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Comparing the super-Eddington accretion of SMC X-3 and RX J0209.6–7427 with Swift J0243.6+6124

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ABSTRACT

We study the giant outbursts of SMC X-3 and RX J0209.6–7427 (hereafter RX J0209) to compare their super-Eddington accretion regime with that of Swift J0243.6+6124 (hereafter Swift J0243). The high double-peak profile of SMC X-3 is found to be 0.25 phase offset from that below 2.3×10^{38} erg s⁻¹, which is similar to Swift J0243 (happened around 0.9×10^{38} erg s⁻¹). The profile of RX J0209 shows a similar 0.25 phase offset between high double-peak and low double-peak around 1.25×10^{38} erg s⁻¹. The 0.25 phase offset corresponds to a 90° angle change of the emission beam and strongly supports for a transition from a fan beam to a pencil beam. Their critical luminosities imply a surface magnetic field $\sim 4 \times 10^{13}$ and 2×10^{13} G for SMC X-3 and RX J0209, respectively, based on the recently measured cyclotron line of Swift J0243. The spin-up rate and luminosity of SMC X-3 follows a relation of $\dot{\nu} \propto L^{0.94 \pm 0.03}$, while that of RX J0209 follows $\dot{\nu} \propto L^{1.00 \pm 0.03}$, which are similar to Swift J0243 and consistent with the prediction of a radiation-pressure-dominated disc. These results indicate that accretion columns are indeed formed above Eddington luminosity, and the population of ultraluminous X-ray pulsars likely corresponds to X-ray pulsars of highest magnetic field.

Key words: accretion, accretion discs – pulsars: individual: RX J0209.6–7427 – pulsars: individual: SMC X-3 – pulsars: individual: Swift J0243.6+6124 – X-rays: binaries.

1 INTRODUCTION

The discovery of ultraluminous X-ray pulsars (ULXPs) revealed the existence of super-Eddington accretion on to magnetized neutron stars with luminosity around 10^{40} erg s⁻¹ (e.g. Bachetti et al. 2014; Fürst et al. 2016; Israel et al. 2017a,b; Carpano et al. 2018). At such a high luminosity, an accretion column will be formed below a radiation shock around the magnetic pole. The accretion disc is most likely geometrically thick, and an outflow driven by the strong radiation field is expected (Shakura & Sunyaev 1973). The process of super-Eddington accretion is relatively poorly studied compared with the sub-Eddington accretion, due to the rareness of targets and limited availability of good data since most ULXPs are located in other galaxies (for a recent review on the topic, refer to Fabrika et al. 2021; Mushtukov & Tsygankov 2022).

The discovery of the Galactic ULXP of Swift J0243.6+6124 (hereafter J0243) during its 2017 giant outburst, which reached a luminosity above 10^{39} erg s⁻¹, has enabled a detailed study of super-Eddington process around its outburst peak (e.g. Tsygankov et al.

2018; Wilson-Hodge et al. 2018; Tao et al. 2019; Zhang et al. 2019; Doroshenko et al. 2020). The detailed temporal study (Liu et al. 2022) showed that the formation of accretion column was evidenced by a 0.25 phase offset, between the double-peak profiles at luminosities below 1.5×10^{38} erg s⁻¹ and above 5×10^{38} erg s⁻¹. The 0.25 phase offset corresponds to a 90° angle change of the emission beam, which is just the transition angle between a pencil beam and a fan beam. The luminosity range within $1.5\text{--}5 \times 10^{38}$ erg s⁻¹ corresponds to an intermediate formation stage of accretion column from appearance to fully dominating the emission beam.

A detailed model of radiation-pressure-dominated (RPD) disc (Chashkina, Abolmasov & Poutanen 2017; Chashkina et al. 2019) indicated that an increasing accretion rate leads to an increased disc thickness, and the pressure balance can be satisfied at a similar radius for different accretion rate. As a result, the magnetosphere size of a RPD disc is almost independent on the mass accretion rate and a linear-like relation of $\dot{\nu} \propto \dot{m}$ is expected. Indeed, the spin-up rate $\dot{\nu}$ of Swift J0243 due to accreted material is found to be linearly correlated with luminosity L within $5\text{--}13 \times 10^{38}$ erg s⁻¹ (Liu et al. 2022). However, above 1.3×10^{39} erg s⁻¹, the $\dot{\nu}$ – L relation of Swift J0243 becomes flattened, coincident with a flattening of spectral hardness and a saturation of the faint peak of the pulse

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profile. These anomalous behaviours above 1.3×10^{39} erg s⁻¹ could be related with a geometry change of accretion disc due to the illumination by the strong radiation of accretion column (Liu et al. 2022). We note that the luminosity of Swift J0243 mentioned above would be reduced by a factor of 0.6 if adopting a distance of 5.2 kpc measured from *Gaia* Early Data Release 3 (see Section 4).

It is interesting to check the temporal behaviour of other ULXP to see whether the behaviour of Swift J0243 is a general feature of super-Eddington process. In this paper, we study the two ULXPs around Small Magellanic Cloud, SMC X-3 and RX J0209.6–7427 (hereafter J0209), the giant outbursts of which reached a peak luminosity around $1\text{--}2 \times 10^{39}$ erg s⁻¹, and they are close enough to allow detailed studies.

