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HAL Id: insu-04155751
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Submitted on 8 Jul 2023

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Comparisons between non-interferometric and interferometric passive surface wave imaging methods—towards linear receiver array

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Accepted 2022 November 29. Received 2022 November 21; in original form 2021 July 20

SUMMARY
Passive seismic methods in highly populated urban areas have gained much attention from the geophysics and civil engineering communities. Linear arrays are usually deployed for passive surface wave investigations because of their high convenience, and passive surface wave imaging methods commonly used for linear arrays can be grouped as non-interferometric methods (e.g. passive multichannel analysis of surface wave, refraction microtremor) and interferometric methods (e.g. multichannel analysis of passive surface waves and spatial autocorrelation). It is well known that the seismic interferometry method is able to retrieve Green’s function between inter-station pairs based on passive seismic data and that is how interferometric methods work. Although non-interferometric methods are also popular and effective in near-surface seismic imaging, particularly in the geotechnical industry, there is no theoretical proof to clarify the accuracy and/or the bias of these methods. In this study, we use numerical derivations and simulations to demonstrate the underlying physics for both non-interferometric and interferometric methods, under two common noise source environments including a homogeneous source distribution and a dominant in-line source distribution. We also prove the strength of interferometric methods for accurate dispersion imaging over the non-interferometric methods, and provide a way to estimate the biases in non-interferometric measurements. Finally, we present comprehensive comparisons between different passive surface wave methods with three typical field examples considering various observation systems.

Key words: Seismic interferometry; Seismic noise; Surface waves and free oscillations; Wave propagation.

1 INTRODUCTION
Shear (S)-wave velocity structure can be determined by inverting the dispersive phase velocity of surface waves (Dorman & Ewing 1962), due to the higher sensitivity of dispersion curve to S-wave velocity than other earth properties, like compressional (P)-wave velocity, bulk density and thickness, for a layered earth model (Xia et al. 1999). Several methods exist for estimating near-surface S-wave velocity utilizing the dispersion characteristics of surface waves, and they can be classified into two groups related to the energy source type: active surface wave methods and passive surface wave methods. The term ‘active’ denotes that the energy sources are not controlled; depending on the frequency band, a wide variety of passive sources can be utilized for surface wave analysis. A commonality of passive techniques is the lack of information on when or where the sources are activated.

The passive surface wave methods use ambient seismic energy from natural or anthropogenic sources (e.g. earthquakes, ocean-seafloor interaction, traffic and industrial activities). During the past few decades, passive surface wave methods have gained much attention from both geophysical and civil engineering communities because of the logistical costs and the exploration depth limitations associated with traditional active seismic surveys, especially in highly populated urban areas. Aki (1957) introduced a passive surface wave method to derive the S-wave velocity ($V_s$) structure by the inversion of spatial autocorrelation (SPAC) curves using microtremor, which is mainly composed of Rayleigh wave energy. Okada & Suto (2003) presented an overview of the SPAC method and further developed a microtremor array measurement (MAM) in order to improve the flexibility of the receiver configuration and explore deeper S-wave velocity structure (Hayashi et al. 2016). Chávez-García et al. (2006) testified the performance of SPAC method in linear array, and proved the possibility of extending this 2-D method into linear array (Margaryan et al. 2009; Kita et al. 2011). Louie (2001) presented the refraction microtremor (ReMi) method as a fast and effective passive seismic method based
Seismic interferometry has been used to estimate Green’s function between two receivers from the ambient seismic field (Lobkis & Weaver 2001; Campillo & Paul 2003; Shapiro & Campillo 2004; Snieder 2004; Wapenaar 2004; Nakata et al. 2014; Fichtner et al. 2019, 2020; Paitez et al. 2019; Sergeant et al. 2020). This approach has been applied to characterize multiple scales of Earth structure: from global scale or continental scale deep-structure imaging in seismology (e.g. Bensen et al. 2007; Yang et al. 2007; Lin et al. 2008; Yao & van der Hilst 2009; Lehujoer et al. 2018; Martins et al. 2019; Planès et al. 2020) to local scale exploration (e.g. Bakulin & Calvert 2006; Wapenaar et al. 2008; Draganov et al. 2009; Nakata et al. 2011; Ali et al. 2013; Behm et al. 2014; Nakata 2016). In recent years, seismic interferometry has found a variety of applications in the near-surface characterization domain (e.g. Foti et al. 2011; O’Connell & Turner 2011; Xu et al. 2013; Cheng et al. 2015; Foti et al. 2017; Cheng et al. 2018a, 2021c; b; Behm et al. 2019; Castellanos et al. 2020). Le Feuvre et al. (2015) introduced the use of cross-correlation and beamforming in the passive multichannel analysis of surface waves to enhance coherent signals. Cheng et al. (2016) proposed a hybrid method by combining seismic interferometry and multichannel analysis of surface wave (MASW) method to attenuate the potential effects from directional noise sources, called multichannel analysis of passive surface waves (MAPS).

