore involve  
407 solving the forward problem 1024 times. On the other hand, the adjoint state technique requires  
408 less computation, by iteratively solving an adjoint operator with the form of the forward  
409 problem on the number of observation wells, but its implementation is quite complex (for more  
410 details see Sykes et al., 1985; Cardiff & Kitanidis, 2008). In this section, we test the  
411 implementation of Gauss-Newton algorithm to invert the synthetic hydraulic data determined  
412 with a hydraulic transmissivity field shown in figure 10, (called True model) and using the  
413 piezometric coverage shown in figure 2. These data were then contaminated with noise with a  
414 variance of  $5 \text{ cm}^2$ . The inversion of these datasets was constrained, as explained in the  
415 introduction, by geostatistical parameters such as covariance and mean. The results of the  
416 inversions with the GN algorithm using the finite difference and adjoint state methods are

417 presented in Figure 10 where we can conclude that the GN algorithm succeeded in both cases  
418 in identifying the main heterogeneities of the real field with certain smoothness. In the case  
419 where the Jacobian is computed with the finite difference method, convergence was achieved  
420 after 7 iterations in 4h10min and solving the forward problem  $7 \times (1024 + 1) = 7175$  times. In  
421 contrast, the adjoint state method took only 1min24s.

422         Regarding the application of the CNN-HT algorithm, we geostatistically generate 11000  
423 transmissivity models with the same variogram as the real model used for the comparison. We  
424 keep 10000 models for the training, and the rest of the models are equally split for validation  
425 and testing. In this case study, we test two different ways of resizing the piezometric data, the  
426 first method called projection method uses the Hessian matrix to reshape the input data into the  
427 same size as the transmissivity field (output) as described in the section 2. This method requires  
428 the calculation of a Jacobian matrix with a homogeneous field for example the mean field value.  
429 The second method is based on a linear interpolation of the hydraulic data recorded during each  
430 pumping test to form 5 maps of size  $32 \times 32$  (i.e., one map for each pumping test), so the input  
431 data has the form of a matrix of 5 channels ( $32 \times 32 \times 5$ ).

432         Both approaches were used to build the SegNet networks and then tested on the test  
433 sample. The analysis of the predictions obtained on this sample shows that both methods  
434 provide almost the same prediction qualities with a small advantage for the projection method  
435 according to the correlation coefficients (see table 2). However, when we reduced the training  
436 size to  $T=500$  and  $T=2500$  models, the network using the projection method was able to provide  
437 accurate generalization while the network with interpolated data failed. We believe this is due  
438 to the fact that the interpolation of piezometric data does not preserve all the information  
439 masking the degree of variability in hydraulic responses, which makes learning difficult with  
440 few training data. In addition, using the interpolation method increases the size of the input

441 data, which requires more memory and makes the training process more cumbersome compared  
442 to the projection technique that brings the data into a compact form.

443       Regarding the application of these two networks on the hydraulic data of the real model,  
444 we find that the network built with compact data provides a better reconstruction of the spatial  
445 variability of transmissivity field with a correlation coefficient  $R=0.80$  (see figure 10).  
446 Moreover, this prediction has almost the same accuracy as the result determined by the Gauss-  
447 Newton algorithm ( $R=0.83$ ). We also mention that the time used in the GN with the finite  
448 difference method is perhaps comparable to the time spent in the DL method mainly devoted  
449 to the construction of the data set. In particular, the DL method becomes faster when we reduce  
450 the amount of training data to  $T=500$  and  $T=2500$  (20 min and 1h41 min), because even with  
451 these small samples, the transmissivity field can be well reconstructed well in a reasonable  
452 amount of time, as demonstrated by the comparison of real and inverted fields (figure10).