The 2016 giant outburst of SMC X-3 was initially reported by the *Monitor of All-sky X-ray Image* (*MAXI*) on 2016 August 8 (MJD 57609; Negoro et al. 2016) and was identified by *Swift*/X-Ray Telescope (XRT; Kennea et al. 2016). It was regularly monitored by *Swift*/XRT, the data of which had been reported in the literature (Townsend et al. 2017; Tsygankov et al. 2017; Weng et al. 2017). Two *Nuclear Spectroscopic Telescope Array* (*NuSTAR*) observations and one *XMM-Newton* observation were also performed for SMC X-3 during the outburst (Tsygankov et al. 2017; Koliopanos & Vasilopoulos 2018; Zhao et al. 2018). The 2019 giant outburst of RX J0209 was detected by *MAXI* on 2019 November 20 (MJD 58807; Negoro et al. 2019) and was localized by *Swift*/XRT (Kennea et al. 2019), and it was regularly monitored by the *Neutron star Interior Composition Explorer* (*NICER*; Iwakiri et al. 2019; Vasilopoulos et al. 2020). It was also observed once by *NuSTAR* during the rising stage and several times by *Astrosat* (Chandra et al. 2020) and *Insight-Hard X-ray Modulation Telescope* (*HXMT*; Hou et al. 2022) during the peak stage. A distance of 62 and 55 kpc is adopted for SMC X-3 and RX J0209, respectively.

2 LIGHT CURVES AND ORBITAL PARAMETERS

2.1 SMC X-3

The binary orbital motion of SMC X-3 has an apparent effect on the observed spin frequency and has to be taken into account to obtain the intrinsic spin frequency. A classical relation of accreting torque of $\dot{\nu} \propto L^{6/7}$ was generally assumed when fitting the orbital solution. Townsend et al. (2017) had found that such a relation cannot provide a reasonable fit to the observed spin frequency of SMC X-3. They noted that if the super-Eddington part of the giant outburst is excluded, a much improved fit is resulted. If the super-Eddington accretion of the giant outburst of SMC X-3 is similar to that of Swift J0243, a linear-like relation of $\dot{\nu} \propto L$ is expected during the peak period, and the general usage of $L^{6/7}$ relation would be improper.

On the other hand, Townsend et al. (2017) used the luminosity only within the XRT band (0.5–10 keV), and Tsygankov et al. (2017) noted that if correcting the 0.5–10 keV luminosity with a bolometric factor, they could obtain a much better fit. Therefore, to obtain a reasonable fitting of the orbital parameters, a bolometric luminosity and a limited time range of the giant outburst period should be used, since the $\dot{\nu}\text{--}L$ relation is expected to change at different accretion luminosity.

Tsygankov et al. (2017) estimated the bolometric correction of SMC X-3 based on the flux ratio in the 0.5–10 keV (*Swift*/XRT) plus 15–50 keV [*Swift*/Burst Alert Telescope (BAT)] bands to that in the 0.5–10 keV band, which is scaled to match two *NuSTAR* observations

(on MJD 57613 and 57704). We note that when converting the *Swift*/BAT rate to flux, they adopted a default converting factor suitable only for a power-law model with index of $\Gamma = 2.15$ (for Crab Pulsar; Krimm et al. 2013), which is quite different from the index of SMC X-3 ($\Gamma \sim 1$). So we estimate a broad-band flux as the sum of the flux in 0.6–10 and 10–70 keV bands. To estimate the unabsorbed flux in 0.6–10 keV, we fit the XRT spectrum with a model of power law plus blackbody subjecting to a fixed absorption column of 0.14×10^{22} cm⁻² (Zhao et al. 2018) and infer the unabsorbed flux by setting the model absorption to zero.

To estimate the flux in 10–70 keV band, we convert the BAT 15–50 keV rate to 10–70 keV flux based on *NuSTAR* spectra. The *NuSTAR* flux on MJD 57613 is 1.41×10^{-9} erg cm⁻² s⁻¹ within 10–70 keV band, while the BAT rate at this time is about 0.0149 counts cm⁻² s⁻¹ (C_1) estimated from a 2-d-binned BAT light curve. So a converting factor of $f_1 = 1.41 \times 10^{-9}/0.0149 = 0.95 \times 10^{-7}$ erg count⁻¹ is resulted. This is 1.65 times larger than the default converting factor mentioned above ($1.26 \times 10^{-8}/0.22 = 0.573 \times 10^{-7}$ erg count⁻¹). For the second *NuSTAR* observation on MJD 57705, the *NuSTAR* flux and BAT rate are 2.48×10^{-10} erg cm⁻² s⁻¹ and 0.0026 counts cm⁻² s⁻¹ (C_2), respectively, and a factor of $f_2 = 0.96 \times 10^{-7}$ erg count⁻¹ is resulted. As the rate is relatively low on MJD 57705, we adopt the factor of 0.95×10^{-7} erg count⁻¹ to convert the BAT rate to the flux within 10–70 keV.

The obtained bolometric fluxes, together with those in 0.6–10 keV (XRT) and 10–70 keV (BAT) bands and their ratio, are plotted in Fig. 1. The soft-band fluxes are a little higher than those in the hard band during the outburst peak between MJD 57620 and MJD 57633, while during other time, the soft-band fluxes were lower than the hard-band fluxes. This reflects the softening of the spectrum near the outburst peak. The hard-to-soft ratio reached a level of 2 around MJD 57670–57680.

The spin frequencies of SMC X-3 had been measured with both *Fermi*/Gamma-ray Burst Monitor (GBM) and *Swift*/XRT data. In principle, the GBM measurements are more accurate as the GBM daily coverage is longer than those for *Swift*/XRT, which generally has only a few snapshots for one observation. Nevertheless, the XRT spin measurements are helpful supplement because the regular GBM measurements only extended to MJD 57673. To measure the spin frequencies from barycentre-corrected XRT light curve, we first estimate a spin period for one XRT observation with epoch folding method, and then we employ a phase connection between the snapshots to obtain a more accurate estimation.

