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Determination of the orbits of 62 moons of asteroids based on astrometric observations.

N. V. Emelyanov[1,2]★ and A. E. Drozdov[3]

[1]*M. V. Lomonosov Moscow State University/Sternberg Astronomical Institute, Universitetskij prospect 13, Moscow, 119234 Russia*
[2]*Institut de mécanique céleste et de calcul des éphémérides – Observatoire de Paris, UMR 8028 du CNRS, 77 avenue Denfert-Rochereau, 75014 Paris, France*
[3]*M. V. Lomonosov Moscow State University/Physics Department, Vorobievy Hills, 1, Moscow, 119234 Russia*

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ABSTRACT

Based on all published astrometric observations, we have determined the moon orbits for asteroids using a model of the fixed Keplerian orbit. We applied 5–114 observations for each moon. As a result, we have determined the orbits of 62 moons. All results, including the orbital parameters obtained, are presented in the tables that are provided as supplementary material, available online. These data can be used to calculate the ephemerides of the moons of the asteroids. Among the moons considered, 13 belong to asteroids of the main asteroid belt, two are the moons of Jupiter Trojan asteroids, while the rest are trans-Neptunian objects. Our results are in good agreement with the corresponding results published in the literature. We argue that reliable estimates of the accuracy for the ephemerides can be only made using parameter covariance matrices. These matrices that we have obtained are also given in the supporting information.

Key words: astrometry – celestial mechanics – minor planets, asteroids: general – planetary systems.

1 INTRODUCTION

The diversity of the bodies in the Solar system and the complexity of its structure are complemented by the presence of moons of asteroids. These objects will be the subject of active research in the field of the dynamics of the Solar system in the future. In fact, the properties of asteroids' moons can provide a lot of information about the formation and evolution of the Solar system. In light of the planned space missions to these objects, we need to know the physics and dynamics of these bodies. Our task is to obtain the necessary data based on observations.

In order to do this, we created a model of the orbital dynamics of moons based on available observations. As the quality of observations improves, the model will need to be improved. Currently, the accuracy of the existing model is considered sufficient if this model is in good agreement with the results of observations. In this paper, we have followed this principle.

We remind the reader that there are already 350 asteroids known to have moons. For most of them, only the presence of the moon is known. Astrometric observations of approximately 90 moons of asteroids have been published. The main difficulty is that the angular apparent distance between the moon and the asteroid is very short. It barely reaches 1 arcsec. In addition, the magnitude of the moon is much smaller than magnitude of its asteroid. For these reasons, it is very difficult to obtain positional data for the moons of asteroids. Generally, there are not enough observations and the observatins available are of low accuracy. Currently, it has only been possible to determine the orbits of about 60 moons of asteroids from observations.

Alternatively, the orbit can be determined by using photometric observations of an asteroid system with a moon during their mutual occultations and eclipses. An example of the determination of such an orbit is presented by Sokova et al. (2019). However, the orbits of only a few moons of asteroids have been determined by this method to date. The resulting accuracy is low. We do not consider such observations in this paper. The astrometric observations of the moons of asteroids were used as the initial data in our work.

In many studies, when researchers create models of the motion of celestial bodies based on observations, they have to provide bibliographic references to all publications of observations. In our case, this would be more than 110 references, and listing these references in the bibliography would increase the length of the paper too much. Indeed, anyone who makes a similar attempt would again have to list all these references.

We suggest a different process. An aggregate data base ABIN of all observations has been created (Emel'yanov, Vashkov'yak & Ural'skaya 2018) containing all the bibliographic references for the sources. This data base is freely accessible on the Internet at the Sternberg Astronomical Institute Moscow State University site[1]

★ E-mail: emelia@sai.msu.ru

[1]See http://www.sai.msu.ru/neb/nss/html/obspos/babine.htm.

and at the mirror site hosted by the Institut de Mécanique Céleste et de Calcul des Éphémérides (IMCCE).[2]

Now, the way to obtain a bibliographic reference is quite simple. Using one of the above-mentioned web sites, choose *Go to Summary*.[3] There, it is possible to find the asteroid with a moon that is of interest, and a full list of portions of observations is available. For each portion, the data base gives, using the link *Content*, the bibliographic reference together with the link to the relevant article in ADS Abstract Service. A further click will give the article itself if available in the ADS Abstract Service.

Nevertheless, it is best to credit the people who first did the work. Without trying to find out who was the first to publish a discovery of an asteroid's moon, we mention here the publications of the earliest observations included in the ABIN data base (Emel'yanov et al. 2018).

