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Research Paper/

# Hydraulic Characterization of Unconfined Aquifers with Oscillatory Excitations: A Numerical Analysis

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## 1 **Abstract**

2 We present a synthetic hydraulic analysis of the propagation of oscillatory perturbations in a  
3 two-dimensional saturated zone bounded above by a capillary fringe. Unsaturated flows are  
4 simulated in a physical model using a hysteretic form of Richards' equation parameterized  
5 using the van Genuchten water retention curve and the Mualem hydraulic conductivity  
6 equation for a sand, loam, and intermediate soil. Soil type affects periodic responses as a  
7 function of their hydrodynamic and van Genuchten properties. We first analyze the effect of  
8 various hydraulic or signal properties on the oscillatory responses simulated at different  
9 locations and depths. We then test the imaging potential of oscillatory responses recorded in  
10 the saturated zone to spatially characterize homogeneous or heterogeneous soils, also  
11 considering the effect of period of oscillation. We find that a separation of these responses  
12 into amplitude/phase couples permits to dissociate the characterization of different hydraulic  
13 properties. Periodic amplitudes decay with distance mainly as a function of hydraulic  
14 conductivity and phase lags decrease with frequency due to storage in the capillary fringe.  
15 Thus, responses amplitude appears to be especially sensitive to conductivity  $K$ , which allows  
16 to isolate this property from the others. Phases can then be used to better characterize the  
17 vertical flows effect, through the Van Genuchten parameters  $\alpha$  and  $n$ . Higher periods will  
18 tend to accentuate the unsaturated flows effect on the responses and can therefore be adopted  
19 for a better characterization of  $\alpha$  and  $n$ . However, using low-period responses can also be  
20 useful, as they provide better estimates of conductivity values in media with low  
21 heterogeneities. We consider this work as a useful preliminary analysis for future sandbox or  
22 field real cases applications and provide advices for such applications in the discussion.

## 23 1. Introduction

24 In hydraulic characterization of unconfined aquifers by pumping tests, identifying the impact  
25 of the capillary fringe on saturated flows remains a non-trivial task. Indeed, the unsaturated  
26 zone can contribute substantially to the flows in the saturated zone through vertical fluxes  
27 (Neuman 1972; Mishra et al. 2012; Lin et al. 2017). Therefore, a rigorous interpretation of  
28 these tests requires a quantification of these vertical fluxes and a characterization of the  
29 hydraulic properties of the unsaturated zone. In addition, a spatial characterization of the  
30 hydraulic properties (such as hydraulic conductivity) of unconfined aquifer including its cover  
31 can substantially improve our comprehension of the processes of recharge and contaminant  
32 transport in the critical zone. This can be performed with a hydraulic tomography approach in  
33 which a physical numerical model is used in an inversion process to retrieve the hydraulic  
34 properties by fitting hydraulic head measurements. Usually, in such approach, these hydraulic  
35 heads are measured simultaneously in multiple wells and at multiple depths in response to  
36 cross-wells perturbations (Yeh and Lee 2007; Yeh et al. 2013; Liu et al. 2002; Zhu and Yeh  
37 2005).

38 In the literature, several hydraulic tomography of unconfined aquifers have already been  
39 performed on field or laboratory applications. In particular on field cases applications, the  
40 Boise Hydrogeophysical Research Site (BHRS), has been particularly studied (Cardiff et al.  
41 2009; Cardiff and Barrash 2011; Cardiff et al. 2012; Cardiff et al. 2013a; Rabinovich et al.  
42 2015; Hochstetler et al. 2016; Cardiff et al. 2019). However, in these applications, the authors  
43 considered that the drainage occurring during their tests was instantaneous and did not  
44 generate delayed responses effect. Thus, they simulated only the saturated flows in the aquifer  
45 in order to characterize the conductivity and specific storage fields. It is shown that this  
46 approximation is acceptable when a specific yield value is employed for the drainage in the  
47 early-time pumping (Cardiff and Barrash 2011; Cardiff et al. 2012). However, to apply this

48 method, the effective porosity must trigger instantaneous drainage at the beginning of the  
49 pumping or the effective porosity must contribute only for a negligible vertical flux compared  
50 to the horizontal fluxes generated by pumping (Cardiff and Barrash 2011). Mao et al. (2013a)  
51 have presented a theoretical analysis of the application of hydraulic tomography on an  
52 unconfined aquifer also taking into account hydraulic properties related to the unsaturated  
53 flows (pore-size distribution parameters and saturated water content). This approach has been  
54 successfully applied theoretically and experimentally in sandboxes with an analysis of  
55 temporal drawdown curves in response to constant-rate pumping (Mao et al., 2013b; Zhao et  
56 al. 2015).. However to date a real field case application following this method has not been  
57 presented. Therefore further recommendations have been provided for adapting such process  
58 in real applications in Mao et al. (2013a) and Zhao et al. (2015). The major concerns in its  
59 application remains effect of noise on the data and the necessity to assess a sufficient amount  
60 of information on both the saturated flows (rather horizontal) and the unsaturated flows  
61 (rather vertical) in the dataset.

62 Oscillatory pumping tests can improve the quality of hydraulic data and thus the  
63 characterization of unconfined aquifers, including the effect of the marginal capillary zone.  
64 During the last decades, applications of oscillatory excitations in the identification of the  
65 hydraulic properties of aquifers have been widely explored, both theoretically and  
66 experimentally (Black and Kipp 1981; Rasmussen et al. 2000; Rasmussen et al. 2003; Renner  
67 and Messar 2006; Fokker et al. 2013; Fischer et al. 2018; Salina Borello et al. 2019; Cardiff et  
68 al. 2019; Fischer et al. 2020a; Fischer et al. 2020b). The imaging potential of such data has  
69 been studied in Cardiff et al. (2013b) and their advantages in field application have been  
70 discussed in Bakhos et al. (2014) in terms of ability to easily eliminate noises from the  
71 measurements and in Cardiff and Barrash (2014) for configuring the period of oscillation in  
72 order to investigate different flowpaths in the aquifer.

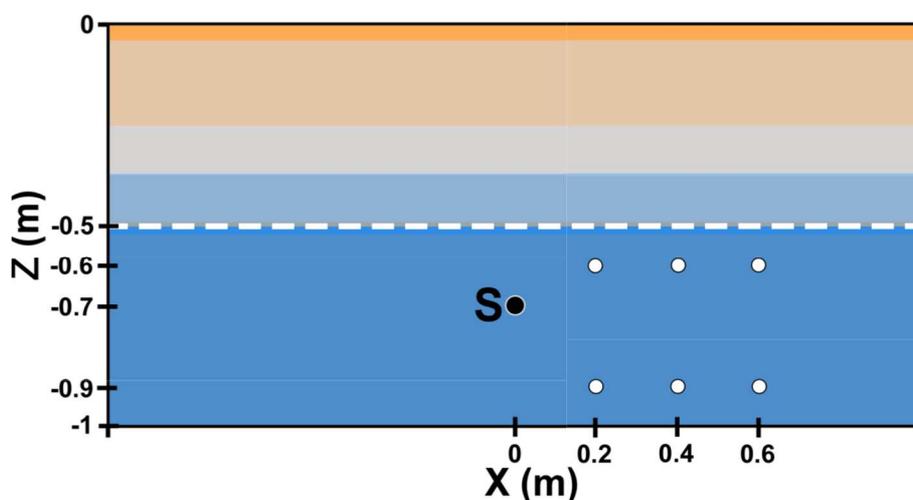
73 The propagation of oscillatory signals in soils under conditions of variable saturation has been  
74 analyzed in several works with numerical data (Revil et al. 2008; Shoushtari et al. 2015) or  
75 laboratory data (Cartwright et al. 2003; Cartwright et al. 2004; Cartwright 2014). All these  
76 works discuss the effect of hysteresis, existing for unsaturated flows and having an impact on  
77 the oscillatory responses. This hysteresis effect is due to the wetting and dewatering processes  
78 of soils that generate loops in the water content curves. These scanning loops can be modeled  
79 with hydrodynamic-dependent unsaturated flows parameters. Effects of air entrapment during  
80 soils wetting, can also temporarily affect the saturated water content value (Kool and Parker  
81 1987).

82 Several analytical solutions have recently been proposed for the estimation of soils hydraulic  
83 property values in unconfined aquifers from oscillatory signal responses (Dagan and  
84 Rabinovich 2014; Huang et al. 2018; Huang et al. 2019; Cheng and Renner 2018). However,  
85 it is still rare for spatial variability to be taken into account in characterization applications  
86 considering both the saturated and unsaturated hydraulic properties. Furthermore the  
87 information brought by oscillatory responses for the spatial characterization of unconfined  
88 aquifers still needs to be investigated. With this work our aim is to further investigate the  
89 effect of oscillatory flows generated at the limit of the saturated zone in an unconfined  
90 aquifer, in order to propose a preliminary analysis of this type of data for future  
91 characterization application, in particular hydraulic tomography. For this purpose we  
92 simulated flows associated to extraction/injection oscillations in a synthetic sandbox with a  
93 physical numerical model. We first present an analysis of the effect of different soil properties  
94 on the oscillatory responses. Then we focus more particularly on the effect of conductivity  $K$   
95 and the unsaturated Van Genuchten parameters  $\alpha$  and  $n$  and study the possibility to  
96 spatially characterize these properties with the information carried in the oscillatory

97 responses. Finally we discuss our analysis and imaging results in order to provide some  
 98 advices for future works of hydraulic tomography using this type of data.

## 99 2. Presentation of the model

100 We developed a physics-based model of an unconfined soils with variable saturation with  
 101 COMSOL Multiphysics (see Figure 1). This model represents a 2-D side-views of a synthetic  
 102 sandbox at a metric scale: 3m wide for 1m depth. We considered the initial water table to be  
 103 50 cm below the surface (thus, initially, half of the model is saturated and the other half  
 104 unsaturated). The modeled sandbox is equipped with a pumping/injection point (hereafter  
 105 noted S) and measurement axes at distances of 20, 40 and 60 cm from the pumping axis, in  
 106 which hydraulic pressure responses are simulated at different depths of the saturated zone (60  
 107 and 90 cm).



108

109 Figure 1: Schematic representation of our side-viewed models of an unconfined aquifer.  
 110 Surface is at  $Z=0$  cm and the water table at  $Z=-50$  cm. The white circles represent the  
 111 pressure simulations points while the black circle represents the perturbation source point  
 112 (also used as pressure measurement). Orange color represents the soil, light blues the  
 113 unsaturated fringe and blue the saturated zone of the aquifer.

114

115 In order to take into account the capillary fringe effect on the saturated responses, we  
 116 simulated water flows with Richards' formulation of the continuity equation for a 2D side-  
 117 viewed temporal model:

$$118 \quad (C_m + S_e \cdot S) \frac{\partial h}{\partial t} + \nabla \cdot (-K \cdot k_r (\nabla H_p + \nabla z)) = q, \quad (1)$$

119 where  $t$  is time (s),  $H_p$  is the hydraulic pressure (m) at a given depth  $Z$  (m),  $K$  is the  
 120 hydraulic conductivity (m/s),  $S$  is the storage coefficient ( $m^{-1}$ ) related to the soil  
 121 compressibility,  $S_e$  denotes the effective saturation (-),  $C_m$  is the specific moisture capacity  
 122 ( $m^{-1}$ ),  $k_r$  is the relative permeability (-) and  $q$  is the Darcy's velocity at the pumping  
 123 locations (m/s). We consider that no vertical flows occur at the upper boundary of the  
 124 sandbox during the simulations. The other boundaries are impervious, implying a finite  
 125 volume of water in the sandbox (no inlet or outlet). The flows in the model are then solved  
 126 within an adaptive-size triangular mesh with a finite-element method.