To fit the orbital parameters, we adopt a torque model of $\dot{\nu} = b L_{\text{bol}}^\alpha$. We use both the GBM and XRT barycentre-corrected spin frequency measurements. For the GBM data, we neglected those data before MJD 57616, which showed some fluctuations. We limit the fitting time range to MJD 57610–57685, which includes the outburst peak and two turn-over of measured spin frequencies. The fitting is performed for frequency versus time with L_{bol} as input. The fitted results are listed in Table 1 and plotted in Fig. 2. In general, the fitted orbital parameters are consistent with those reported in previous studies. Compared with that obtained by *Fermi*/GBM pulsar team, the most different parameter is the eccentricity: we got $e = 0.26 \pm 0.1$, while their value is 0.22. An eccentricity of $e = 0.26$ is also obtained by Weng et al. (2017), who used a high-order polynomial to follow the evolution of the intrinsic spin instead of a torque model. We note that a different e will change the intrinsic $\dot{\nu}$ apparently near periastron. The fitted index of $\alpha = 0.95 \pm 0.02$ is larger than the standard index of 6/7, similar as Swift J0243.

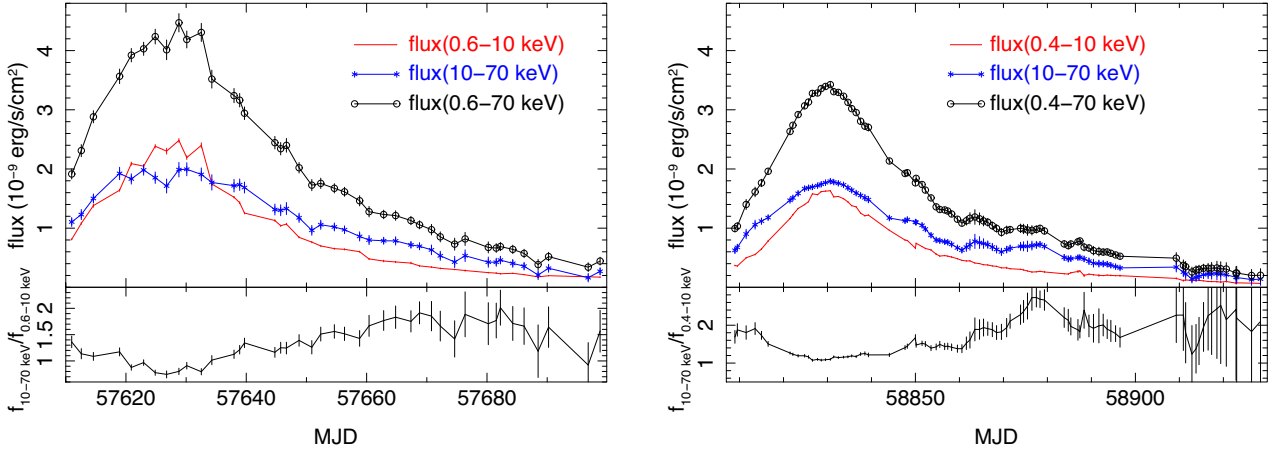


Figure 1. Bolometric fluxes of SMC X-3 (left) and RX J0209 (right), together with the fluxes in the soft and hard band and their ratio. The soft fluxes of SMC X-3 are from *Swift*/XRT, and those of RX J0209 are from *NICER*, while the hard fluxes are from *Swift*/BAT.

Table 1. Fitted orbital parameters.

Parameters	This work SMC X-3	GBM team ^a SMC X-3	This work RX J0209	GBM team ^a RX J0209
Orbital period (d)	45.2 ± 0.2	45.383	47.8 ± 0.3	47.4
Time of periastron (MJD)	57677.2 ± 0.3	57677	58782.3 ± 0.5	58782.56
$a \sin i$ (light-second)	195.2 ± 1.5	196.2	132.3 ± 2.4	169.8
Long. of periastron ($^\circ$)	208.0 ± 2.5	208.77	71.5 ± 1.5	65.7
Eccentricity	0.26 ± 0.01	0.22	0.36 ± 0.02	0.32
α	0.95 ± 0.02	–	0.97 ± 0.02	–