The discovery of a moon of the asteroid (45) Eugenia was announced in Merline et al. (1999). Five astrometric positions were shown in their fig. 3 as a composite of individual observations made from 1998 November 1–10. The exact dates of observations were not given in this paper although they are included in the ABIN data base (Emel'yanov et al. 2018) to confirm that the observations were made and published.

Storrs et al. (2001) reported the discovery of an apparent companion to (107) Camilla in *Hubble Space Telescope* (*HST*) images made during March 1.242–1.252 UT. One astrometric position was published. Merline et al. (2001) reported the detection of a moon of (22) Kalliope on September 2.6 UT from *H*-band direct imaging with the 3.6-m Canada–France–Hawaii Telescope on Mauna Kea. Three astrometric positions were published and included in the ABIN data base. Brown et al. (2001) reported the discovery of a moon for (87) Sylvia. This has given one astrometric position. Other observations included in the ABIN data base were published in 2002 and later.

We used observations from two sources of information. The main source is the ABIN data base (the data base of all published astrometric observations of moons of asteroids; Emel'yanov et al. 2018). Another source of observational data is presented on the web page 'Orbit Status of Known Binary TNOs'.[4] This data base was created by Grundy et al., and is regularly updated.

A fixed Keplerian orbit of the moon of an asteroid is considered in this paper as a model of the orbital motion. For most moons of asteroids, the accuracy of observations is such that a Keplerian orbit model is sufficient for an adequate representation of motion based on observations. However, a number of moons had to be excluded from our study in cases where the asteroid has more than one moon. In fact, for these cases, the model becomes more complex because the moons have a strong mutual attraction, which must be taken into account in the motion model. For only a few asteroids with two moons, the independent motion of each moon can be represented by the Kerplerian orbit with an accuracy comparable to the observation accuracy. We provide a list of moons excluded from our study.

Our results are provided as Supporting Information, available online, and are presented in four tables. Table S1 contains the main characteristics of asteroid systems with moons, the number of observations used and the estimated accuracy of adjusting the model with observations. Table S2 presents the obtained orbital parameters. A comparison of the parameters obtained in our paper with the results for 38 moons of trans-Neptunian objects published in Grundy et al. (2019a,b) and in the data base of Grundy et al. is presented in Table S3. In Table S4, we give the covariance matrices of parameters for the obtained orbits. A detailed description of the data of these tables follows below.

[2]See http://nsdb.imcce.fr/obspos/babine.htm.

[3]It is also possible to go directly to http://www.sai.msu.ru/neb/nss/html/obspos/OBS_COLL/abin/abinSummary.html.

[4]Available online at http://www2.lowell.edu/users/grundy/tnbs/status.html.

2 METHOD FOR THE DETERMINATION OF THE ORBIT AND REQUIRED PARAMETERS

In this paper, we take a fixed Keplerian orbit as a model of the motion of the moon of an asteroid. Because the mass of an asteroid system with a moon usually is unknown, we assume that the semimajor axis of the orbit a and the mean motion n are independent parameters determined from observations.

Other orbital parameters are the eccentricity e, the inclination to the main plane i, the mean anomaly at the epoch M_0, the argument of pericentre ω and the longitude of the ascending node Ω. Obviously, the value M_0 is associated with the epoch of the mean anomaly t_0.

We have assumed that the main plane is the plane of the Earth's equator fixed for the J2000 epoch. No declaration of a coordinate system, such as the International Celestial Reference Frame, for a moon's orbit can be made in the theory itself. The binding of the model to the coordinate system can only follow through observations. Below, we provide a brief analysis of the possibilities of accounting for various perturbations in the motion of moons of asteroids.

During astrometric observations, the differences of the right ascension of the moon α_s and the asteroid α_a and the differences of their declinations δ_s and the asteroid δ_a are measured. Published observational data provide us with the values

$$X = (\alpha_s - \alpha_a)\cos\delta_a, \quad Y = (\delta_s - \delta_a), \tag{1}$$

with the angular distance between the bodies s and the position angle P defined by the relations

$$s = \sqrt{X^2 + Y^2}, \quad \tan P = \frac{X}{Y}. \tag{2}$$

The method used in our work for determining the orbit is a differential refinement of orbital parameters based on observations. Corrections to the parameters were made using a linear least-squares method (LSM). A brief description of this method can be found in Emelyanov, Safonov & Kupreeva (2019).