127 This model simulates, in time, the hydraulic pressures in the variably saturated soil in  
 128 response to a sinusoidal oscillating perturbation, injecting and extracting successively a same  
 129 amount of water.

$$130 \quad Q = -Q_A \cdot \sin\left(\frac{2\pi}{Pe} \cdot t\right) \quad (2)$$

131 where  $Q$  is the extraction/injection flowrate ( $m^3/s$ ),  $Q_A$  is the oscillating flowrate amplitude  
 132 ( $m^3/s$ ) and  $Pe$  is the oscillating flowrate period (s).

133  $S_e$ ,  $C_m$  and  $k_r$  are calculated with the Van Genuchten equations. These equations are  
 134 dependent on the groundwater pressure  $H_p = h - z$ ; i.e. if the water, at a given location  $(x, z)$   
 135 , is at an atmospheric pressure or not:

$$136 \quad \left\{ \begin{array}{ll} S_e = \frac{1}{\left(1 + |\alpha \cdot H_p|^n\right)^{1-1/n}} & \text{if } H_p < 0 \\ S_e = 1 & \text{if } H_p \geq 0 \end{array} \right. \quad (3)$$

$$137 \quad \left\{ \begin{array}{ll} C_m = \frac{\alpha \left(1 - \frac{1}{n}\right)}{1 + \frac{1}{n}} \cdot (\theta_s - \theta_r) \cdot S_e^{\frac{1}{1-1/n}} \cdot \left(1 - S_e^{\frac{1}{1-1/n}}\right)^{1-1/n} & \text{if } H_p < 0 \\ C_m = 0 & \text{if } H_p \geq 0 \end{array} \right. \quad (4)$$

$$138 \quad \left\{ \begin{array}{ll} k_r = S_e^\ell \cdot \left(1 - \left(1 - S_e^{\frac{1}{1-1/n}}\right)^{1-1/n}\right)^2 & \text{if } H_p < 0 \\ k_r = 1 & \text{if } H_p \geq 0 \end{array} \right. \quad (5)$$

139 where  $\alpha$  ( $m^{-1}$ ),  $n$  (-) and  $\ell=0.5$  (-) are the Van Genuchten parameters and  $\theta_s$ ,  $\theta_r$  are the  
140 saturated and residual water content (-).

141 A hysteretic effect occurs in unsaturated soils, inducing a difference in the dewatering and the  
142 wetting of the soil. In this study, a spatial and temporal frequency-dependent hysteresis effect  
143 is implemented in the model, based on the formulation of Revil et al. (2008) for periodic  
144 sources and the observations made in Shoushtari et al. (2015) that hysteresis effects attenuates  
145 with distance to the signal source. The hysteresis is generated by computing dynamic values  
146 of  $\alpha$  and  $n$ , following the induced periodic movement, over space and time:

$$147 \quad \alpha(t) = \alpha_{\text{mean}} \times \left[ 1 - \left( M_{\text{hyst}} - \min\left(\frac{|z_p - z|}{z_{\text{hyst}}}, 1\right) \cdot M_{\text{hyst}} \right) \cdot \sin\left(\frac{2\pi t}{Pe}\right) \right] \quad (6)$$

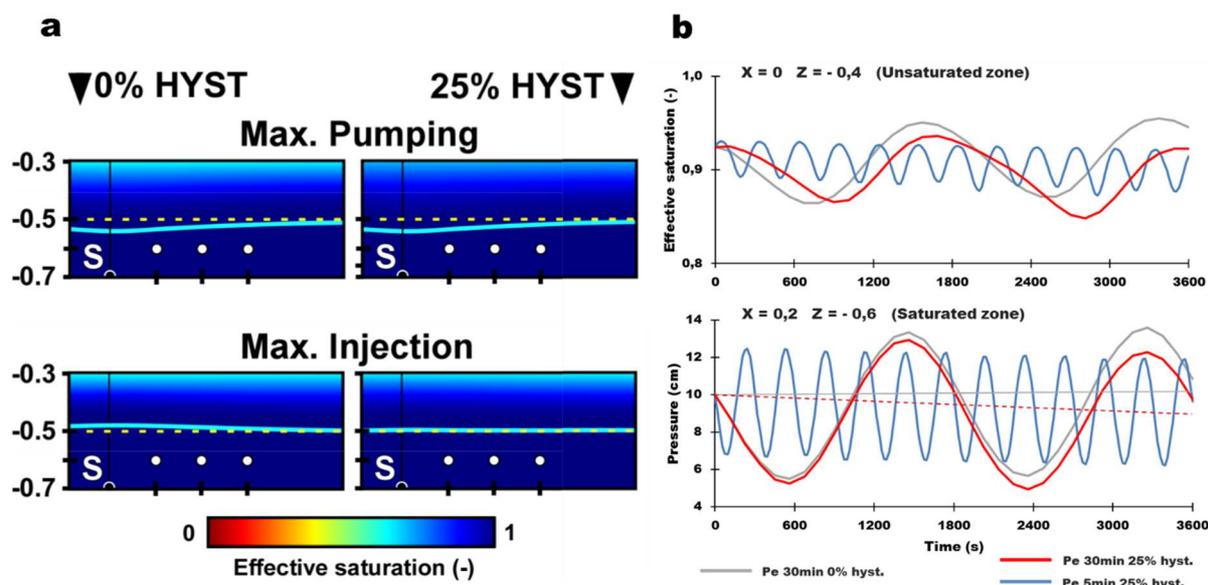
148 where  $M_{\text{hyst}}$  is the magnitude of the hysteresis effect (%),  $\alpha_{\text{mean}}$  represents the mean value of  
149  $\alpha$  ( $1/m$ ),  $z_p$  is the pumping depth (m) and  $z_{\text{hyst}}$  is the distance of attenuation of the hysteresis

150 effect (hereafter 50 cm). A similar formulation is also used to compute  $n(t)$ , considering a  
 151  $n_{\text{mean}}$  value (-).

152 We briefly present in Figure 2 the effects of the simulated hysteresis on the saturation level  
 153 and the simulated pressures in our model. After one cycle of pumping/injection the saturation  
 154 level will not come back to its origin when incorporating a hysteresis effect. A part of the  
 155 mobilized water will remain in the unsaturated zone, generating a lowering of the saturation  
 156 level. After several cycle, the effect generates a global decrease of the saturated/unsaturated  
 157 interface. In the unsaturated zone, the shape of the signal becomes deformed by the hysteresis  
 158 effect (especially observable for the period of 30 min), wherein it cannot be considered as a  
 159 sinusoidal form anymore. The effect of the period of oscillation is also more important in the  
 160 unsaturated zone, where the oscillatory response tends to vanish faster (in terms of amplitude)  
 161 than in the saturated zone when the period of the signal is decreased. In the saturated zone, the  
 162 hysteresis effect does not seem to modify the shape of the signal directly. Thus the pressure  
 163 responses in the saturated zone contain a sinusoidal part ( $h_{\text{osc}}$ ) in response to the oscillating  
 164 signal that can be written:

$$165 \quad h_{\text{osc}} = h_A \cdot \sin\left(\frac{2 \cdot \pi}{\text{Pe}} \cdot t - h_p \cdot \frac{\pi}{180}\right) \quad (7)$$

166 where  $h_A$  is the oscillating response amplitude (m) and  $h_p$  is the oscillating response phase  
 167 lag to the signal source ( $^\circ$ ). These oscillatory parts can be extracted from the head responses  
 168 through a Fast Fourier Transform (FFT) directly in terms of timeless amplitudes ( $h_A$ ) and  
 169 phases ( $h_p$ ).



170

171 Figure 2: (a) Effect of simulated 25% hysteresis on the soil effective saturation and the  
 172 saturation level (cyan line) during one cycle of pumping and injection. The initial saturation  
 173 level is indicated as a dotted line. (b) Effect of the hysteresis on the effective saturation  
 174 simulated at position (0;-0.4) in the unsaturated zone and the pressure simulated at position  
 175 (0.2;-0.6) in the saturated zone after several cycles of injection/pumping and for different  
 176 periods of oscillation (5 and 30 min)

177

178 We simulated and observed the oscillatory responses in the saturated zone for different types  
 179 of soils, by modifying the property values inputs in the model. Hereafter in this work we  
 180 principally compare these responses in sand, in loam or an intermediate soil with properties  
 181 between sand and loam ones. The hydraulic property values employed for these two  
 182 configurations are presented in Table 1 and were chosen mainly according to the UNSaturated  
 183 SOil hydraulic DATabase (UNSODA) in Leij et al. (1996) and to previous works in the  
 184 unconfined aquifers literature.

185

186

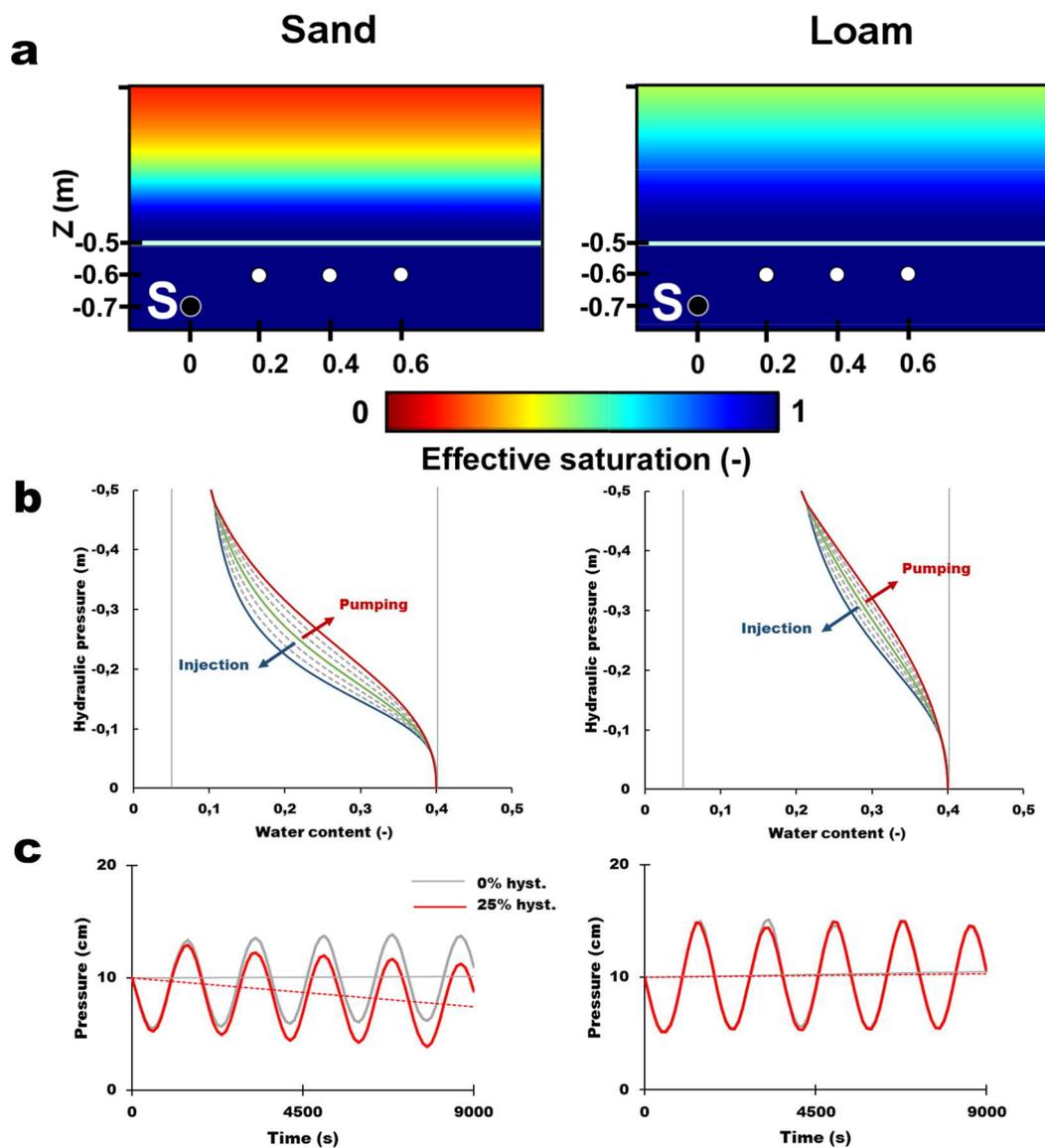
187 Table 1: Property values used in our model for the two types of soils and the intermediate soil  
 188 considered.