^a<https://gammaray.msfc.nasa.gov/gbm/science/pulsars>

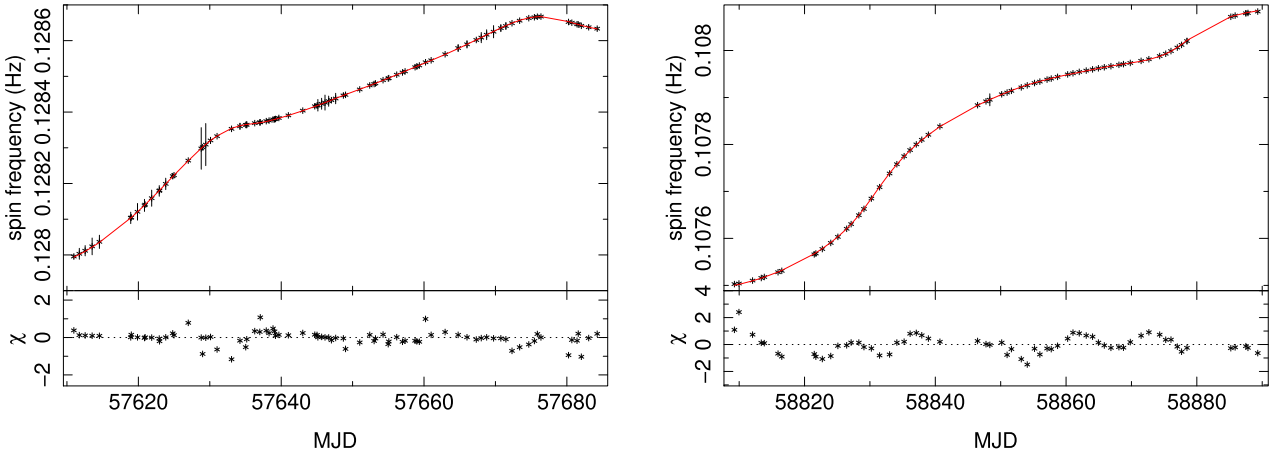


Figure 2. The barycentre-corrected spin frequencies of SMC X-3 (left) and RX J0209 (right), together with the fitted value from a binary orbit plus pulsar torque model.

2.2 RX J0209

We did a similar fitting to the observed spin frequencies of RX J0209 measured with *NICER* data (Hou et al. 2022). The *NuSTAR* observation of RX J0209 on MJD 58813 provides a conversion factor (BAT rate to flux) $f_1 = 1.01 \times 10^{-7}$ erg count $^{-1}$ for a BAT rate of $C_1 = 0.01$ counts cm $^{-2}$ s $^{-1}$. The *HXMT* observations near the outburst peak around MJD 58828 provide $f_2 = 1.1 \times 10^{-7}$ erg count $^{-1}$ for a BAT rate of $C_2 = 0.016$ counts cm $^{-2}$ s $^{-1}$. The differences of f_1 and f_2 reflect the spectral softening above 10 keV at higher fluxes, which was also found for *Swift* J0243. So we adopt a conversion

factor linearly interpolated over BAT rate C : $\frac{f_2 - f_1}{C_2 - C_1}(C - C_1) + f_1$. As the public BAT light curve of RX J0209 only started after MJD 58818, we use a BAT light curve produced by Amy Lien. We note that the bolometric correction is different from that in Hou et al. (2022), who did not use the BAT light curve. To estimate the *NICER* fluxes, we used the *NIBACKGEN3C50* tool to extract background and source spectra. The corresponding light curves are plotted in the right-hand panel of Fig. 1. The 0.4–10 keV fluxes of RX J0209 do not exceed the level of the 10–70 keV fluxes even during the outburst peak.

The fitting time range for RX J0209 is limited to MJD 58808–58890. The fitting results are presented in the right-hand panel of Fig. 2 and listed in Table 1. Compared with the orbital parameters by *Fermi*/GBM pulsar team, we obtain a smaller $a \sin i$ and also a larger eccentricity, similar to that of SMC X-3. The fitted torque model index of $\alpha = 0.97 \pm 0.02$ is also larger than the standard value of 6/7.

3 EVOLUTION OF THE PULSE PROFILE

3.1 SMC X-3

The XRT pulse profiles of SMC X-3 had been presented in Weng et al. (2017), in a normalized sense, which is not sensitive to the growth/decline detail of the absolute fluxes. Here we present a phase-coherent XRT profile study, as did for Swift J0243 in Liu et al. (2022). We assign a pulse phase $\phi(t)$ to the event time t as $\phi(t) = \int_{t_0}^t \nu(t) dt$, where $\nu(t)$ is the pulse frequency interpolated from measurements using a cubic spline function. The event arrival time t is barycentre corrected and binary corrected with the orbital parameters obtained in previous section. The GBM-measured spin frequencies showed some fluctuations before MJD 57616, so we use the GBM-measured spin frequencies within MJD 57616–57673 and XRT-measured values in other time. On MJD 57618, 57634, 57664, and 57667, the obtained pulse profiles show apparent phase offsets, and we modify the measured spin frequencies a little bit around these times to make the profiles aligned. The resulted pulse profiles within 0.6–10 keV are plotted in Fig. 3.

As can be seen, during the peak period of the outburst, the profiles are dominated by two peaks around phase 1 and 0.5. After MJD 57652 (the second panel), the main peak becomes more left tilted, and the phase separation between the two peaks is about 0.35. Around MJD 57667 (the third panel), a minor peak appeared about 0.25 phase right to the main peak. Around MJD 57680–57682, the phase separation of the original two peaks reduced to about 0.25. Finally, around MJD 57690, the original two peaks merged together and became a broad bump around phase 0.7, and the profiles looked double peaked again. Note that the two low peaks after MJD 57690 are about 0.25 phase offset from those during the outburst peak period, a phenomenon just as that for Swift J0243.

Taken the high double peaks as from a fan beam, the low double peaks would be from a pencil beam. The transition of the fan beam to the pencil beam, i.e. the disappearance of accretion column, happened around MJD 57686–57690, when the two peaks of the fan beam merged into one. The corresponding luminosity is around $2.3 \times 10^{38} \text{ erg s}^{-1}$. The high double peaks (the fan beam) take their full shape before MJD 57662, corresponding to a luminosity of $6 \times 10^{38} \text{ erg s}^{-1}$. In between, the fan beam gradually changes to the pencil beam.

3.2 RX J0209

The corresponding *NICER* pulse profiles within 0.4–10 keV of RX J0209 are presented in the right-hand panels of Fig. 3. (The *NICER* profile of RX J0209 was studied in a normalized sense in Hou et al. 2022.) During the outburst peak, the profiles are also composed of two peaks, although the minor peak is relatively fainter compared with the main peak. Around MJD 58871–58872 (the second panel), the previous left-tilted main peak becomes more symmetric, and the minor peak becomes more left titled. Around MJD 58885–58888, the minor peak merged with the main peak and the profile became a broad bump.