We have used predetermined values for some parameters. In many cases, these values were taken from other works. In those situations when the preliminary orbit was unknown, we used some kind of heuristic approach. In this way, some parameters were fixed at the most probable values, and an attempt was made to determine other parameters. Then, the list of refined parameters was updated. Finally, we found all the required parameters.

We recognize that the question of the truth of the result is very complicated. We use an objective function similar to that denoted by χ^2 in other publications. If this function has a well-defined minimum, then our iterations after walks will surely find this minimum. If there are several equally good solutions, then we find one of the good solutions following a predefined prompt from external sources. We do not know why this solution is chosen and we say let it be: this is one of the good solutions. If there are few observations, then we do not obtain the most correct solution, but the solution that best fits the observations used. We have no other option until new observations have appeared.

After all the above seven parameters are found, we can calculate the gravitational parameter of the system μ by the formula

$$\mu = n^2\, a^3. \quad (3)$$

Almost all observational data in the available publications are provided with measurement errors. We use all these additional data to assign weights to the conditional equations.

3 OBSERVATIONS

We used astrometric observations from two sources. The first source is the ABIN data base of all published astrometric observations of moons of asteroids (Emel'yanov et al. 2018). This data base is regularly updated with new publications of observations. At the time of writing, the ABIN data base included 1492 observations of 86 moons of asteroids (Emel'yanov et al. 2018). The second source of observational data is the web page 'Orbit Status of Known Binary TNOs', which is a data base that is regularly updated by Grundy et al.

By individual observations, we mean data consisting of three quantities: the moment of observation t and a pair of coordinates X, Y or s, P. The meaning of the notation has already been explained above. To determine the orbital parameters, a set of N observations is used. The number of observations is denoted by $N_{\rm A}$ if observations were taken from the ABIN data base (Emel'yanov et al. 2018), and denoted by $N_{\rm G}$ if observations were taken from the data base of Grundy et al. When refining the orbital parameters, some observations were initially discarded because of their unsatisfactory quality. The number of discarded observations is denoted by $N_{\rm d}$. As a result, the number of observations used is $N = N_A + N_{\rm G} - N_{\rm d}$.

In Table S1, we give in separate columns the number of observations for each moon: $N_{\rm A}$, $N_{\rm d}$, $N_{\rm G}$ and N. The number $N_{\rm d}$ refers only to observations from the ABIN data base. No observations were discarded from the second source.

Bibliographic references for the publications of observations that we used can be found in the ABIN data base (Emel'yanov et al. 2018), a service that was initially provided for in the concept for the ABIN data base.

We used only the results of ordinary astrometric observations from ground-based observatories and from the *HST*.

In addition to the above observational data, photometric observations of an asteroid system with a moon during their mutual occultations and eclipses can be used to determine orbits. The difficulties in extracting positional and orbital data from such observations are significant as changes in the brightness of the system due to the rotation of the non-spherical components of the system must be separated from the changes caused by occultations and eclipses. An example of using photometric observations to determine a moon's orbit is published in Sokova et al. (2019), where the orbit of the moon of asteroid (2121) Sevastopol is determined. Unfortunately, we did not find astrometric observations of this moon.

For the purposes of our study, it would be advisable to find such photometric observations of moons of asteroids during their mutual occultations and eclipses, which could be combined with the results of ordinary astrometric observations in order to determine the orbits. In some cases, only the fact that there is a mutual occultation of the moon and the asteroid at a given time gives us positional information. This is because during such events the relative coordinates of the moon can be considered equal to zero if the size of the bodies with respect to the distance between them can be neglected. However, the fact is that the time of the light-curve minimum in a mutual event does not necessarily correspond well to the time of minimum separation as seen from Earth.

4 CHOICE OF MOONS FOR DETERMINING ORBITS

To determine the orbits, we first examined 86 moons from the ABIN data base. After this, those moons that have fewer than five observations were excluded from this set. Next, we considered asteroids that have two or more moons. The mutual gravitational influence of moons in pairs was estimated from publications on these asteroids. We found that for several such pairs of moons, the mutual influence can be significant, and the independent determination of the fixed Keplerian orbits of the moon in pairs is doubtful. According to this criterion, the following asteroids with pairs of moons were excluded from our study: (93) Minerva, (130) Elektra, (216) Kleopatra, (47171) Lempo and (136108) Haumea. At the same time, for the Remus and Romulus moons of the asteroid (87) Sylvia, as well as for the two moons of the asteroid (107) Camilla, we were able to independently determine the fixed Keplerian orbits. It was also possible to determine the orbit of the moon Petit-Prince of the asteroid (45) Eugenia, neglecting the gravitational attraction of the second Princesse moon. The Keplerian orbit of the Princesse moon could not be determined.