	K (m/s)	Anisotropy ( $K_x / K_z$ )	$\alpha$ (1/m)	n (-)
'Loam' soil	$1.10^{-5}$	4	3.2	2.4
'Intermediate' soil	$5.10^{-5}$	3	4.0	2.7
'Sand' soil	$1.10^{-4}$	2	5.0	3.0

189

190 For the hydrogeological soil properties we chose constant values, also based on what can be  
 191 usually found in the literature:  $\theta_r = 0.05$ ,  $\theta_s = 0.40$  and  $S = 10^{-5} \text{ m}^{-1}$ . We considered a  $M_{hyst} =$   
 192 25 % magnitude hysteresis effect in the unsaturated zone, based on the recommendations in  
 193 Kool and Parker (1987).

194 Figure 3 presents the differences between our simulated sand and loam soils in terms of  
 195 unsaturated flows (i.e. effective saturation profiles, water content curves and hysteresis effects  
 196 on the responses in the saturated zone). It can be noted that this hysteresis effect, shown in  
 197 Figure 2, is more important in sand than in loam.



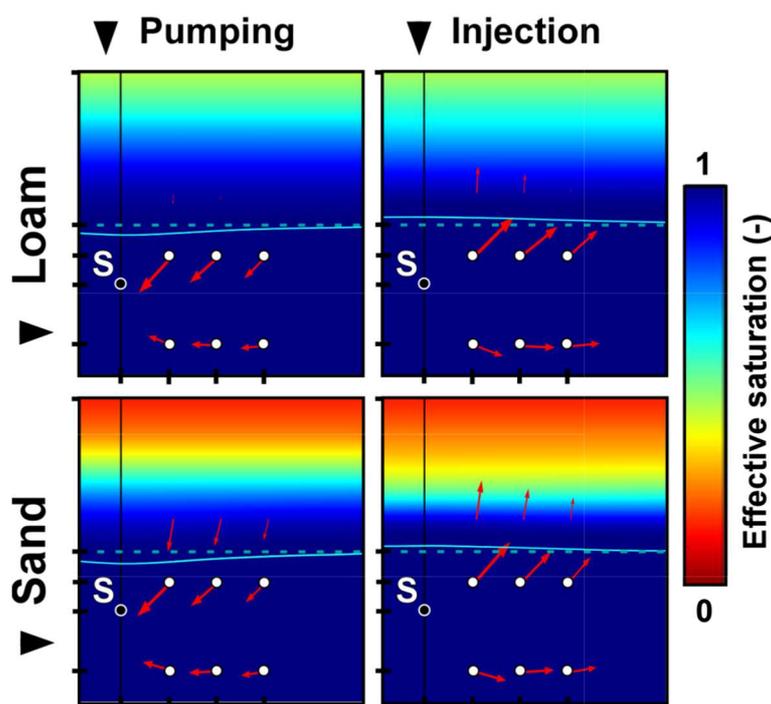
198

199 Figure 3: (a) Effective saturation in sand and loam between the surface and the saturated zone,  
 200 at 50cm depth. (b) A spatial and temporal-evolving hysteresis effect ('scanning loop') has  
 201 been taken into account in the water content curves during the successive pumping and  
 202 injection phases. (c) This hysteresis effect does not substantially affect the oscillation  
 203 response, but rather the mean pressure level, especially in sands (here the pressures simulated  
 204 at position (0.2;-0.6) with 0% and 25% hysteresis effects for a period of oscillation of 30  
 205 min).

206

207 **3. Comparative analysis of the effect of various soil properties on the**  
 208 **oscillatory responses**

209 By simulating flows during pumping and injection phases in sand and loam (Figure 4), we can  
 210 clearly observe the different types of flows that will be mobilized in these two configurations.  
 211 Vertical flows will occur in the unsaturated zone, horizontal in depths of the saturated zone,  
 212 and mixed vertical/horizontal flows near the saturated/unsaturated interface. It can also be  
 213 observed from the arrows sizes (proportional to the flow intensity) in the unsaturated zone in  
 214 Figure 4 that vertical flows are, in proportion, more important in sand than in loam.

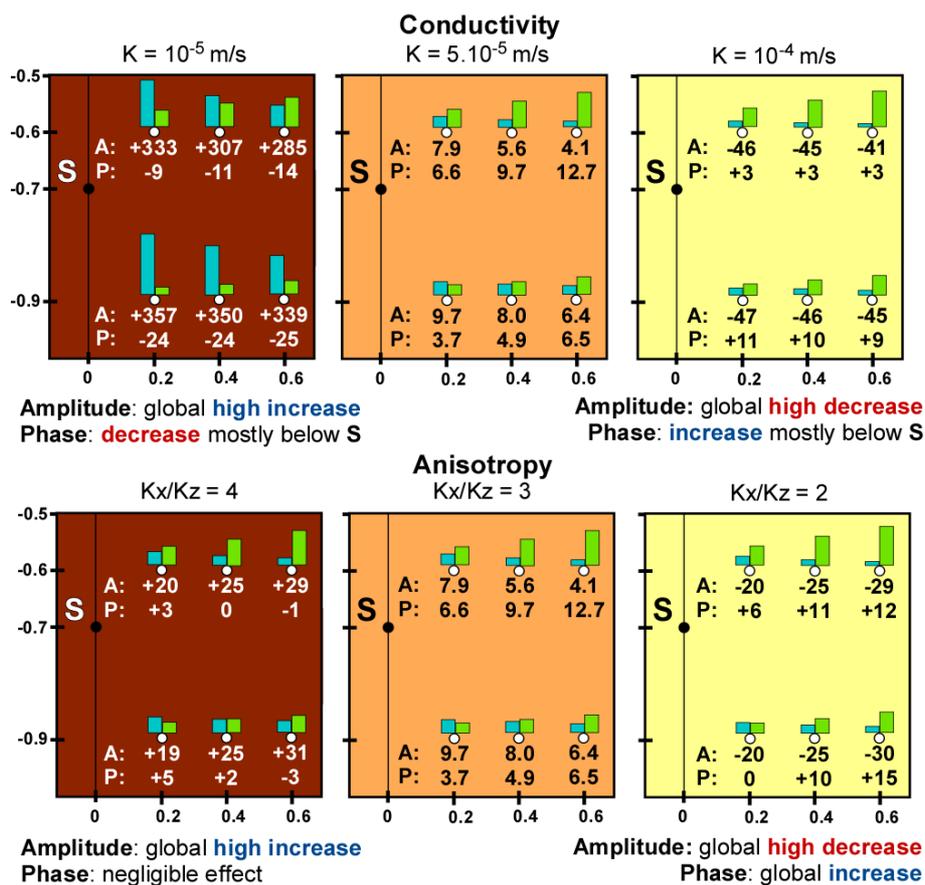


215  
 216 Figure 4: Flow velocity vectors (red arrows) in a log10 scale when pumping and injecting in  
 217 loam or in sand. Current and initial saturation levels are indicated as blue lines and dotted  
 218 lines respectively.

219  
 220 We first evaluated how oscillatory responses (i.e., amplitudes, phase lags) are affected by  
 221 variations of individual parameters values for the intermediate soil (Table 1) using a pumping  
 222 amplitude of  $Q_A = 1$  mL/s and period of oscillation  $Pe = 15$  min.

223 In Figure 5, we present the variations induced in the amplitude and phases responses at the  
 224 different locations when modifying the conductivity and anisotropy values of our  
 225 ‘intermediate’ soil (to values corresponding rather to ‘sand’ soil ones or rather to ‘loam’ soil

226 ones). The absolute responses (presented for the ‘intermediate’ soil in the middle column of  
227 the figure) globally show an amplitude decrease and phase increase with the radial distance to  
228 the perturbation source point S. However the oscillation better propagates in the deep  
229 saturated zone (below S) than near the unsaturated one, as amplitudes are higher and phases  
230 lower below S, even if these simulation points are more distant to the source than the ones  
231 above S. It clearly appears that the oscillatory responses are affected by the conductivity and  
232 its anisotropy, especially in terms of amplitudes. For a similar perturbation the amplitude  
233 responses will be higher in loam than in sand. Oppositely, the phases will become globally  
234 lower when applied in loam and higher in sand. However, in this case the conductivity seems  
235 to more particularly affect the phases of responses associated to horizontal flows (below S).  
236 The degree of anisotropy will have a similar but less important effect than conductivities on  
237 amplitude responses, becoming more important in loam with higher anisotropy. The phase  
238 responses are especially affected when lowering the anisotropy, generating higher phases in  
239 sand when moving away from the perturbation source.