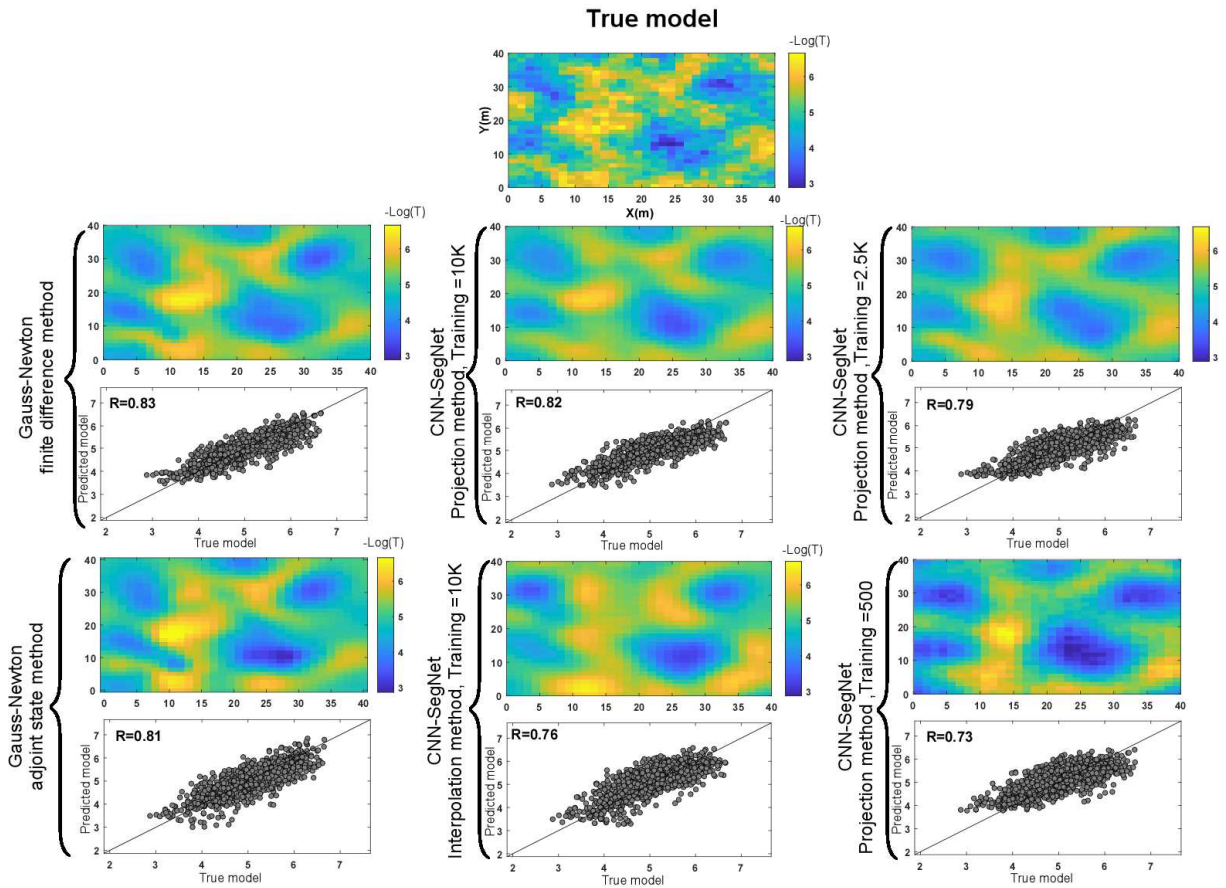
453       At the end of this comparison, we conclude that both the CNN-HT and Gauss Newton  
454 codes are based on the forward problem for which the computation time depends on the  
455 numerical method and the degree of mesh refinement used to solve the groundwater flow  
456 equation. The machine learning technique uses this forward problem to generate a training  
457 database to build the CNN network. The time spent in this construction can be equivalent or  
458 less to the time spent in finding a local minimum with the Gauss Newton using the finite  
459 difference method to calculate the Jacobian matrix over a large number of unknown parameters.  
460 However, it is difficult to have an idea in advance of the time that will be spent in building this  
461 database because the size needed to get accurate results is related to the degree of complexity  
462 of the input-output relationship and the type of network. The optimal size as well as the rest of  
463 the network parameters (epochs, number of encoders) are only established by a trial-and-error  
464 analysis, which can take a considerable amount of time. Therefore, the data size used in this  
465 paper cannot be generalized to all cases treated by this network or others.

466 Both methods also share their dependence on prior information. In the Gauss-Newton  
 467 algorithm, this prior information is introduced as a constraint to guide the optimization towards  
 468 realistic models and reduce the uncertainties associated with the non-uniqueness of the solution.  
 469 On the other hand, the CNN-HT algorithm uses it to generate the training models. The CNN-  
 470 HT offers the possibility of incorporating multiple prior models into the construction of the  
 471 training dataset, which is difficult to perform in a deterministic algorithm where the  
 472 mathematical forms of these constraints must be differentiable. In addition, both methods are  
 473 sensitive to the uncertainties associated with the measured data. The CNN-HT allows  
 474 simultaneous learning with data affected by several degrees of uncertainty by adding noise to  
 475 the training data, then combining this set with the original to double the size of the training data  
 476 and take into account the impact of noise. Regarding the quantification of the prediction  
 477 uncertainties, in the Gauss Newton method, we use the a posteriori covariance formulation to  
 478 get an idea of the uncertainty interval concerning the local minimum (Tarantola and Valette,  
 479 1982). However, in the CNN-HT algorithm it is difficult to construct an uncertainty map of the  
 480 predicted model. We can only build an uncertainty map on the model used in the test.