For moon S/2015(136472)1 of the asteroid (136472) Makemake, there are five observations, but these were performed during a narrow time interval of 30 d, which is a small part of the estimated orbital period of this moon. As a result, it is not possible to determine the orbit of this moon.

To the remaining 47 moons that have five or more observations from the ABIN data base, and the 13 moons with observations taken from both sources, we added two more moons with observations that are available only in the data base of Grundy et al.

As a result, we determined the fixed Keplerian orbits for 62 moons of asteroids. The names of these asteroids and their moons are given in the first two columns of Table S1, available online. The total number of observations taken for each moon from each of the two data bases are shown in the corresponding columns. The third column of Table S1 shows the value a_a of the semimajor axes of the asteroid's orbit when the asteroid is moving around the Sun. These data show that among the 62 moons, 13 moons belong to asteroids of the main asteroid belt, two are the moons of Jupiter Trojan asteroids while the rest are trans-Neptunian objects.

5 RESULTS

The results obtained in this paper are presented in Tables S1 and S2. In Table S1, in addition to the data indicated above, we also present some specific information.

We give the root mean square (rms) value σ of the angular distances of the moon's observed positions from those calculated by the model after the refinement of the parameters. This value characterizes the accuracy of the moon's positions relative to the main component. It indicates the accuracy of observations, but does not show how this accuracy relates to the size of the visible orbit. In turn, the accuracy of the observations with respect to the moon's orbit is characterized by a different value. The angular distance between the moon's observed position and the calculated position was divided by the apparent semimajor axis of the orbit. Thus, we obtained a relative deviation of the observed position from the calculated position. As a result of the calculations, the rms value σ_r of such deviations was determined for all the observations used.

Both values (i.e. σ, expressed in arcsec, and the dimensionless value σ_r) are given in columns 8 and 9 of Table S1, respectively.

In addition, we calculated the weighted values of the squared deviations of the moon's observed positions from the calculated positions. The rms value σ_w of such weighted deviations always turns out to be less than σ. Therefore, in the final results in Table S1 we present only σ.

As is known, the accuracy of the ephemerides, which is calculated from the obtained parameters of the orbit, significantly depends on the time interval of the observation. The time interval of the observations used for each moon, expressed in yr, is given in column 10 of Table S1.

In turn, columns 11 and 12 give the moon's orbital period T (in d) and the semimajor axis of the moon's orbit a (in km). Based on these values, we can calculate the gravitational parameter μ of an asteroid system with a moon. This parameter, expressed in km^3 c^{-2}, is given in column 13 of Table S1.

Column 14 of Table S1 also gives the dimensionless parameter μ', characterizing the perturbing effect of the Sun on the moon's motion. The meaning of this parameter is explained in the next section.

Table S2 presents the orbital parameters of each moon determined from observations. The first two columns identify the asteroid and moon, respectively. The third column contains the epoch t_0 of the mean anomaly in MJD units defined in the TT time-scale. The following are the orbital parameters: n, e, i, M_0, ω, Ω and a. In the next two columns, we present the ecliptic coordinates (longitude and latitude) of the pole of the orbit, calculated based on the values i and Ω. By the pole of the orbit, we mean the direction from the attracting centre (of the asteroid) perpendicular to the plane of the fixed Keplerian orbit of the moon. Among the two possible directions, the direction of the 'right' rotation of the moon in orbit was chosen. In Table S2, all angular values are expressed in deg, while the semimajor axis of the orbit a is expressed in km.

The epoch t_0 of the mean anomaly can be chosen arbitrarily. We chose it closer to the beginning of the observation's time interval. For those moons for which the orbit has been determined and published in both Grundy et al. (2019a,b) and the data base of Grundy et al., the epoch t_0 was taken as given in both sources, so that the results of determining the parameters could be compared.

6 ESTIMATES OF UNACCOUNTED PERTURBATIONS

Taking the model of a fixed Keplerian elliptical orbit for the moon's orbital motion, we neglect the perturbations acting on the moon. The major perturbations are related to the non-sphericity of the main component, the attraction of the Sun and the mutual attraction of the moons, if the asteroid has more than one moon.

In this paper, we have excluded from consideration the asteroid systems known to have several moons when their mutual attraction cannot be neglected.