240

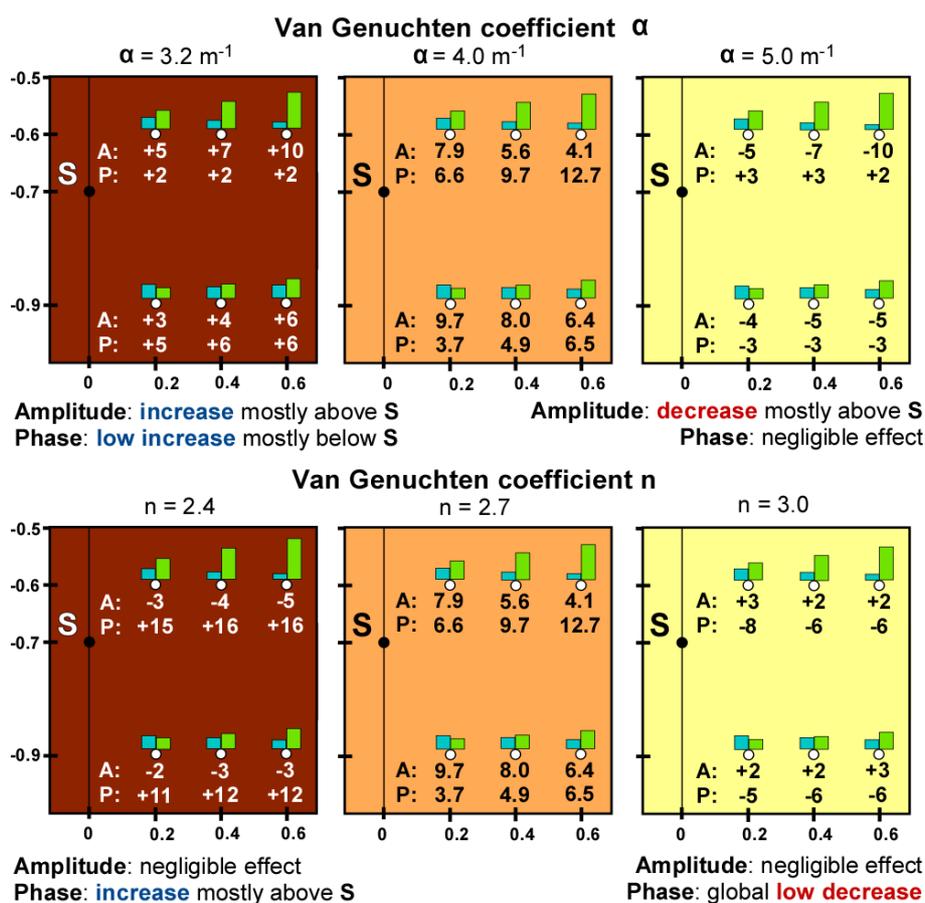
241 **Figure 5:** Lateral columns: spatial variations (in %) of amplitudes (blue, A) and phases (green, P) at each measurement point and for the saturated flows properties ( $K$  and anisotropy) 242 243 244 245

245

246 Figure 6 presents a similar analysis with the Van Genuchten parameters  $\alpha$  and  $n$ , associated 247 248 249 250 251 252 253 254

246 Figure 6 presents a similar analysis with the Van Genuchten parameters  $\alpha$  and  $n$ , associated 247 to the unsaturated flows. These parameters also affect the oscillatory responses, but in a 248 different manner than conductivity. Parameter  $\alpha$  appears to affect the differences in the 249 responses above and below S. In fact, its modification will generate more important variations 250 in terms of amplitude above S (denoting an effect on the vertical flows impacts). It will also 251 affect the phase responses but rather reducing the vertical differences in loam, and 252 accentuating it in sand, thus, again, generating responses variations vertically. Parameter  $n$ , in 253 contrary, generates variations affecting essentially the phases of the responses, accentuating 254 them and their vertical differences in loam and lowering them in sand. The phase variations

255 remain constant with distance to S. This suggests a more local impact at the perturbation point  
 256 that is then propagated to the other responses. As for the previous conductivity analysis, the  
 257 effects on the responses induced by  $\alpha$  and  $n$  are opposites when tending to one or another  
 258 sand or loam property value. This already suggest that quantitatively analyzing the responses  
 259 spatially and vertically could allow to assess a soil's saturated and unsaturated hydraulic  
 260 property values.



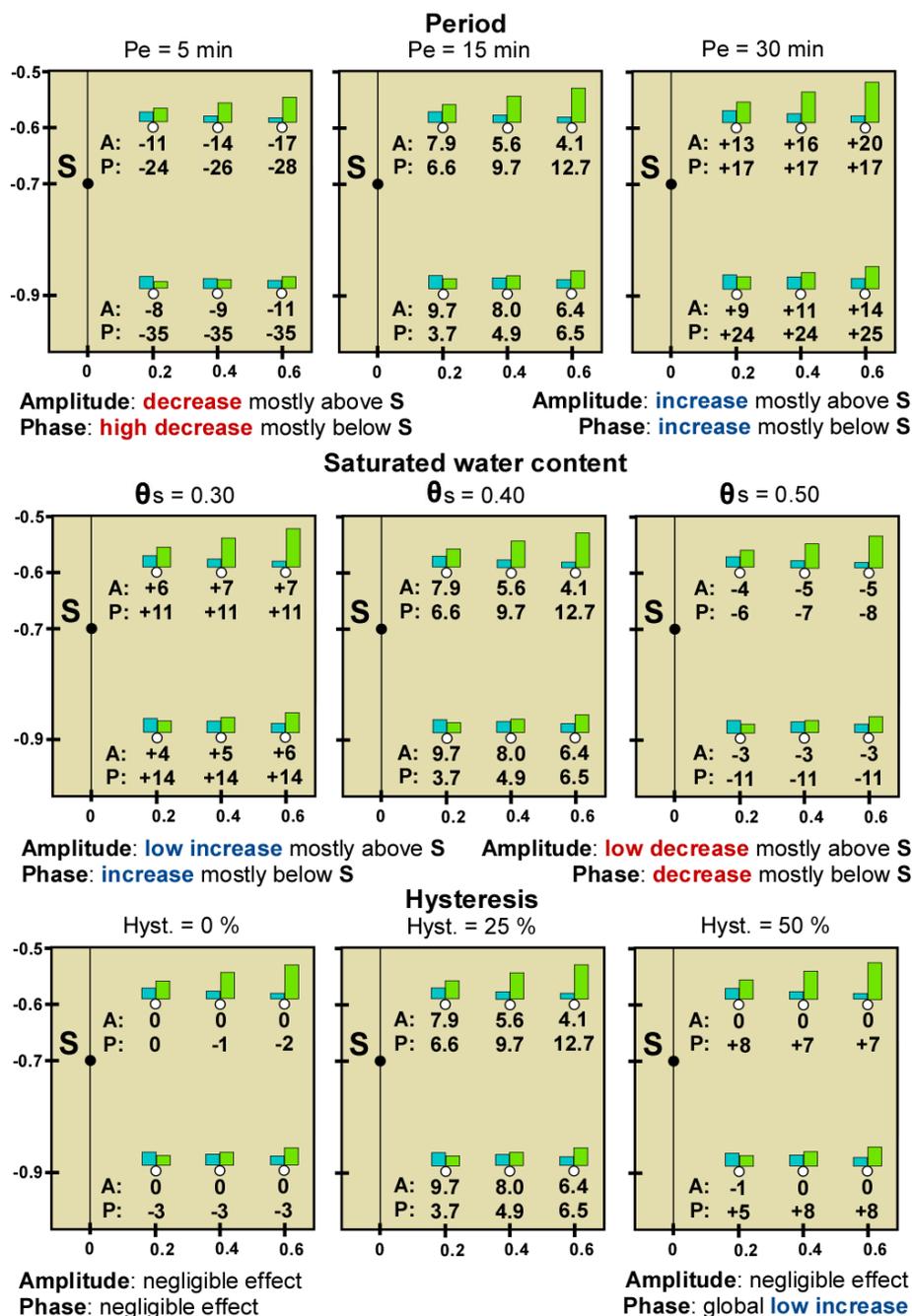
261

262 Figure 6: Lateral columns: spatial variations (in %) of amplitudes (blue, A) and phases (green, P) at each measurement point and for the unsaturated flows properties ( $\alpha$  and  $n$ ) tending to their 'sand' soil or 'loam' soil values. Middle column: absolute amplitude (in mm) and phase responses (in  $^{\circ}$ ) obtained with medium property values of the 'intermediate' soil.

266

267 A third similar analysis has been conducted on other parameters, not necessarily linked to the  
 268 two types of soil (loam and sand) considered in this study. This analysis is presented in Figure  
 269 7 with the effect of the period of oscillation, the saturated water content of the soil and the

270 magnitude of hysteresis effect. The period of oscillation influences the distance of  
271 propagation of the perturbation in the soil. A lower period will attenuate more rapidly with  
272 distance to S. An opposite effect is observed when increasing the period. A vertical effect of  
273 the period can also be observed. The period tends to accentuate the vertical difference  
274 observed in the responses above and below S when it is decreased (more important horizontal  
275 flows), and to reduce this difference for higher periods. The saturated water content generates  
276 a similar effect, but with opposite variations. In fact, a higher saturated content will accentuate  
277 the vertical differences in the responses, while a lower saturated content will reduce the  
278 vertical differences. The hysteresis effect occurring in the unsaturated zone does not  
279 substantially affects the oscillatory responses in the considered range of periods of oscillation  
280 (5 to 30 min) except a global low increase of the phase responses that can be observed for a  
281 high hysteresis magnitude. Nevertheless, Figure 2 shows that this effect affects the mean  
282 saturation level over time. However, when performing a FFT analysis on the pressure  
283 responses, this mean level can be isolated.



284

285 Figure 7: Lateral columns: spatial variations (in %) of amplitudes (blue, A) and phases (green,  
286 P) at each measurement point and for different parameters (oscillation period, hysteresis  
287 magnitude and saturated water content) tending to minimal and maximal values. Middle  
288 column: absolute amplitude (in mm) and phase responses (in °) obtained with medium  
289 parameter values between these extrema.

290

291 This first comparative analysis shows that the spatial oscillatory responses are differently  
292 affected by the different soil properties and signal parameters. One advantage of oscillatory

293 responses is that they can be easily dissociated into amplitudes and phases, which are also  
 294 differently affected by each properties. Therefore, this type of responses could be used to  
 295 assess the values of different soil properties.

## 296 **4. Imaging potential of the oscillatory responses in the saturated zone**

### 297 **4.1. Inverse problem**

298 In order to analyze if, and how, responses to oscillatory signals may provide information in  
 299 order to image the spatial distribution of hydraulic properties in an aquifer, we have re-  
 300 interpret generated oscillatory responses in our ‘sand’ or ‘loam’ soils models through  
 301 inversion processes. The inverse problem is based on objective functions, which quantify the  
 302 gap between responses simulated through a physical model for given property values and a set  
 303 of observed responses that one wants to interpret. These objective functions also incorporate a  
 304 priori information on the property values in order to overcome the issue of non-unicity of the  
 305 solution.

306 We use our model presented in section 2 to generate oscillatory responses for different values  
 307 of conductivities  $\mathbf{K}$  and Van Genuchten parameters  $\alpha$  and  $n$ . This operation can be  
 308 mathematically written:

$$309 \quad \mathbf{d} = f(\mathbf{p}) \quad (8)$$

310 where  $\mathbf{d}$  is a  $(n \times 1)$  vector containing the amplitude and phase responses simulated at each  
 311 measurement location,  $f$  is the solver of the numerical model and  $\mathbf{p}$  is a  $(m \times 1)$  vector  
 312 containing distributed values of  $\mathbf{K}$ ,  $\alpha$  and  $n$  in logarithmic scale. Using logarithmic values  
 313 permits avoiding negative values during the optimization process.