481 **Table 2:** We report the metric evaluations (correlation coefficient R) and computational  
 482 time of inversions obtained with the CNN-HT and Gauss-Newton algorithms. The CNN-HT  
 483 network was trained with hydraulic data introduced either by the linear interpolation method or  
 484 by the projection method. We tested 3 different sizes of training data (500, 2500, 10000), but  
 485 the network with the interpolation method failed in the training process when the data size  
 486 became small (500 and 2500). We also present the evaluation of the Gauss-Newton metric using  
 487 the finite difference method and adjoint state on a test model shown in figure 10.

		Computation Time	R True Model (figure 10)	R Test Sample min mean max		
CNN-HT	T1=500 Projection method	20min	0.73	0.54	0.69	0.79
	T2=2500 Projection method	1h41	0.79	0.65	0.77	0.86
	T3=10000 Projection method	6h	0.82	0.69	0.80	0.88
	T3=10000 Interpolation method	6h38min	0.76	0.68	0.80	0.89
GN	Finite Difference	4h 10 min	0.83			
	Adjoint state method	1 min 24s	0.81			

488



489

490 **Figure 10:** The results obtained when applying CNN-HT and Gauss-Newton algorithm  
 491 in the reconstruction of the “True model”. The results obtained with GN using the finite  
 492 difference and adjoint state methods recovered the main heterogeneities of the field. The same  
 493 reconstruction quality was obtained with CNN-HT, especially using the projection method for  
 494 the preparation of the input data. Even with a small amount of data (  $T=500$  and  $T=25000$ ), this  
 495 CNN-HT network provided satisfactory reconstructions.

496

## 497 5. Conclusion and Summary

498 In this work, we explore the relevance of a deep learning tool called the SegNet network  
 499 in the mapping of spatial variability of hydraulic parameters by interpreting the piezometric  
 500 data recorded during pumping tests. This approach relies on the approximation of the non-linear  
 501 inverse operator connecting the hydraulic head data to the hydraulic transmissivity field. This  
 502 approach is the outcome of an adaptation of the SegNet network that has been successfully  
 503 applied in the realm of image segmentation by identifying the objects studied on the pixels of  
 504 images. The SegNet is designed to establish a relationship between two images sharing the

505 same size and certain similarities, which is not the case for hydraulic head data, which are less  
506 numerous than the hydraulic transmissivity values to be estimated. To meet this point, we test  
507 the projection and interpolation methods. Projection method is based on the classical iterative  
508 formulation of a least-square method at the initial iteration to build a projection operator with  
509 the Hessian and Jacobian matrices derived from the mean value of transmissivity fields used in  
510 the training data to resize the hydraulic data into the hydraulic transmissivity field size.  
511 Interpolation method involves reshaping the data size by interpolating the hydraulic head data  
512 to form a map with the same size as the output (transmissivity field) for each pumping test. As  
513 a result, the interpolation significantly increases the size of the input data, which can slow down  
514 the numerical calculation without improving the quality of the prediction compared to the  
515 projection method, which better preserves the hydraulic information in compact form.

516         In the projection method, the operator is applied to the totality of the hydraulic pressure  
517 data used in training which are determined numerically as responses to pumping tests carried  
518 out on geostatistically generated hydraulic transmissivity fields with certain statistical  
519 properties. Once the training data has been gathered, the inverse operator is established by  
520 identifying the different filters used in the SegNet, which is composed of encoders and decoders  
521 networks. The encoder network allows, through multiple application of filters and sequential  
522 calculations of convolution, Batch normalization, ReLU and max-pooling operations to extract  
523 the main features of the input data. This encoder process induces a drop in spatial resolution,  
524 which will be recovered in the decoder network, which is designed to prepare the output with  
525 the applications of up-sampling, convolution, Batch normalization and ReLU operations. This  
526 network ends with a regression layer to evaluate the performance of the inverse operator  
527 approximation.