The influence of the non-sphericity of the main component in the motion of the moon is manifested primarily in the precession of the line of nodes and the apsidal line. Other perturbations are insignificant in comparison with the accuracy of modern ground-based observations of the moons of asteroids. Precession of the line of nodes can be detected by the rotation of the poles of the orbit. As for the effect of the non-sphericity of the main component, in this case the pole of the orbit should rotate around some axis that is fixed in space with a constant inclination to it. The direction of this precession axis can only be found from astrometric observations. We tried to implement such a determination of perturbations for the moon Linus of the asteroid (22) Kalliope in a previous paper (Emelyanov et al. 2019).

In order to determine the precession of the moon's orbit from observations, a number of conditions must be satisfied. First, the time interval of the observations should be large enough. It should significantly exceed the moon's orbital period. Secondly, the precession itself at this time interval should be significant enough to appear in the results of observations. The orbit's precession can be detected as follows. We divide the observation's time interval into several subintervals. At each such subinterval, the orientation of the fixed axis of the orbit should be independently determined, neglecting the movement of this pole inside the selected subinterval. Then, the obtained differences in the positions of the pole of the orbit at different time points can indicate a precession of the orbit. To perform this procedure, it is necessary that the observations are more or less evenly distributed over time, the time interval is large enough and the precession is noticeable.

We tested this technique on moons that are most suitable for this purpose. The results have shown that for all considered moons except the moon Linus of asteroid (22) Kalliope, the initial data and properties of motion do not allow us to determine the precession of the orbit.

The most significant displacements of the orbit's pole in the observation's time interval did not exceed 1.5 deg. It is clear that such small displacements do not allow us to determine the axis around which the orbit's pole rotates.

The determination of the precession of the axis of the moon's orbit was made in Emelyanov et al. (2019) for the asteroid system (22) Kalliope in the observation's time interval of approximately 16 yr. The positions of the orbit's axis determined from observations show that the axis rotates in the same direction for the first 7 yr, and then turns in the opposite direction. This behaviour of the axis of the orbit cannot be explained in terms of the non-sphericity of the asteroid.

In order to simulate the orbit of the moon Linus of asteroid (22) Kalliope, additional astrometric observations are required. Possible displacements of the axis of the moon's orbit in this system represent rotations around some precession axis with an inclination to this axis not exceeding 1.5 deg. Therefore, in the present paper, for the system (22) Kalliope–Linus, we also determined the fixed Keplerian orbit, as for other moons. As a result, we found the average orbit among those that can be obtained by taking into account the precession of the axis of the orbit.

Another disturbing factor that was not taken into account in the present research is the attraction of the Sun. The influence of this factor is very difficult to describe in a change in the moon's orbit over time. The main effect of the attraction of the Sun is manifested in the precession of the node and the pericentre of the orbit. The task is complicated by the fact that we have to consider the combined effect of the attraction of the Sun and the influence of the non-sphericity of the main component. Estimates of the influence of the attraction of the Sun in the orbital motion of the moon of an asteroid were made in Emelyanov et al. (2019). As can be clearly seen from the formulas shown in Emelyanov et al. (2019), the magnitude of the perturbations from the attraction of the Sun is proportional to the parameter μ' defined by the relation

$$\mu' = \frac{\mu_s}{\mu}\left(\frac{a}{a_s}\right)^3, \tag{4}$$

where μ_s is the gravitational parameter of the Sun and a_s is the semimajor axis of the moon's orbit around the Sun. The meanings of μ and a are explained above. From equation (4), it also follows

that the perturbation due to the attraction of the Sun is greater, the smaller the mass of the asteroid system with the moon and the larger the semimajor axis of the moon's orbit.

To evaluate the possible influence due to the attraction of the Sun for all considered moons, in column 14 of Table S1, the values μ' are given. We found that this parameter has the largest value for systems (379) Huenna – S/2003 (379) 1, (3749) Balam – S/2002 (3749) 1 and 2000 CF105 – companion.

7 NOTES ON THE ACCURACY OF DETERMINING THE ORBITAL PARAMETERS OF MOONS OF ASTEROIDS AND THE RELIABILITY OF THE RESULTS

Errors of observation inevitably affect the errors of orbital parameters. We apply the least-squares method, which allows us not only to obtain the parameters themselves, but also to evaluate their errors.

It is common practice for publications to present the results with their errors. We also obtained from the least-squares method estimates of the errors of orbital parameters. However, at this stage, we refrain from presenting these errors in the paper for the following reasons.