314 A simultaneous optimization of different properties in the iterative inversion process can lead  
 315 to a divergence of the process. Furthermore, as shown in previous section, the amplitude

316 responses appear to be especially sensitive to conductivity. Therefore, we decided to perform  
 317 our inversion process sequentially, with two different objective functions  $\Psi_K$  and  $\Psi_{\alpha,n}$ , that  
 318 are conceived to successively optimize the hydraulic conductivity  $K$ , only with the amplitude  
 319 responses, and then the Van Genuchten parameters  $\alpha$  and  $n$  by adding the phase responses in  
 320 the process:

$$321 \quad \Psi_K = (\mathbf{d}_{\text{amp}}^{\text{obs}} - \mathbf{d}_{\text{amp}})^T \cdot \mathbf{R}_{\text{amp}}^{-1} \cdot (\mathbf{d}_{\text{amp}}^{\text{obs}} - \mathbf{d}_{\text{amp}}) + (\mathbf{p}_K^{\text{prior}} - \mathbf{p}_K)^T \cdot \mathbf{C}_K^{-1} \cdot (\mathbf{p}_K^{\text{prior}} - \mathbf{p}_K) \quad (9)$$

$$322 \quad \Psi_{\alpha,n} = (\mathbf{d}^{\text{obs}} - \mathbf{d})^T \cdot \mathbf{R}^{-1} \cdot (\mathbf{d}^{\text{obs}} - \mathbf{d}) + (\mathbf{p}_{\alpha,n}^{\text{prior}} - \mathbf{p}_{\alpha,n})^T \cdot \mathbf{C}_{\alpha,n}^{-1} \cdot (\mathbf{p}_{\alpha,n}^{\text{prior}} - \mathbf{p}_{\alpha,n}) \quad (10)$$

323 where  $\Psi$  designates the objective functions for determining  $K$  or  $\alpha$  and  $n$  from the  
 324 information carried by the amplitude and phase responses contained in a  $n$ -vector  $\mathbf{d}^{\text{obs}}$ .  
 325  $\mathbf{R} = \sigma^2 \cdot Id(n)$  is a  $n \times n$  diagonal matrix holding variances  $\sigma^2$  on the amplitude ( $\sigma^2 = 0.1$   
 326 cm) and phase responses ( $\sigma^2 = 1^\circ$ ).  $\mathbf{d}_{\text{amp}}^{\text{obs}}$  and  $\mathbf{R}_{\text{amp}}$  designate the sub-vector and sub-matrix  
 327 associated only to the amplitude responses.  $\mathbf{p}^{\text{prior}}$  is a  $m$ -vector containing a priori values of  
 328  $K$  (in sub-vector  $\mathbf{p}_K^{\text{prior}}$ ) and  $\alpha$  and  $n$  (in sub-vector  $\mathbf{p}_{\alpha,n}^{\text{prior}}$ ).  $\mathbf{C}$  is a  $m \times m$  matrix holding  
 329 covariances constraining the hydraulic property values and their distribution.  $\mathbf{p}_K$ ,  $\mathbf{p}_{\alpha,n}$ ,  $\mathbf{C}_K$   
 330 and  $\mathbf{C}_{\alpha,n}$  designate the sub-vectors and sub-matrices associated to  $K$  or  $\alpha$  and  $n$ .

331 The inversion process consists in an iterative process aiming to minimize the values of these  
 332 objectives function successively, by updating deterministically the property values in the  
 333 model via the computation of a Jacobian matrix (Tarantola and Valette 1982):

$$334 \quad \mathbf{F}_i = \frac{f_i(\mathbf{p}_j + \Delta\mathbf{p}) - f_i(\mathbf{p}_j)}{\Delta\mathbf{p}} \quad (11)$$

335 where  $\mathbf{F}_i$  represents the sensitivity of a data  $i$  to modification  $\Delta p = 10^{-2}$  on the property  
 336 value  $j$ .

337 During the first inversion sequence, the update of the conductivity values from an iteration  
 338  $k - 1$  to  $k$  is given by:

$$339 \quad \mathbf{p}_K^k = \mathbf{p}_K^{k-1} + \left( \mathbf{F}_K^T \cdot \mathbf{R}_{\text{amp}}^{-1} \cdot \mathbf{F}_K + \mathbf{C}_K^{-1} \right)^{-1} \cdot \left( \mathbf{F}_K^T \cdot \mathbf{R}_{\text{amp}}^{-1} \cdot (\mathbf{d}_{\text{amp}}^{\text{obs}} - \mathbf{d}_{\text{amp}}) + \mathbf{C}_K^{-1} \cdot (\mathbf{p}_K^{\text{prior}} - \mathbf{p}_K) \right) \quad (12)$$

340 where  $\mathbf{F}_K$  is the Jacobian matrix described in Eq. 11, performing the sensibility analysis  
 341 between the  $\mathbf{K}$  values and amplitude responses.

342 During the second inversion sequence, the update of the Van Genuchten parameters values  
 343 from an iteration  $k - 1$  to  $k$  is given by:

$$344 \quad \mathbf{p}_{\alpha,n}^k = \mathbf{p}_{\alpha,n}^{k-1} + \left( \mathbf{F}_{\alpha,n}^T \cdot \mathbf{R}^{-1} \cdot \mathbf{F}_{\alpha,n} + \mathbf{C}_{\alpha,n}^{-1} \right)^{-1} \cdot \left( \mathbf{F}_{\alpha,n}^T \cdot \mathbf{R}^{-1} \cdot (\mathbf{d}^{\text{obs}} - \mathbf{d}) + \mathbf{C}_{\alpha,n}^{-1} \cdot (\mathbf{p}_{\alpha,n}^{\text{prior}} - \mathbf{p}_{\alpha,n}) \right) \quad (13)$$

345 where  $\mathbf{F}_{\alpha,n}$  is the Jacobian matrix described in Eq. 11, performing the sensibility analysis  
 346 between the  $\alpha$  and  $n$  values and amplitude and phase responses.

347 At the end of the inversion, the standard deviations on the inverted property values can be  
 348 obtained from the square-roots values of the diagonal entries of matrix  $\mathbf{C}^{\text{post}}$ , using the last  
 349 iterations Jacobian matrices:

$$350 \quad \mathbf{C}^{\text{post}} = \left( \mathbf{F}^T \cdot \mathbf{R}^{-1} \cdot \mathbf{F} + \mathbf{C}^{-1} \right)^{-1} \quad (14)$$

351 Performing an inversion in this way, by isolating the  $\mathbf{K}$  values in the process, we could  
 352 improve the stability of the inversion and accelerate its convergence to more realistic  
 353 solutions. In fact, the results presented hereafter show that the amplitude responses can be  
 354 closely reproduced by optimizing the  $\mathbf{K}$  field, regardless of the Van Genuchten parameters.  
 355 The remaining amplitude and phase differences can then be adjusted by optimizing the Van

356 Genuchten parameters. In fact, we have also tested inversions simultaneously optimizing  $K$ ,  
357  $\alpha$  and  $n$  that did not converge. In particular it seems that a good assumptions on  $K$  is  
358 required prior to the optimization of the Van Genuchten parameters.

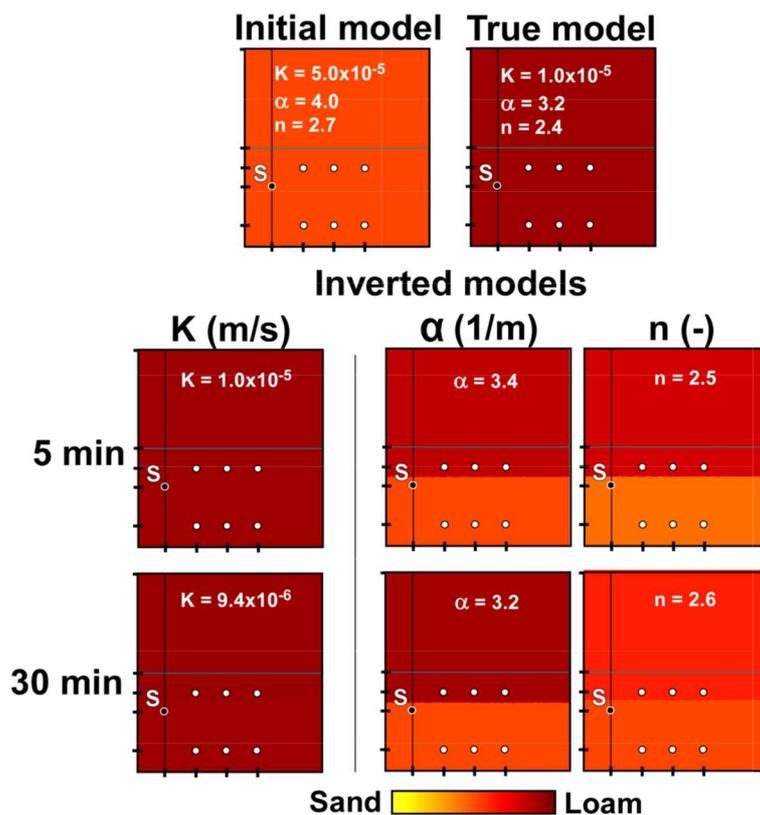
359 For all inversion applications presented hereafter we did not consider any hysteresis effect on  
360 the responses simulated in the inversions, even if the observed data were generated with a  
361 25% hysteresis magnitude.

## 362 **4.2.Homogeneous cases applications**

363 Inversions have been performed with oscillatory responses generated in homogeneous loams  
364 or sands (using property values presented in Table 1). For this generation we applied an  
365 injection/extraction rate aiming to generate similar pressure amplitudes (10 cm) at the  
366 perturbation point in both type of soil (i.e.  $Q_A = 1$  mL/s in loam and  $Q_A = 10$  mL/s in sand).  
367 The generated responses have been noised (2% added noise) for the imaging applications.

368 The hydraulic properties were optimized within two blocks (one ‘superficial’ block between 0  
369 and 65 cm depth and one ‘deep’ block between 65 and 100 cm depth). Thus, in our inversions  
370 the number of property values to be inverted was  $m = 6$  (2 distributed values for each  
371 property) for a dataset composed of  $n = 7$  amplitude/phase responses couples. We considered  
372 a diagonal covariance matrix for  $C$  with a 0.01 variance on each property value. We  
373 initialized our inversions with the ‘medium’ soil properties (see Table 1) so that we would be  
374 able to estimate if the information carried by the oscillatory responses would permit to  
375 partially or totally recover the loam and sand properties used for their generation. The  
376 anisotropy of the medium was considered as known.