Because of the lack of data within MJD 58896–58909, we cannot align well the pulse profiles after MJD 58909 to those before MJD 58896. Nevertheless, it is clear that the broad bump on MJD 58895 evolved to a right-tilted peak around MJD 58910. A likely evolution path is that the intensity of the left-hand part of the bump (the main peak of the high double peak) decreased, and that produced a right-tilted peak, if assuming the valley phase of the profile is not changed much within MJD 58896–58909. This would make the peak on MJD 58910 located near the valley of the high double peak. The profiles show two peaks again after MJD 58923. We note that there seems already a feature of minor peak around MJD 58911–58918, although not as apparent as those after MJD 58923. Thus, we can see that the high double peaks changed to one bump around MJD 58888, and then changed to low double peaks after MJD 58911, which is also likely 0.25 phase offset from the high double peaks. The corresponding critical luminosity (around MJD 58911–58918) is about $1.25 \times 10^{38} \text{ erg s}^{-1}$.

4 THE $\dot{\nu}$ - L RELATION

For the super-Eddington regime of Swift J0243, we found a linear relation of $\dot{\nu} \propto L$ within $5\text{--}13 \times 10^{38} \text{ erg s}^{-1}$. A linear-like $\dot{\nu}$ - L relation is also implied during the fitting process of the orbital parameters of SMC X-3 and RX J0209. To obtain the $\dot{\nu}$ - L relation of SMC X-3, we convert the GBM measured spin frequencies with the fitted orbital parameters listed in Table 1. We calculate an averaged bolometric fluxes between two successive XRT observations, and calculate $\dot{\nu}$ at the intermediate times. The calculated spin-up rate against the bolometric flux is plotted in Fig. 4. The data are limited to MJD 57681, beyond which the high two peaks started to merge together and the inferred $\dot{\nu}$ showed larger fluctuations.

The $\dot{\nu}$ - L relation of SMC X-3 can be fitted with a power-law model of $\dot{\nu} \propto L_{\text{bol}}^{0.94 \pm 0.03}$, consistent with that obtained during the fitting process of the orbital parameters. On the other hand, the $\dot{\nu}$ - L relation of Swift J0243 shows a flattening at the highest luminosity. The $\dot{\nu}$ - L relation of SMC X-3 shows no such a flattening.

The $\dot{\nu}$ - L relation of RX J0209 is presented in the right-hand panel of Fig. 4. It can be fitted with a power-law model of $\dot{\nu} \propto L_{\text{bol}}^{1.00 \pm 0.03}$. The maximum luminosity of RX J0209 is about $1.2 \times 10^{39} \text{ erg s}^{-1}$, about half of that of SMC X-3.

To compare the $\dot{\nu}$ - L relation for all three sources, the observed $\dot{\nu}$ - L data for Swift J0243, SMC X-3, and RX J0209 are plotted together in Fig. 5. The luminosity of Swift J0243 is within 0.5–150 keV band, a little broader than the 70 keV limit for SMC X-3 and RX J0209. We note that the previous estimation of the luminosity of Swift J0243 assumed a distance of 6.8 kpc from *Gaia* Data Release 2 (Bailer-Jones et al. 2018), while the *Gaia* Early Data Release 3 provided a distance of 5.2 kpc for Swift J0243 (Bailer-Jones et al. 2021). If taking this new distance, the luminosity of Swift J0243 would be 0.6 times smaller than previous estimation. We have applied this reduction of the luminosity of Swift J0243 in Fig. 5. (If not applying this reduction, the data points of Swift J0243 will locate among those of RX J0209.) As can be seen, the data points of SMC X-3 are lying below those of RX J0209, which are below those of Swift J0243. That is, for a given observed luminosity, the spin-up rate of the neutron star of Swift J0243 is higher than that of SMC X-3 and RX J0209.

5 DISCUSSION AND CONCLUSION

We analysed the 2016 giant outburst of SMC X-3 and 2019 giant outburst of RX J0209, with the aim to compare their super-Eddington