First, we consider obtaining orbital parameters only as an intermediate stage in the procedure. As a final result, we need the ephemerides of moons of asteroids, which are in demand for various purposes. Using these ephemerides, we verify newly performed observations. The ephemerides of the moons of these asteroids are needed to calculate the moments and the configuration of the mutual occultations and eclipses of the asteroid and its moons. Ephemerides might be required in the future for planning space missions. It should be noted that when the parameters themselves are needed, in addition to calculating the ephemerides, one can simply use the most probable values.

Further, it is important for us to know the accuracy of the ephemerides calculated from the obtained orbital parameters. We found that to estimate the accuracy of the ephemerides, it is not enough to know the parameter errors. The error of the ephemeris substantially depends on the time interval of the observations. Such information is contained in the covariance matrix of parameter errors. In turn, this matrix can be obtained when determining the parameters by the least-squares method. It would be advisable to publish this matrix, but it would result in a significant volume of data that is more than for a typical paper. Another possibility is to place the matrices in an appendix or an additional supplement to the paper, which can be made available to the user.

However, we will do otherwise. At this stage, we hope in the future to calculate and publish estimates of the accuracy of the ephemerides of asteroid moons. It is such estimates that may interest some users.

In order to draw conclusions about the reliability of our results, it is necessary to take into account the following circumstances.

The results of this paper were obtained using the least-squares method. This is a big risk, as this method gives a reliable result only if certain conditions are met. In particular, we accept the very bold hypothesis about the randomness of observation errors. In addition, in many cases our task turns out to be ill-conditioned. There is also a strong dependence of the result on the observation content. Poor conditionality is manifested in the degeneracy of the matrix of normal equations in the least-squares method. These circumstances are often due to poor observations. Especially, the reliability of the results is worsened by the smallness of the observation's time interval. If the observation's time interval is a small part of the moons's orbital period, then the error of the obtained mean motion value can subsequently cause an ephemeris error of the order of one moon orbit. In this case, the ephemerides generally lose their meaning. Poor conditionality of the problem is also caused by the small eccentricities and inclinations of the moon orbits.

The adopted moon motion model is an approximate model. Differences in real motion from the model's motion lead to systematic errors arising in the differences between the measured and calculated values in observations. It also breaks the conditions for the applicability of the least-squares method.

The least-squares method consists of minimizing the sum of the squared deviations of the measured coordinates from their model values on the set of the desired parameters. There is no guarantee that such a minimum is unique, and the most reliable of the minima has been found.

Regarding the problem of finding the minimum of the above objective function, an interesting method was proposed by Vachier, Berthier & Marchis (2012). This is a quick method that allows us to go over a large space of possible values of the desired parameters and to find the most reliable solution without a priori information about the moon's orbit. This method can analyse many minima of the objective function.

Other solutions can be looked at, especially in cases where the number of observations is small. In some cases, we cannot determine any orbit even from five observations.

Given these criticisms, we still publish the results that we were able to obtain.

8 ESTIMATES OF THE ACCURACY OF THE EPHEMERIDES

Given the importance of the ephemerides of asteroid moons, we propose a method for assessing the accuracy of ephemerides. We base our proposal on the results of Emelyanov (2010). Three different methods have been proposed in this paper. The first method consists of variations in the errors of observations. The second method is the variation of the orbital parameters. The third method uses a lot of bootstrap samples of observations from the available data. The hypothesis of a Gaussian distribution of the probability density of errors of the initial parameters is also usually accepted. It was shown in Emelyanov (2010) that these three methods give the same results. Only the bootstrap method gives suspicious results when there are very few observations. The first method requires a lot of calculations. The third method is not suitable because we have few observations for moons of asteroids. Therefore, we propose using the second method. Emelyanov (2010) described how to apply this method in detail. The nominal ephemeris and each ephemeris sample according to random data should be calculated for a number of time points in the interval of interest to us. After that, the rms of the deviations of the ephemeris for each time point give estimates of the accuracy of the ephemeris.

It is very important to calculate the deviations of the ephemeris separately along and across the apparent orbital trajectory of the moon. Because observations from the Earth are carried out at different distances from the asteroid, the calculated apparent deviations should be normalized to the apparent value of the semimajor axis of the moon's orbit.

The hypothesis of randomness of source data errors will be satisfied by choosing an appropriate random number generator. Note that in this way we obtain a model very close to reality and the dependence of the accuracy of the ephemeris on the observation's time interval is shown here directly.

Table 1. Orbital parameters of the moon S/2000(90)1 of asteroid (90) Antiope determined in Descamps et al. (2007) and in the present paper. The orbit's inclination, the longitude of the ascending node Ω and the pericentre argument ω refer to the Earth's equator (J2000). The semimajor axis of the orbit is denoted by a. Errors are given at the 1σ level following the convention adopted from the previously published paper.