528         The quality of this approximation is highly dependent on the amount of data used in the  
529 training with an accurate approximation when the dataset is large. This makes the assembly of

530 the training data the most expensive step in time, requiring several hours compared to the  
531 learning process that in this case does not exceed 30 minutes. In practice, it is preferable to  
532 launch the learning with few data and check whether or not the desired accuracy is achieved  
533 before generating large amounts of data unnecessarily. In terms of comparison, the time spent  
534 in forming a training database can be equivalent to the time spent to reach a local minimum  
535 with Gauss Newton using the finite difference method to compute the Jacobian matrix

536 In addition to the amount of training data, the nature of this data also controls the  
537 prediction results of the network. Indeed, the trained network is only intended to handle certain  
538 types of models with similar characteristics to the models used in training. Thus, it is by no  
539 means a universal inversion operator, which makes the choice of the nature of the training  
540 models a crucial step that must be based on realistic prior information. In real application case,  
541 it should also be verified that the transmissivity fields used in the training allow to generate  
542 hydraulic data with the magnitude of the piezometric data observed in the field. It is also  
543 necessary to ensure that the degree of heterogeneity of the generated models can be easily  
544 mapped to the piezometric data during training process. Because if the piezometric coverage  
545 does not allow to cover transmissivity models with high heterogeneity, the learning process will  
546 fail to establish this relation. The result of the inversion with CNN-HT depends on the prior  
547 information and this particularity is also found with classical inversion techniques such as  
548 Gauss-Newton.

549 Noise can alter the information carried by the piezometric data and is a frequent problem  
550 in inversion algorithms. The deep learning algorithms are no exception, but its impact can be  
551 reduced by integrating it into the learning step by contaminating the input of training data. The  
552 number of wells and their spatial configurations used in the study also had a determining effect  
553 on the effectiveness of the CNN-SegNet inversion tool, with more reliable reconstructions when

554 these piezometers are well distributed over the study area, which was also observed in the use  
555 of conventional deterministic and stochastic methods.

556 At the end of this conclusion, we enumerate and discuss the main advantages and  
557 limitations of the proposed method. Regarding the advantages, we point out that:

558 ✓ The CNN-HT method uses the forward problem as a black box to generate the training  
559 data with user-selected transmissivity fields, thus avoiding numerical instability and outliers in  
560 hydraulic head simulations related to its solving. These numerical problems can be encountered  
561 when using conventional inversion methods (stochastic and deterministic) if they are ill-  
562 constrained.

563 ✓ Once the inverse operator is trained, the network permits to interpret the piezometric  
564 data in a few seconds without the user having to add new parameters.

565 ✓ The CNN-HT code provides predictions that require any initial hydraulic transmissivity  
566 models or computation of sensitivity matrix, as is the case with deterministic methods. This  
567 advantage makes CNN-HT an effective inversion tool for dealing with highly nonlinear inverse  
568 problems with large-scale parameterization that are difficult to handle with deterministic codes,  
569 especially when the computation of sensitivity is complex.

570 ✓ The CNN-HT code performs reconstructions of the transmissivity field that depend on  
571 the a priori model used in the generation of transmissivity field for the training data. This  
572 dependence to the a priori model is a property shared with other inversion methods. However,  
573 in this CNN-HT tool, the mathematical form of this a priori model is not subject to any  
574 condition, which is not the case with deterministic methods where the model must have a  
575 quadratic and differentiable form.

576



577 ✓ The CNN-HT algorithm also provides great flexibility for the user to incorporate  
578 multiple prior models, especially when in doubt about which a priori information to use.

579 ✓ The proposed method is based on a simple network that can be generalized and applied  
580 to handle hydraulic tomography in 3D or in time domain in order to identify the storativity  
581 coefficient as well, by only modifying the training dataset. It is also easily adaptable to deal  
582 with hydraulic tomographies with complex heterogeneities having for example multiple  
583 discontinuous hydrofacies with contrasting transmissivities, or parameterized in discrete  
584 fracture network mode. These types of problems can be treated as a classification task by  
585 replacing the regression layer of the network with the Softmax layer. In this way, the network  
586 can identify fracture geometries or hydrofacies shapes.

587 ✓ The code allows the incorporation of uncertainties on the boundary conditions imposed  
588 in the numerical solving of the forward problem by randomly choosing their values within a  
589 confidence interval when building the training data set.

590 ✓ This code, like other deep learning algorithms that use the convolutional neural network  
591 technique are able to better reduce the effects of noise in the data. This is thanks to the  
592 convolution concept that processes the input data with some local connectivity and not in a  
593 pointwise manner as in other conventional inversion methods (Vu and Jardani, 2021). The  
594 impact of noise could be minimized and accounted for in the inversion results by contaminating  
595 the hydraulic data used in the training data with a noise signal.

596 Regarding the disadvantages of this method, we mention that

597 ✓ It is based on a repetitive and time-consuming numerical resolution of the forward  
598 problem, which is involved in the construction of the training database. The size of this database  
599 conditions the quality of the predictions, and its optimal size can only be obtained by a long  
600 trial-and-error procedure.

601 ✓ The method requires important computing resources to carry out the learning.