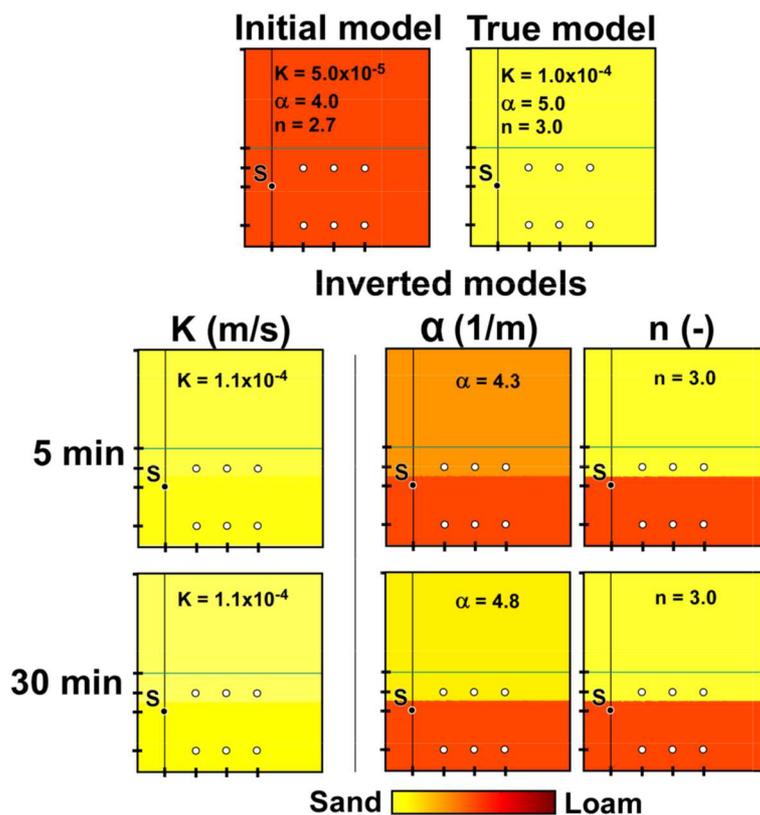
377 Figure 8 and Figure 9 present the inversion results obtained with oscillatory responses  
378 generated with different oscillation periods (5 and 30 min) respectively in loam and in sand.



379

380 Figure 8: Inversion results obtained with responses simulated in a homogeneous loam with  
 381 different periods of oscillations (5 and 30 min). Conductivities are firstly inverted only with  
 382 the amplitude responses, then the Van Genuchten coefficients are inverted with both  
 383 amplitude/phase responses.

384



385

386 Figure 9: Inversion results obtained from responses simulated in a homogeneous sand with  
 387 different periods of oscillations (5 and 30 min). Conductivities are firstly inverted only with  
 388 the amplitude responses, then the Van Genuchten coefficients are inverted with both  
 389 amplitude/phase responses.

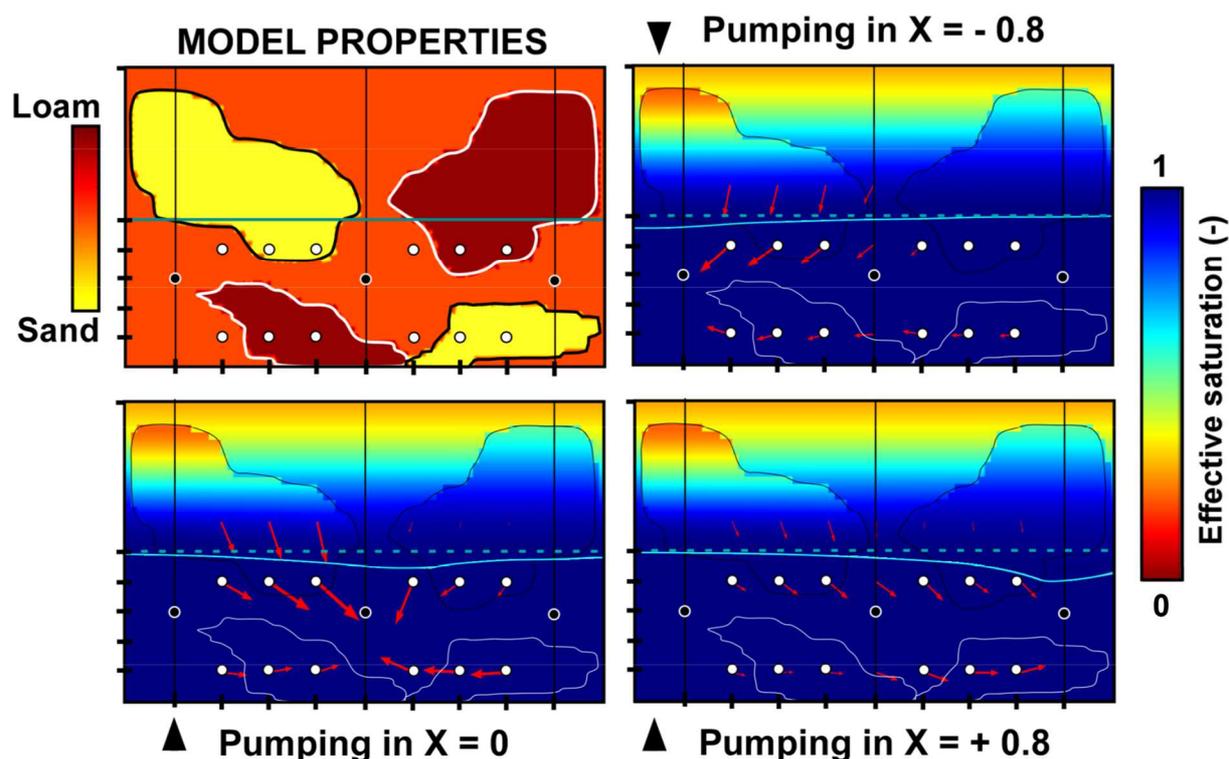
390

391 Accurate conductivity values can be obtained from the amplitude responses alone, for both  
 392 type of soils and any oscillation period. A slightly more accurate result is obtained with lower  
 393 periods. Thus, most of the amplitude responses can be reconstructed by optimizing the  
 394 conductivities. Then, the remaining responses differences, in terms of amplitudes and phases,  
 395 can be essentially linked the unsaturated flows and the Van Genuchten parameters.  
 396 Unsurprisingly, these properties were optimized only in the upper blocks of the models,  
 397 associated to the unsaturated zone. For  $\alpha$ , the accuracy of the characterization seems more  
 398 period-dependent, better assumptions being systematically obtained with the 30 min periodic  
 399 responses (i.e. a lower frequency). Regarding  $n$ , the accuracy appears to be more related to  
 400 the soil, with better quantifications obtained in sands rather than in loams.

401 Globally, we observe that the oscillatory responses measured in the saturated zone provide  
 402 sufficient information to distinguish the type of soil type in the medium. They can also allow  
 403 the quantification of accurate properties values associated to the saturated and the unsaturated  
 404 flows without necessarily having to simulate a hysteresis effect.

### 405 **4.3.Simplified applications for heterogeneous cases**

406 We have then performed inversions with oscillatory responses simulated in simplistic  
 407 heterogeneous cases (see Figure 10) in a wider sandbox (3.6 m width). These cases consisted  
 408 in 3 perturbations points (black dots in Figure 10) in a ‘medium’ soil (as described in Table  
 409 1), surrounded by sand or loam areas in the saturated or unsaturated zones. The velocity  
 410 vectors simulated at each observation point (white dots in Figure 10) show that the generated  
 411 flows are differently affected in each areas of the model. Most important velocities are  
 412 observed in the sand areas, both in saturated and unsaturated zones. Therefore, in order to  
 413 efficiently characterize the whole heterogeneity, it is necessary to cross-interpret data  
 414 generated by signal sources located at different locations (hydraulic tomography principle).



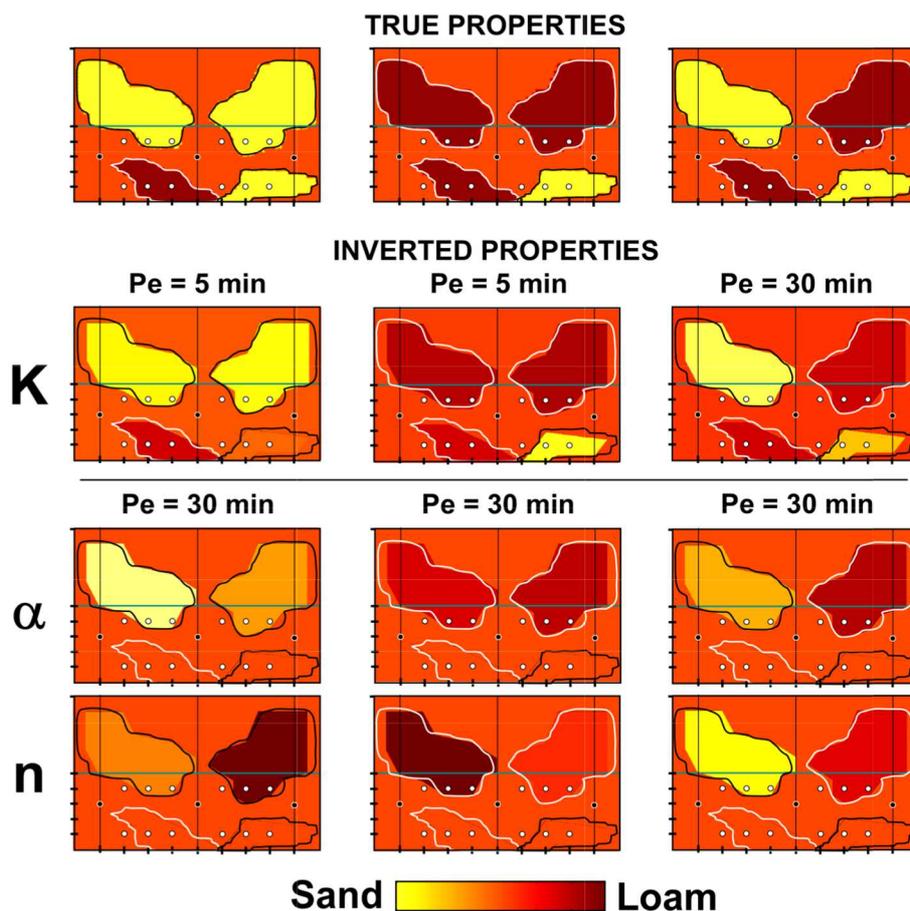
415

416 Figure 10: Flow velocity vectors (red arrows) in a log100 scale when pumping at different  
417 locations ( $X=-0.8$ ,  $X=0$  and  $X=+0.8$ ) in the heterogeneous model (shown in the upper left  
418 corner). Current and initial saturation levels are indicated as blue lines and dotted lines  
419 respectively.

420

421 Thus, for the inversions we have simultaneously inverted the responses generated with each  
422 of the three perturbation sources. These responses have been noised (2% added noise) for the  
423 imaging applications. We considered the anisotropy of the medium and also the properties of  
424 the soil at the perturbation locations to be known. We have performed two types of inversions.  
425 In a first inversion we have considered having a certain prior knowledge on the existing  
426 contrast in the soil, and therefore invert properties within delineated zones. In a second  
427 inversion, we did not consider this knowledge and inverted the properties within a grid  
428 regularly distributed in 2D in the saturated zone and in 1D in the unsaturated zone (see dotted  
429 lines delineations in Figure 12).

430 For our zoned inversions the number of property values to be inverted was  $m = 12$  (4 zoned  
431 values for each property) for a dataset composed of  $n = 45$  amplitude/phase responses couples  
432 (15 responses obtained for each perturbation). We considered a diagonal covariance matrix  
433 for  $\mathbf{C}$  with a 0.01 variance on each property value. We have tested different combinations of  
434 responses generated with lower or higher periods of oscillation (5, 15 or 30 min) in terms of  
435 imaging efficiency, and for different heterogeneity configurations (3/4 sand, 3/4 loam and half  
436 sand/half loam). The best imaging results obtained are presented in Figure 11.