Parameter	Descamps et al. (2007)	Present work
Period, d	16.5051 ± 0.0001	16.4925 ± 0.0001
a, km	171 ± 1	173.6 ± 4.8
Inclination, deg	63.7 ± 2	62.13 ± 1.88
Ω, deg	303.1 ± 2	302.1 ± 1.6
ω, deg	60 ± 30	276.6 ± 31

The covariance matrices of the parameters required for the proposed calculations for all 62 moons are given in Table S4. A complete report on the accuracy of the ephemerides obtained for all 62 moons using the proposed method will be the subject of our next paper.

9 COMPARISON WITH THE RESULTS PUBLISHED IN THE LITERATURE

The orbital parameters of asteroid moons have already been determined in many previous investigations. A full review of all such results would take up considerable space. Therefore, we restrict ourselves to only a few cases of comparing our results with those published by other authors.

The orbit of the moon S/2000(90)1 of the asteroid (90) Antiope was determined in Descamps et al. (2007) based on the same observations as in our paper. The orbital parameters of this asteroid moon determined in Descamps et al. and in the present paper are given in Table 1 for comparison. It can be seen that the values of almost all the parameters differ slightly, within the error limits. However, the value of the pericentre argument ω obtained in our work is very different from the value in Descamps et al. (2007). This can be explained by the smallness of the eccentricity ($e = 0.0159$ in our results). The fact that eccentricity is small leads to an uncertainty in the value of the pericentre argument ω. In addition, it is assumed in Descamps et al. (2007) that the moon's orbit is circular. Furthermore, Descamps et al. (2007) concluded that, based on the observations used, it is not possible to detect orbital precession caused by the non-sphericity of the asteroid.

The orbits of the two moons of the asteroid (107) Camilla were determined in Pajuelo et al. (2018). In turn, we also made a similar determination of the orbits. However, for moon S/2001(107)1, we used 106 observations in a time interval of 15.45 yr, while Pajuelo et al. (2018) used only 80 observations in the same time interval. For moon S/2016(107)1, in both cases, the same number of observations was used (i.e. 11 observations) over a time interval of 1.17 yr. The results for the moon S/2001(107)1 are presented in Table 2 and those for S/2016(107)1 are shown in Table 3. Comparing the results, we can conclude that the

differences between the orbital parameters determined by Pajuelo et al. (2018) and by us are within the error limits at the 3σ level.

It would also be interesting to compare the orbital parameters obtained in our paper with the results for 38 moons of trans-Neptunian objects published in both Grundy et al. (2019a,b) and the data base of Grundy et al. Such a comparison shows that the corresponding values differ very little. The differences are mainly due to either different observations or the different methods used to determine the parameters. At least we can conclude that we found the same solutions that are given in Grundy et al. (2019a) and the data base of Grundy et al.

Table 2. The orbital parameters of the moon S/2001(107) of asteroid (107) Camilla determined in Pajuelo et al. (2018) and in the present work. The orbit's inclination, the longitude of the ascending node Ω and the pericentre argument ω refer to the Earth's equator (J2000). The semimajor axis of the orbit is denoted by a. Errors are given at the 3σ level.

Parameter	Pajuelo et al. (2018)	Present work
Period, d	3.71234 ± 0.00004	3.71234 ± 0.00001
a, km	1247.8 ± 3.8	1250.6 ± 10.9
Eccentricity	$0.0 + 0.013$	0.0059 ± 0.0059
Inclination, deg	16.0 ± 2.3	15.77 ± 0.49
Ω, deg	140.1 ± 4.9	140.2 ± 2.6
ω, deg	98.7 ± 6.5	32.6 ± 56.9

Table 3. The orbital parameters of the moon S/2016(107)1 of asteroid (107) Camilla determined in Pajuelo et al. (2018) and in the present paper. The orbit's inclination, the longitude of the ascending node Ω and the pericentre argument ω refer to the Earth's equator (J2000). The semimajor axis of the orbit is denoted by a. Errors are given at the 3σ level.