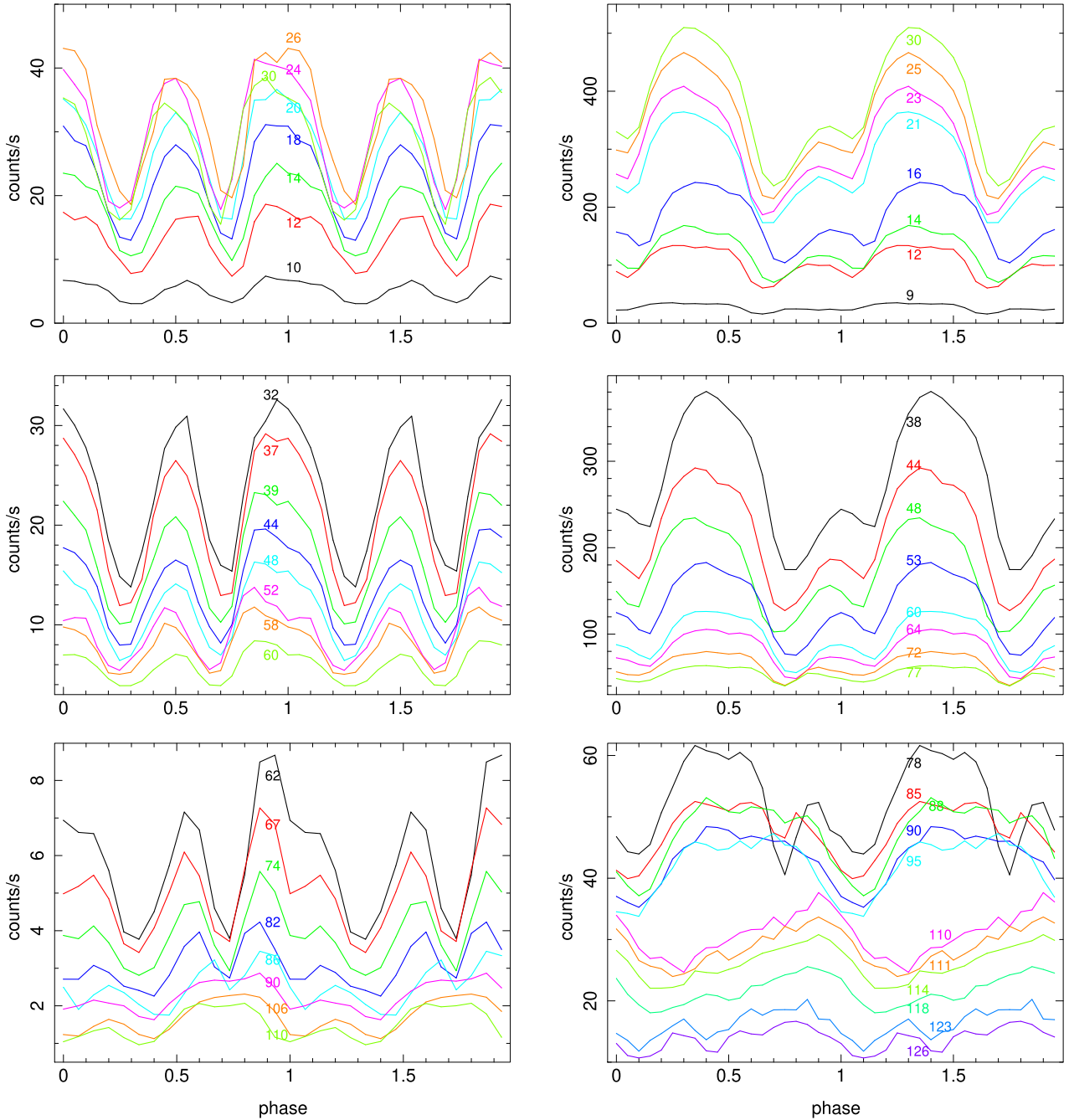


Figure 3. Time evolution of the *Swift*/XRT pulse profile of SMC X-3 during its 2016 giant outburst (left) and the *NICER* profile of RX J0209 during its 2019 outburst (right). The observed date of the profile since MJD 57600 and MJD 58800 are marked for SMC X-3 and RX J0209, respectively. The high double peaks of SMC X-3 (before MJD 57650) are 0.25 phase offset from the low double peaks (after MJD 57700), indicating a 90° change of emission pattern.

accretion regime with that of Swift J0243 in 2017. To find the orbital parameters of SMC X-3 and RX J0209, we estimated their bolometric fluxes based on *Swift*/BAT fluxes in 15–50 keV and *Swift*/XRT and *NICER* fluxes below 10 keV. We have limited the fitting time range of observed spin frequencies to the outburst period, as the accretion torque is dependent on the accretion luminosity. The high double-peak profile of SMC X-3 is found to be 0.25 phase offset from that below 2.3×10^{38} erg s $^{-1}$. The profile of RX J0243 shows a similar high double-peak to low double-peak transition around 1.25×10^{38} erg s $^{-1}$, although the profile phase at low luminosity is not well aligned. These pulse profile behaviours are just the same

as that for Swift J0243, the high double peak of which showed a 0.25 phase offset from that below 0.9×10^{38} erg s $^{-1}$, if assuming a distance of 5.2 kpc for Swift J0243.

In the standard accretion scenario of magnetized neutron star (e.g. Davidson 1973; Gnedin & Sunyaev 1973; Basko & Sunyaev 1976), at low accretion rates, X-ray radiation is emitted in a pencil beam along the magnetic pole; above a critical luminosity, an accretion column formed below a radiation-dominated shock, and X-ray photons are emitted mainly from the sidewall of the column in a fan beam. The angle difference between a pencil beam and a fan beam is 90° . Therefore, the 0.25 phase offset between the high and low double-