437

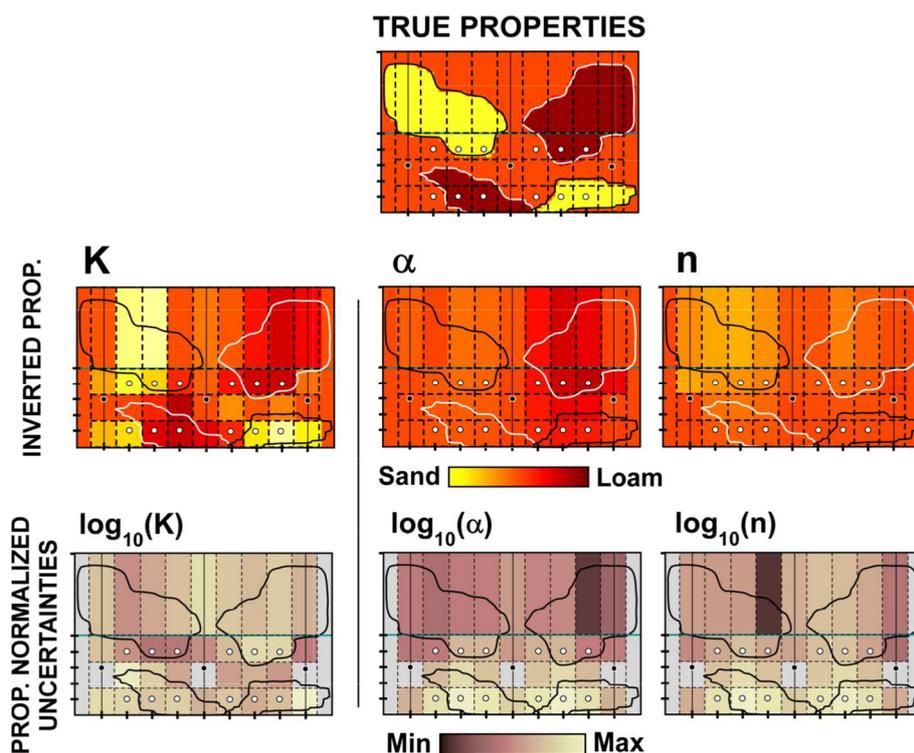
438 Figure 11: Results obtained for the best inversion configurations with responses simulated in  
 439 heterogeneous soils with different periods of oscillations (5, 15 and 30 min). Conductivities  
 440 are inverted only with the amplitude responses, while the Van Genuchten coefficients are  
 441 inverted with both amplitude/phase responses.

442

443 Here again, amplitude responses alone are sufficient to reconstruct the heterogeneous contrast  
 444 in the conductivity field for each period of oscillation tested. The inverted conductivity  
 445 values, however do not always exactly match the true ones. It appears that a contrasted soil  
 446 (half sand/half loam zones) is better characterized with higher periods of oscillation, while a  
 447 less contrasted medium is better characterized with lower periods. Concerning the Van  
 448 Genuchten parameters, optimized with the remaining differences in the amplitude and phase  
 449 responses that could not be reproduced with conductivity alone, they are better characterized  
 450 with the higher period of oscillation in each case. As for the homogeneous cases, they can be  
 451 characterized only in the unsaturated zone of the soil and do not image heterogeneities in the

452 saturated zone. Parameter  $\alpha$  appears to be more reliable than  $n$  for the interpretation of  
 453 heterogeneity in the capillary fringe. In fact, in our results, inverted  $n$  values are good for the  
 454 upper left zone, but not always for the upper right one, while  $\alpha$  values are always coherent in  
 455 both corners.

456 For our distributed inversions the number of property values to be inverted was  $m = 123$  (41  
 457 distributed values for each property). We constrained the spatial distribution of the inverted  
 458 property values with an exponential variogram model with a range of 2 to compute the  
 459 covariance values in matrix  $C$ . We inverted the responses generated in the half sand/half  
 460 loam heterogeneous configuration with a period of oscillation of 30 min (according to the best  
 461 result configuration in the zoned inversions). The inverted property fields and their associated  
 462 normalized standard deviations are presented in Figure 12.



463

464 Figure 12: Property values and their associated normalized standard deviations obtained from  
 465 the inversion of amplitudes (for  $K$ ) and amplitudes/phases (for  $\alpha$  and  $n$ ) responses generated  
 466 in an heterogeneous soil (true properties) solicited with 30 min period oscillations. The  
 467 inversion grid is delineated with dotted lines.

468

469 The inverted property values globally do not match the true ones, they are mostly  
470 underestimated. This can be explained by the lack of spatial constraints in this inversion that  
471 allows optimization process to generate local equivalent property values for the reproduction  
472 of the data, especially for  $K$ , which is optimized first. However, the trends and localizations  
473 of the heterogeneous contrasts can be imaged in both the saturated and unsaturated areas in  
474 the conductivity field. The conductivity field is mainly sensible to the sand zones in the  
475 saturated area, according to the distribution of its uncertainties. On the other hand, again  
476 unsurprisingly, the Van Genuchten parameters image only the heterogeneities in the  
477 unsaturated area. However, in this distributed inversion it appears that  $\alpha$  rather image the  
478 loam areas, while  $n$  rather image the sand areas. The uncertainty analysis shows a globally  
479 higher local sensitivity, near the perturbation sources for  $n$ , and a more homogeneous  
480 sensitivity among the whole unsaturated area for  $\alpha$ .

## 481 **5. Discussion**

### 482 **5.1. Effects of hydraulic properties on the saturated responses**

483 Our hydraulic analysis, presented in this article, tends to indicate that the different hydraulic  
484 properties considered  $K$ ,  $\alpha$  and  $n$  affect differently the measured oscillatory responses in the  
485 saturated zone and that their relation to such type of responses is also conditioned to different  
486 factors, such as the depth of the source or the measurement locations.

487 The conductivity  $K$  globally appears as the most interesting hydraulic property to be  
488 characterized with oscillatory responses. In fact, it is highly linked to the amplitude of the  
489 responses, more than the other studied properties, and, thus, can be easily isolated from these  
490 other properties when analyzing the responses. Basing our inversions of  $K$  on the amplitudes  
491 alone resulted, indeed, in good characterizations of its distributions in each configuration,

492 even without considering accurate Van Genuchten parameters. This observation is in  
493 agreement with other works on unconfined media. Huang et al. (2019) analyzed the sensitivity  
494 of oscillatory responses in unconfined aquifers through an analytic solution and noted the  
495 same high sensibility of the responses to  $K$ . However this observation does not seem specific  
496 to oscillatory data. In fact, in their hydraulic tomography applications taking account  
497 unsaturated flows, Mao et al. (2013b) and Zhao et al. (2015) also observed that the contrasts  
498 in conductivity fields could be globally well imaged with responses to constant-rates  
499 pumping. Mao et al. (2013b) even noticed the same fact that saturated flows properties could  
500 be well characterized even without accurate guesses on the unsaturated ones.

501 Concerning the Van Genuchten parameters, there is currently no previous studies taking them  
502 into account in a characterization of an unconfined aquifer with oscillatory signals. Mao et al.  
503 (2013b) and Zhao et al. (2015) employed the Gardner-Russo's model to simulate the  
504 unsaturated flows and found that its parameters could be well characterized only in the  
505 unsaturated area. In our study we observed that the oscillatory responses in the saturated zone  
506 are affected by both  $\alpha$  and  $n$ , with various intensities depending on the period of oscillation,  
507 the type of soil and the type of flows generated. The effect of  $n$  appears to be more related to  
508 the local response at the perturbation source (local fluxes), while  $\alpha$  generates a more  
509 distributed effect on the responses (vertical flows affecting the whole capillary fringe). The  
510 imaging potential of  $\alpha$ , therefore, appears as more reliable at the scale considered in our  
511 results (metric scale). Trying to link this hydraulic behavior to geological explanations is a  
512 complex topic that is discussed in the literature. It is mainly accepted that  $\alpha$  is related to the  
513 air entry suction of the soil and  $n$  to the distribution of the pore size. Benson et al. (2014)  
514 observed that, for a uniform sand, the breadth of the particle size influenced  $n$ , while  $\alpha$  was  
515 more linked to a median value of the particle size, indicating a more integrative effect.  
516 Porebska et al. (2006) noted that  $n$  was directly dependent to their soil composition (sand,

517 clay, silt), which we also observe in our results, while parameter  $\alpha$  appeared to be more  
518 related to the granulometry of the sample.

## 519 **5.2. Advantages of oscillatory signals and period-dependent effects**

520 Separating the responses signals into amplitude and phase allow for a better separation of the  
521 information contained in the data. Absolute amplitudes can be used to specifically  
522 characterize the conductivity  $K$  and appear to carry already a sufficient amount of  
523 information to interpret its variations around the perturbation points, which can then be used  
524 to guide the characterization of the unsaturated flows parameters. This possibility of  
525 separating the responses into amplitudes and phases in real cases can be achieved only if the  
526 responses reach a simple harmonic motion, mainly after several periods. The conditions  
527 allowing to reach this conditions are described in Bakhos et al. (2014) and in Huang et al.  
528 (2019). Oscillatory signals also offers the possibility to be easily extractable from other  
529 sources of signals or noises. Techniques to efficiently process field oscillatory responses are  
530 proposed in Bakhos et al. (2014). It can also be noted that oscillatory responses generated  
531 with wetting/dewatering of an unsaturated soil potentially allow to assess mean values of  $\alpha$   
532 and  $n$  in the water content curves, while responses associated to constant pumping would  
533 characterize only the Van Genuchten values associated to the dewatering part of the curve due  
534 to the hysteresis ‘scanning loops’ effect.

535 The effect of the period of oscillation on the flows generated in an aquifer has been discussed  
536 in previous works (Rabinovich et al. 2015; Cardiff et al. 2013b; Fischer et al. 2018; Fischer et  
537 al. 2020b). Globally, according to these works, decreasing the period of oscillation accentuate  
538 flows in the most conductive and mobilizable areas of the field and in the vicinity of the  
539 pumping/injection location. As water in the unsaturated zone represents a less mobilizable  
540 source of water than the volume in the saturated area, it appears as logical that a lower period

541 of oscillation accentuate the horizontal flows and does not allow for a good characterization  
542 of the unsaturated area. In their field applications, Rasmussen et al. (2003) observed, in fact,  
543 that with a very low period of oscillation, the responses generated in an unconfined aquifer  
544 could be assimilated to confined-medium responses. Therefore, low periods of oscillations  
545 may be employed with care for unconfined aquifers according to several previous works on  
546 this topic (Rasmussen et al. 2003, Rabinovich et al. 2015; Cardiff et al. 2019). Cardiff et al.  
547 (2019) also suggested that, for high frequencies (period < 1 min), the existing models such as  
548 the Richards-based may become inaccurate.