602 ✓ The code also uses a large number of parameters that intervene in the construction of  
603 the networks (such as: size of the decoder-encoder, number of filters and their size), and in the  
604 optimization algorithm used in the training phase (number of epochs, regularization parameters,  
605 initialization of filters, Batchsize, Dropout). The determination of all these parameters implies  
606 a tedious sensitivity analysis with several training operations to choose the best parameters.

607 ✓ The method does not provide an uncertainty assessment of the predicted transmissivity  
608 fields. This is a major weakness of this method compared to classical approaches. There are  
609 some attempts in the literature to estimate the uncertainty of the prediction determined with DL  
610 algorithms by performing the learning in a probabilistic way (Kendall et al., 2016; Gal and  
611 Ghahramani, 2016). However, this type of approach requires a lot of computational time and it  
612 does not take into account all the uncertainties that can come from all the parameters used in  
613 the learning.

614 ✓ The method is highly dependent on prior information. This dependence can be an  
615 obstacle to its application, especially when the network fails in the learning phase to relate the  
616 transmissivity fields to their piezometric data. This can occur when the transmissivity fields  
617 have complex heterogeneities that are not captured by low piezometric resolution. This  
618 dependence can also lead to a misinterpretation, even with a well-trained inversion operator,  
619 when the magnitudes of the piezometric data used in the training are so different from those we  
620 desire to predict. To avoid both of these problems, it is necessary to ensure that generated  
621 transmissivity fields have a degree of heterogeneity that can be mapped with the piezometer  
622 coverage and that they produce hydraulic head data that include the magnitude of the hydraulic  
623 observations to be interpreted.

624 In conventional inversion methods, the predicted transmissivity distribution is the result of a  
625 compromise between the a priori model and the piezometric data. However, here, the result is

626 strongly linked to the a priori model and the piezometric data to be interpreted come only at the  
627 end of the learning process.

628 .

629

630 **Acknowledgements :** We thank Normandy region for its financial support to our consortium  
631 on the hydraulic characterization of aquifers.

## 632 **Appendix**

633 Here we present the mathematical formulation to analytically calculate the Jacobian matrix of  
634 a homogeneous transmissivity field. This matrix is used in the projection method to transform  
635 the hydraulic data to the same size as the transmissivity field to form the Segnet network.

636 We recall that in the joint state method, the sensitivity matrix can be expressed as an integral  
637 (Sykes et al.,1985):

$$638 \quad \frac{\partial h_i}{\partial m_j} = -\log 10 \iint_{\Omega_j} [T \nabla h \nabla \phi] d\Omega \quad ; \quad \text{A1}$$

639 Where  $h$  is the hydraulic head measured at location  $O(x_{\text{obs}}, y_{\text{obs}})$  due to a pumping source

640 placed at  $P(x_p, y_p)$  with a rate  $q_p$  .  $\phi$  is the adjoint operator with a source term  $q_{\text{adj}} = -1$

641 placed at the observation point  $O(x_{\text{obs}}, y_{\text{obs}})$  .  $T$  is transmissivity hydraulic with  $m = -\log(T)$ .

642 When the transmissivity is constant,  $h$  and  $\phi$  can be determined analytically with Thiem's

643 equation for a confined aquifer

$$644 \quad h = h_0 + \left( \frac{q_p}{2\pi T} \right) \cdot \ln \left( \frac{R}{r_p} \right) \text{ and } \phi = \phi_0 + \left( \frac{q_{\text{adj}}}{2\pi T} \right) \cdot \ln \left( \frac{R}{r_{\text{obs}}} \right) ; \quad \text{A2}$$

645 with  $r_{\text{obs}} = \sqrt{(x_{\text{obs}} - x)^2 + (y_{\text{obs}} - y)^2}$  and  $r_p = \sqrt{(x_p - x)^2 + (y_p - y)^2}$

646  $h_0$  and  $\phi_0$  are the values imposed as the boundary of the domain located at a distance  $R$ , away  
 647 from the study area.  $r$  is radial distance. Then the sensitivity of hydraulic head at point  $O$  to a  
 648 variation of the transmissivity at cell  $\Omega_j$  can be approximated with this integral:

649 
$$\frac{\partial h_i}{\partial m_j} = -\log(10) \frac{q_p q_{\text{adj}}}{(4\pi^2 T)} \iint_{\Omega_j} \frac{(x_{\text{obs}} - x)(x_p - x) + (y_{\text{obs}} - y)(y_p - y)}{r_{\text{obs}}^2 \cdot r_p^2} d\Omega \quad \text{A3.}$$

650

651 **References:**

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