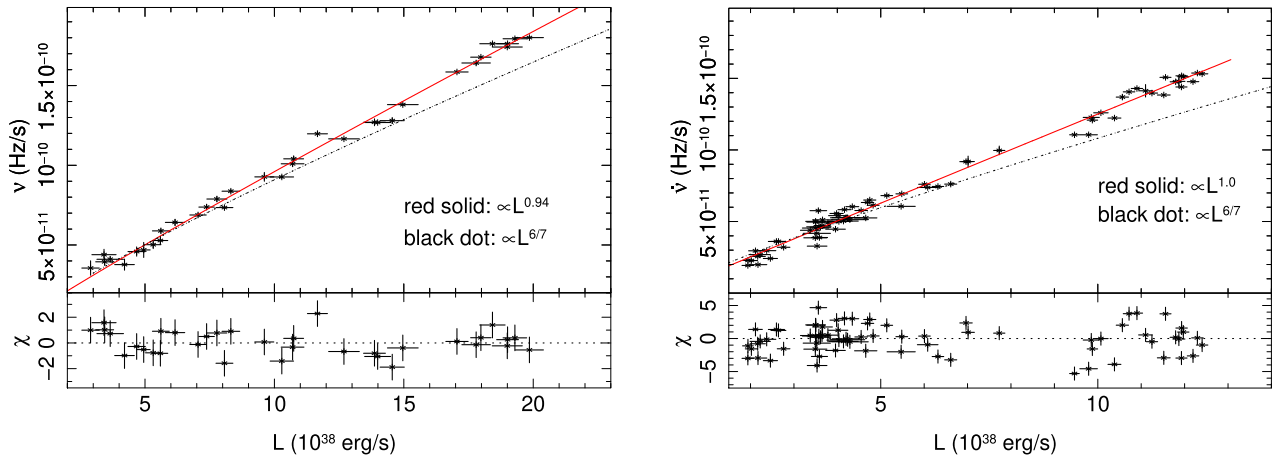


Figure 4. Spin-up rate versus bolometric flux for SMC X-3 (left) and RX J0209 (right). The data can be fitted with a power law of $\dot{\nu} \propto L_{\text{bol}}^{0.94 \pm 0.03}$ and $\dot{\nu} \propto L_{\text{bol}}^{1.00 \pm 0.03}$ (the red solid line), respectively. The classical relation of $\dot{\nu} \propto L^{6/7}$ is also plotted (the black dot line).

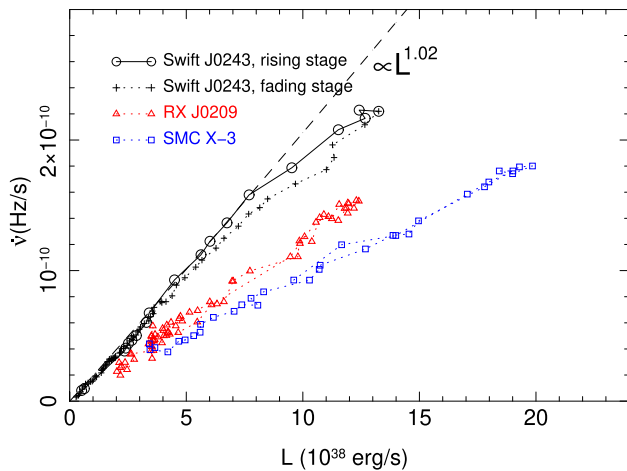


Figure 5. Spin-up rate versus bolometric flux for Swift J0243, SMC X-3, and RX J0209.

peak profiles, which corresponds to a 90° change of the emission beam, is a strong evidence for a transition from a high fan beam to a low pencil beam and for the existence of accretion column during the high double-peak regime. The critical luminosity depends on the accretion geometry and the magnetic field. Above $\sim 10^{37}$ erg s $^{-1}$, the critical luminosity grows with the magnetic field because of the drop of the effective cross-section below the cyclotron energy. It roughly increases linearly with the field strength (Becker et al. 2012; Mushtukov et al. 2015). Recently, a measurement of cyclotron line around 130 keV was reported during the main pulse at the outburst peak of Swift J0243 with *HXMT* observations, which corresponds to a surface field above 1.6×10^{13} G (Kong et al. 2022). Taken it as the face value and a distance of 5.2 kpc for Swift J0243, the surface magnetic field strength of SMC X-3 and RX J0209 would be around 4×10^{13} and 2×10^{13} G, respectively.

Above the profile transition (critical) luminosity, the profiles are not dominated by the fan beam immediately, but there is an intermediate stage for the fan beam to grow up and to dominate the emission pattern. During this intermediate stage, the profile could look like one-peaked (Swift J0243), broad-bump (RX J0243), or three-peaked (SMC X-3) shape. Such intermediate profile shapes would depend on our viewing angle towards the emission beam and

the bending of light near the neutron star, and may also depend on the magnetic field. Detailed investigations are needed to see what kind of constraints these intermediate profiles may provide. In principle, the pulse profile could also be affected by the growth of accretion column, and a sudden change may be expected when the column exceeds a height and the column cannot be eclipsed by the neutron star anymore (Mushtukov et al. 2018). Nevertheless, the predicted dip structure of the eclipse is not observed for both SMC X-3 and RX J0209, and there also seems no a characteristic 0.25 phase offset that was expected by a sudden appearance of eclipsed column.

The observed $\dot{\nu}$ – L relation of SMC X-3 follows as $\dot{\nu} \propto L^{0.94}$, while that of RX J0209 follows as $\dot{\nu} \propto L^{1.00}$. Such a relation is similar to that of Swift J0243 within $3\text{--}8 \times 10^{38}$ erg s $^{-1}$. In the RPD regime as modelled by Chashkina et al. (2017, 2019), the magnetosphere radius is almost independent of the accretion rate. As a result, a linear-like relation of $\dot{\nu} \propto L$ is predicted, which is consistent with the observed $\dot{\nu}$ – L relation. Note that a larger magnetosphere radius is expected for a stronger field: $R_{\text{in}} \propto \mu^{4/9}$ in the model of Chashkina et al. (2019). If this is true, one would expect a larger $\dot{\nu}$ at similar luminosity ($2\pi I \dot{\nu} = \dot{m} \sqrt{GM R_{\text{in}}}$, neglecting the braking torque) for a stronger magnetic field. In contrast, the observed $\dot{\nu}$ of SMC X-3, with the highest critical luminosity (magnetic field), lies below those of Swift J0243 and RX J0209.

One possibility is that the dipole component of SMC X-3 is much weaker than the multipole component in the vicinity of the neutron star, as proposed by Tsygankov et al. (2017). Such an idea has also been proposed for other ULXPs (Israel et al. 2017a,b), and also for Swift J0243 (Kong et al. 2022) and RX J0209 (Hou et al. 2022). If this is true, SMC X-3 would have the largest multipole field, but with a smallest accretion torque among all three sources. We note that a multipolar magnetic structure might increase the accreting area of the neutron star and affect the critical luminosity as discussed recently for Her X-1 (Mönkkönen et al. 2022).