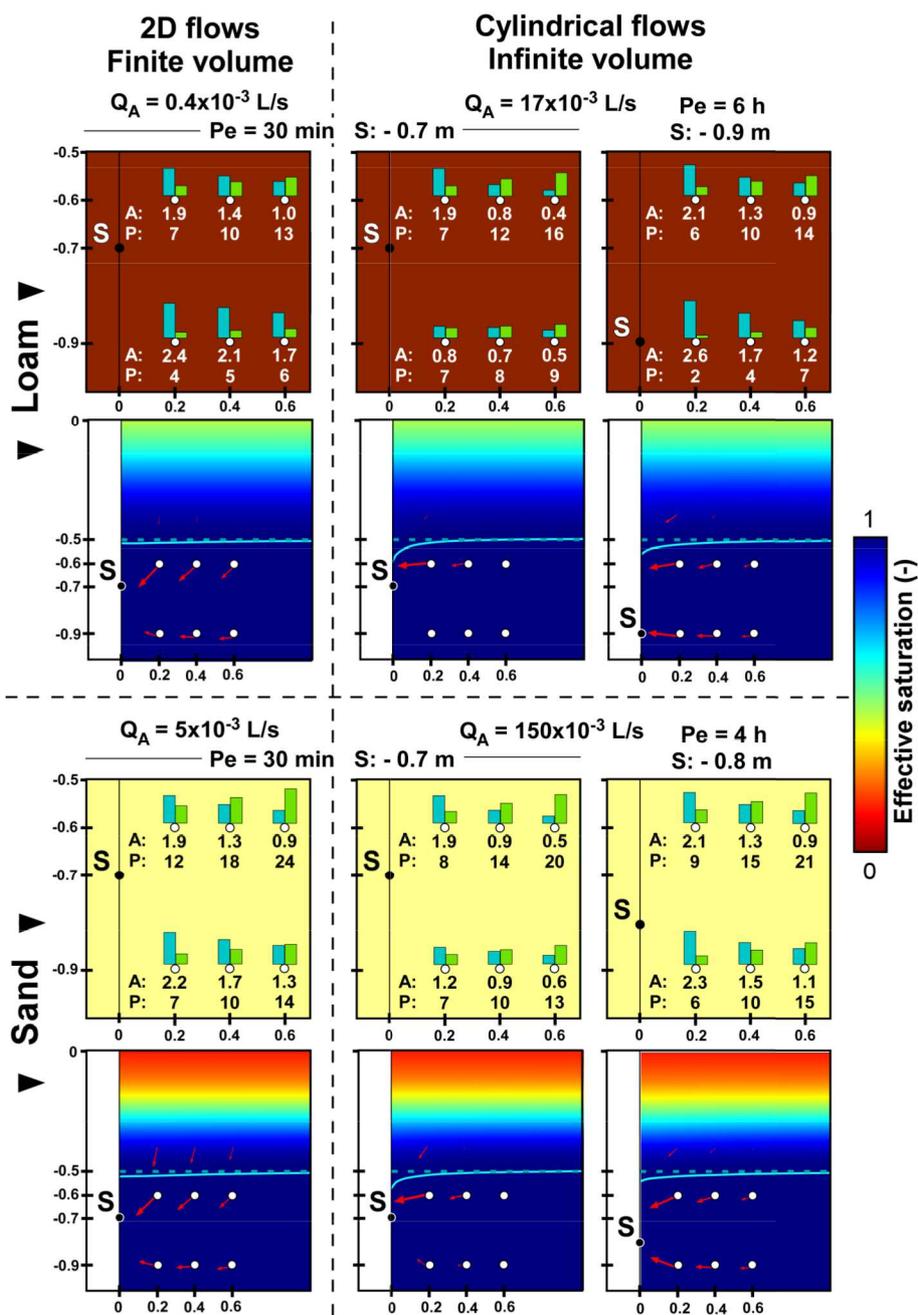
549 According to our results, if lower periods provided better imaging results for  $K$  when the  
550 medium was homogeneous or with lowly contrasted heterogeneities, a higher period of  
551 oscillation should be adopted if one wants to characterize draining effects parameters,  
552 especially for  $\alpha$  ( $n$  being more related to local responses seems less dependent to the period).  
553 However, when dealing with higher period signals, hysteresis effects should be considered  
554 with unsaturated flows. In our results, for periods between 5 and 30 min and at a metric scale,  
555 we show that we can neglect its effects on the responses (only for the responses in the  
556 saturated zone). Experiments made in Cartwright (2014) and Shoushtari et al. (2015) are in  
557 agreement with this observation when the period of oscillation remain relatively low.  
558 However the authors noted that for high periods of oscillations reaching several hours or  
559 more, the hysteresis effect could become important even on the responses in the saturated  
560 zone and should be taken into account.

### 561 **5.3. Toward real cases**

562 The analysis proposed in this article represents a preliminary work for future real cases  
563 tomographic applications. It remains theoretical and real cases applications would face other  
564 problematics not considered in our models, such as the impact of environmental noise on the

565 data. However, it provides some information and methods that may be helpful for a better  
566 setup of oscillatory data interpretation in an unconfined aquifer.

567 A first adaptation for a field application would be to consider 3-D flows with boundary  
568 conditions often not affecting the perturbed volume. Therefore we have also simulated flows  
569 in a cylindrical homogeneous model with a similar geometry but a constant head boundary  
570 condition located far away from the perturbation source point S (50 m away). This  
571 configuration thus considers volumetric effects and a theoretical infinite volume (as water can  
572 enter or leave the model through the constant head boundary). These consideration are more  
573 realistic for a field case. We compare the responses simulated in this way in sand and loam to  
574 our 2-D model previously discussed in Figure 13. In order to generate a signal of similar  
575 amplitude at the perturbation source it is necessary to increase its flowrate amplitude (40  
576 times in loam, 30 times in sand). However, the spatial responses simulated in this way will be  
577 different from the ones obtained in the 2-D finite-volume model: the attenuation of amplitude  
578 responses and the phase lags increase with distance become more important in the cylindrical  
579 infinite-volume model. In order to generate responses in the cylindrical model that are closer  
580 to the 2-D ones, it is necessary to also increase the period of oscillation and the signal source  
581 depth (relatively more in loam than in sand). If the adopted cylindrical model remains very  
582 simplistic compared to real field, it permits to provide insights on what configuration  
583 parameters should be especially considered for fields applications (i.e. flowrate amplitude,  
584 signal source depth and oscillation period) and how to adapt to different type of soils.



585

586 Figure 13: Observed amplitude (A in cm) and phase (P in °) responses and simulated saturated interface during pumping phase in homogeneous sand or loam considering either 2D  
587 flows in a finite volume or cylindrical flows in an infinite volume and different signal  
588 amplitudes ( $Q_A$ ), depths (S) and periods (Pe).  
589

590

591 **5.4.Imaging potential in tomographic applications**

592 According to the results presented in this work, having measurements and perturbation  
593 locations distributed spatially and also at different depths of the aquifer might provide  
594 different information on the different hydraulic properties. Horizontal flows effects might be  
595 better characterized by deeper responses or perturbation, while the vertical flows effects can  
596 be better characterized by perturbation and measurements near the unsaturated zone. This  
597 accumulation of distributed perturbations and information is, furthermore, a well-known and  
598 widely used technique for the characterization of aquifers in hydraulic tomography.  
599 Furthermore, we could observe that the localization of the signal source in depth was crucial  
600 for the characterization of the unsaturated area. It is, in fact, necessary to generate a  
601 movement of the saturated/unsaturated interface, in order to generate vertical flows, mainly  
602 with a source close to this interface, or at this interface directly (such as tide periodic sources).  
603 Mao et al. (2013a) provided several conditions on the dataset to perform accurate hydraulic  
604 tomography of unconfined aquifers. In particular they recommend to measure fluxes in  
605 addition to pressure data and to couple different type of data in order to better characterize the  
606 heterogeneity of the medium. Ideally, the different data should also identify flow processes in  
607 the saturated and the unsaturated zone separately. Coupling geophysical data, allowing to  
608 image water content variations in the soil and heterogeneity contrasts, could greatly improve  
609 the imaging results. Indeed, our inversions constrained with zones (that can be typically  
610 delineated from geophysical interpretations) showed better results than the regularly  
611 distributed ones.

612 In the method and the imaging applications presented in this work, we consider prerequisites  
613 knowledges on the properties. In particular we assume the storage coefficient, the anisotropy,  
614 and the saturated and residual water content of the medium to be known. Therefore, for a real  
615 application case using the inversion method proposed in this paper, one would need to  
616 estimate these values. For this purpose, Cheng and Renner (2018) proposed analytical

617 solutions comparing pressures amplitude and phase responses measured above and below an  
618 oscillatory source to its flow-rate signal allowing to estimate the anisotropy and storage  
619 coefficient at the perturbation location. These solutions provide local estimates that can then  
620 be used for the imaging application. However, even if the storage coefficient remains  
621 unknown it is possible to image diffusivity ( $D = K/S_s$ ), thus inverting  $K$  and  $S_s$  together.  
622 Estimations of water contents constants would be more complicate in field applications, but  
623 important impacts on the oscillatory responses can be observed only for large variations in  
624 their values, according to our analysis in section 3. Thus, an average guess may be sufficient  
625 for an imaging application.

## 626 **6. Summary and conclusion**

627 This article presents a numerical analysis for the hydraulic interpretation of oscillatory  
628 responses measured in an unconfined media at a metric scale and including the effect of the  
629 capillary fringe. It represents a useful preliminary work for future real cases tomographic  
630 applications in unconfined media (sandbox or field cases), as it permits to point out several  
631 comments concerning the characterization of the hydraulic properties  $K$ ,  $\alpha$  and  $n$  with such  
632 data:

633 1) Conductivity  $K$  is highly correlated to the amplitude of the oscillatory responses compared  
634 to the other hydraulic properties. Thus, a good characterization of the conductivity  
635 distribution in an unsaturated media can be obtained separately with the amplitudes of the  
636 responses alone and independently from the unsaturated properties;

637 2) The Van Genuchten parameters  $\alpha$  and  $n$ , related to unsaturated flows, also affect the  
638 responses in the saturated zone. They can be assessed, in a second step, from the amplitude  
639 and phase responses that could not be reproduced by the previously reconstructed  
640 conductivity;

641 3) In a heterogeneous soil, interpreting oscillatory responses in the aforementioned way  
642 allows for a coherent reconstruction of the  $K$  field (that can then be used to guide the Van  
643 Genuchten parameters characterization). The  $\alpha$  and  $n$  values can be characterized only in the  
644 unsaturated zone, and  $\alpha$  appears to better image the heterogeneities than  $n$  at this scale;

645 4) The period of oscillation of the signal tends to accentuate rather horizontal flows for low  
646 periods and generate more vertical flows effects on the responses for high periods. Low  
647 periods signals are useful to an accurate assessment of the conductivity values in the saturated  
648 area. However, for the characterization of the unsaturated area, high periods of oscillations  
649 provide more information but may also be subject to more important hysteresis effects.  
650 Nevertheless, for the periods considered in this study (5 to 30 min), these effects could be  
651 neglected.

652 These comments may represent interesting advices for real cases applications. However, other  
653 problematics not taken into account in this work, might also be considered. Real cases  
654 applications are naturally more complex than the model studied in this work, and effect of  
655 noise on the measurements also represents a common problematic in such cases. A commonly  
656 employed and efficient technique that reduce uncertainties in real field applications is  
657 hydraulic tomography. This technique, based on a crossed interpretation of responses to  
658 perturbations obtained at several locations and depths, should therefore be considered in  
659 association with periodic signals for the spatial characterization of unconfined aquifers  
660 properties in future field applications. Addition of other types of data, such as geophysical  
661 information could also greatly improve the inversion results in real-cases applications.

662

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