Parameter	Pajuelo et al. (2018)	Present work
Period, d	1.376 ± 0.016	1.3762 ± 0.0002
a, km	643.8 ± 3.9	599.397 ± 67.9
Eccentricity	0.18 ± 0.18	0.22 ± 0.19
Inclination, deg	27.7 ± 21.8	23.2 ± 6.7
Ω, deg	219.9 ± 67.0	176.4910 ± 23.3
ω, deg	199.4 ± 37.6	161.965157 ± 22.3

We present our results along with the results of Grundy et al. in Table S3, where three rows are given for each of the 38 moon orbits: the first row is the names of the asteroid and its moon, the second row is the orbital parameters from Grundy et al. (2019a,b) and the data base of Grundy et al., and the third row is the orbital parameters from the present work. If data for the same moon appear in both Grundy et al. (2019a,b) and the data base of Grundy et al., we take the parametres from Grundy et al. (2019a,b).

In each row, we successively indicate the following parameters: n is the mean motion (degrees per day), e is the eccentricity, i is the inclination to the Earth's equator according to the J2000 epoch (deg), M_0 is the mean anomaly at the J2000 epoch (deg), ω is the angular distance of the pericentre from the node (deg), Ω is the longitude of the ascending node in the Earth's equator system of the J2000 epoch (degrees), and a is the semimajor axis of the orbit (km). The epochs of the orbit elements in our paper are the same as in Grundy et al. (2019a,b) and the data base of Grundy et al., which allows us to compare the obtained values for the mean anomaly of the moons M_0.

The mutual Keplerian orbit of binary Jupiter Trojan asteroid (617) Patroclus and Menoetius was determined using the *HST* and ground-based Keck observatory observations (Grundy et al. 2018). The exclusivity of this work is that not only is the orbit defined but on this basis the mutual occultations and eclipses times in 2019 were predicted and calculated. For each event, the moments of beginning and ending were determined and published in Grundy et al. (2018).

The mutual Keplerian orbit of binary Jupiter Trojan asteroid (617) Patroclus and Menoetius was determined using the *HST* and ground-based Keck observatory observations (Grundy et al. 2018). Unlike

our solution, a circular orbit solution was found. Grundy et al. (2018) used only one observed position in 2013 and six s in 2017. Two observations made in 2001 were not taken into account.

To test our calculations, we made a comparison with the results published in Grundy et al. (2018). To make our solution consistent, we also adopted a circular orbit solution and deleted the observation made in 2001. With ephemerides obtained from such a solution, we also calculated the mutual occultations and eclipses times in 2019. The durations of these events are mostly of 100 min. The differences between our results and the results of Grundy et al. (2018) turned out to be small. Moments differ no more than 5 min. These differences are mainly due to the method used to fix the moments of events.

10 CONCLUSION

Many previously published works on the dynamics of the moons of asteroids contain determinations of orbits only for individual moons based on observations. It is time to give a complete set of orbits that can be determined from astrometric observations, which is what is done in this paper. Preparation for this procedure was done in Emel'yanov et al. (2018), where the ABIN data base of all published observations of moons of asteroids was created. This allowed the determination of all possible orbits, referring to the ABIN data base.

As a result, we have determined the Keplerian orbits of 62 moons of asteroids. We note that those moons whose motion cannot be represented by a fixed Keplerian orbit were excluded from our study. There were also asteroid moons for which so far there are not enough observations to determine the orbits.

Among the 62 moons with calculated orbits, 13 belong to asteroids of the main asteroid belt, two are the moons of Jupiter Trojan asteroids while the rest are trans-Neptunian objects.

Our results are presented in the four tables provided as supporting information, available online. In Table S1, we give information about the objects considered, the observations used, and the quality of the theory's agreement with the observations. In turn, Table S2 presents the obtained orbital parameters that can be used for various purposes, including the calculation of ephemerides.

Our results are in good agreement with the corresponding results published in the literature, for example, the results of the determination of the orbits of 38 asteroid moons given in Grundy et al. (2019a,b) and the data base of Grundy et al.

At subsequent stages of our work, we will have to make estimates of the errors of the ephemerides calculated on the basis of the determination of the orbits. To this end, we plan to use the covariance error matrices of those parameters that are obtained when determining the orbits, and we will apply the methods for assessing the accuracy of ephemerides described in Emelyanov (2010).

In this context, it would be interesting to calculate, for several years ahead, the ephemerides and configurations of possible mutual occultations and eclipses in asteroid–moon systems, for all 62 moons for which orbits are now determined.

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SUPPORTING INFORMATION

Supplementary data are available at *MNRAS* online.

Table S1. The main characteristics of asteroid systems with moons.
Table S2. The orbital parameters of each moon determined from observations in the present work.
Table S3. Solutions given in Grundy et al. (2019a, b) along with results of this work.
Table S4. Covariance matrix for each moon. Because the covariance matrix is symmetric, only the lower triangular part is given here.

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