On the other hand, we noted that the corotation radius of SMC X-3 is about 6.7×10^8 cm, which is comparable with the predicted magnetosphere radius (2×10^8 cm) of a source with a magnetic moment $\sim 2 \times 10^{31}$ G cm 3 around 10^{39} erg s $^{-1}$ (Chashkina et al. 2019, their fig. 13). If we take the braking torque as on the order of μ^2/R_c^3 (e.g. Lipunov 1992), it is indeed similar to the accreting torque with a luminosity of 10^{39} erg s $^{-1}$. Thus, one needs to take into account the braking torque in the situation of SMC X-3. Chashkina et al. (2019) have assumed that the disc–magnetosphere interaction is

confined to a narrow region near the inner radius of the disc. Adopting a wide interaction/threading region might modify the prediction of a RPD disc, and further modelling is needed to check whether a smaller torque would be resulted for accretion through a RPD disc with a strong dipolar field.

Above $8 \times 10^{38} \text{ erg s}^{-1}$ (with a distance of 5.2 kpc), the minor peak of Swift J0243 showed a reduced intensity compared with the main peak, and the $\dot{\nu}$ - L relation becomes flattened (Liu et al. 2022). The peak luminosity of RX J0209 reached a level of $1.2 \times 10^{39} \text{ erg s}^{-1}$, while that of SMC X-3 reached $2 \times 10^{39} \text{ erg s}^{-1}$. It is interesting to see that neither flattening of $\dot{\nu}$ - L relation nor the reduction of faint peak was observed for SMC X-3 and RX J0209. On the one hand, this is consistent with the assumption that these two phenomena of Swift J0243 are connected somehow, like through irradiation of the accretion disc by the accretion column (Liu et al. 2022). On the other hand, it indicates no apparent irradiation (feedback) from the accretion column of SMC X-3 at its highest luminosity. In principle, the irradiation of the disc depends on the geometry of the accretion column relative to the accretion disc, which is hardly constrained. If taking $R_{\text{in}} \propto \mu^{4/9}$, the magnetosphere radius of SMC X-3 during the RPD regime is estimated to be about 1.5 times larger than that of Swift J0243. This will make the irradiation flux per unit area on the inner region of the disc relatively fainter for SMC X-3, reduced by a factor of ~ 2.3 . If the disc and column configuration of SMC X-3 is similar to that of Swift J0243, one would expect some changing of the pulse profile and $\dot{\nu}$ - L relation above $2.3 \times 8 \times 10^{38} = 1.84 \times 10^{39} \text{ erg s}^{-1}$, which is just reached by the peak luminosity of SMC X-3. It seems a luminosity higher than $2 \times 10^{39} \text{ erg s}^{-1}$ is needed to probe the irradiation effect in SMC X-3. While for RX J0209, a luminosity higher than $1.2 \times 10^{39} \text{ erg s}^{-1}$ is needed.

We note that there are some uncertainties not accounted for when estimating the observed $\dot{\nu}$ - L relation. For example, we have included all the soft-band fluxes when calculating the bolometric luminosity. In reality, part of the soft emission could come from the accretion disc (although the fraction should be small as the disc is truncated far away), and one may only use the luminosity from the accretion column to estimate mass accretion rate. This would likely make the index of $\dot{\nu}$ - L relation a little larger. For sources of relatively lower luminosity (below $10^{38} \text{ erg s}^{-1}$), indexes of $\dot{\nu}$ - L relation around 1 had been reported in the literature (e.g. Bildsten et al. 1997; Filippova et al. 2017). If this is true, it would be interesting to compare the origin of linear-like $\dot{\nu}$ - L relation below and above Eddington luminosity. We noted that the bolometric correction is critical to estimate the index, and a broad-band spectroscopy is necessary to check and calibrate the correction. The intense broad-band monitoring of Galactic X-ray pulsars by *HXMT* provides a chance to study these less-luminous sources, and we plan to study them in a future work.

In summary, the high double peaks of SMC X-3 are found to be 0.25 phase offset from the low double peaks, just as that for Swift J0243, and it seems also the case for RX J0209. These results support for the formation of an accretion column during the high-luminosity state in these three sources. Among them, SMC X-3 has a highest critical luminosity around $2.3 \times 10^{38} \text{ erg s}^{-1}$, which leads to an estimation of a highest magnetic field $\sim 4 \times 10^{13} \text{ G}$. Such a high magnetic field is strong enough to support electron-positron annihilation and may produce a neutrino pulsar (Mushtukov et al. 2019). These results indicate that the population of ULXPs likely corresponds to X-ray pulsars of highest magnetic field. The $\dot{\nu}$ - L relation of all three sources follows a linear-like relation, consistent with accretion through a RPD disc. Among all sources, only Swift J0243 showed a flattening of $\dot{\nu}$ - L relation and reduction of the faint

peak at the highest luminosity. Nevertheless, it does not mean these anomalous behaviours are unique to Swift J0243, because the peak luminosity of SMC X-3 and RX J0209 may be not high enough to probe this behaviour. One should be cautionary when dealing with the $\dot{\nu}$ - L relation of ULXPs.

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DATA AVAILABILITY

The data underlying this paper are publicly available at <https://swift.gsfc.nasa.gov/>, <https://heasarc.gsfc.nasa.gov/docs/nicer/index.html>, and <http://archive.hxmt.cn/>.

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