Based on the data processing schemes of the previously mentioned passive surface wave methods, they can be roughly divided into two groups: non-interferometric methods (e.g. ReMi and PMASW) and interferometric methods (e.g. MAPS and SPAC). Fig. 1 provides a general flowchart for non-interferometric and interferometric passive surface wave methods. Non-interferometric methods directly extract dispersion measurements from ambient seismic records (Louie 2001; Park et al. 2004), while interferometric methods calculate interferograms before dispersion spectra measurement, where interferograms are either empirical Green’s function (Cheng et al. 2016; Xu et al. 2017) or spatial autocorrelation coefficients, also known as spatially averaged coherency (Asten 2006; Chávez-García et al. 2006). Several studies have provided explicit relationships between Green’s function (or cross-correlation functions) and spatial autocorrelation results (Asten 2006; Nakahara 2006; Tsai & Moschetti 2010; Haney et al. 2012). Interferometric methods appear more reasonable for utilization in passive seismic surveys. Because they retrieve interpretable signals with a specified virtual source (e.g. Green’s function or spatially averaged coherency) from ambient seismic records, the process of dispersion analysis is analogous to active source surface wave analysis (e.g. MASW). Recent studies have argued that interferometric methods have advantages over non-interferometric methods (O’Connell & Turner 2011; Le Feuvre et al. 2015; Hayashi et al. 2015; Cheng et al. 2018b). Cheng et al. (2016) demonstrated that MAPS would be more accurate than PMASW in the presence of directional ambient source effects. Xu et al. (2017) identified how to improve ReMi by combining seismic interferometry and τ − p transform.

There is no theoretical proof to clarify the accuracy and/or the bias of these methods, although they are popular and effective in near-surface seismic imaging, particularly in the geotechnical industry (Pullammanappillil et al. 2003; Stephenson et al. 2005; Thelen et al. 2006; Pancha et al. 2008; Park & Miller 2008; Rosenblad & Li 2009; Strobbia & Cassiani 2011; Louie et al. 2022). In this study, we use numerical derivations and simulations to improve our understanding of how these methods work, what their strengths are, and how to estimate the biases that exist in surface wave measurements. We first introduce a numerical framework for passive surface wave imaging based on two typical source distribution models, random and linear distribution models. We derive approximate solutions for both non-interferometric and interferometric passive surface wave methods within this framework, and present a way to estimate the bias of non-interferometric imaging method using array smoothing functions. Next, we show three field examples to make further comparisons between different passive surface wave methods. Finally, we summarize the advantages and disadvantages for both non-interferometric and interferometric passive surface wave methods.

We should note that this work focuses on the linear receiver array, which is often deployed for passive surface-wave investigations because of its high efficiency and convenience. In populated urban areas, it is challenging to construct dense 2-D arrays due to the spatial restrictions imposed by existing infrastructure (Liu et al. 2020). Linear receiver arrays are a natural geometry for road-side investigations utilizing receivers deployed on shoulders or median strip areas. Therefore, SPAC method discussed in this work is also the linear-array based SPAC method (Chávez-García et al. 2006; Margaryan et al. 2009; Kita et al. 2011), rather than the traditional SPAC (Aki 1957) using a circle array or the two-station SPAC (Ekström et al. 2009; Hayashi et al. 2013), although they all share the same mathematical base of fitting the Bessel function (the function itself or the zero-crossing of the function) with the spatial autocorrelation coefficient; under the considering of 2-D array, for example dense nodal array, SPAC method is flexible for various geometry configurations (Asten & Hayashi 2018; Cho & Iwata 2021) and can be extended to multicomponent recordings (Haney et al. 2012). With increasing attentions paid on the powerful train sources, linear arrays will definitely contribute to shallow structure seismic imaging and monitoring (Quiros et al. 2016; Fuchs & Bokelmann 2018; Cheng et al. 2019; Liu et al. 2021; Yan et al. 2021). Linear array techniques are also useful when processing distributed acoustic sensing (DAS) data sets, a recently developed technique which utilizes subsurface fibre-optic to capture earth vibrations for seismic imaging (Dou et al. 2017; Ajo-Franklin et al. 2019; Zhan 2020; Cheng et al. 2021a, 2022). However, attention should also be paid to the limitations (or inaccuracy) of the linear array configuration in terms of handing irregular ambient noise source distributions (Cheng et al. 2016, 2017; Foti et al. 2017; Liu et al. 2020). Highly directional ambient sources are not considered in this work; under such case, all of above mentioned linear array-based passive surface wave methods require additional azimuthal adjustment to avoid apparent velocity overestimation (Le Feuvre et al. 2015; Cheng et al. 2016). One possible solution is the use of 2-D adaptive arrays, for example, T-shape, L-shape or crossing-shape arrays along road-ways of interest, for better azimuthal averaging (Foti et al. 2014; Darvasi 2021; Morton et al. 2021); another solution is the use of irregular 2-D arrays combined with the specific algorithms, like kRSPAC (Stephenson et al. 2019). We should also clarify that the ReMi method discussed in this study does not base its estimates on the subjectively picking the lower borders of the dispersion energy.
as suggested by Louie (2001) and Louie et al. (2022) but on the objectively defined energy peaks, considering the poor definition of the ‘lower’ contours which depend on the subjectively chosen colour map (Mulargia & Castellaro 2013).

2 A NUMERICAL FRAMEWORK FOR PASSIVE SURFACE WAVE IMAGING

In order to provide a quantitative understanding of the different imaging techniques, we use a numerical framework to derive the various imaging schemes for these passive surface wave methods, including PMASW, ReMi, MAPS and SPAC. For a stratified medium, the contribution of the time–space unrelated ambient noise sources to a seismogram can be written as (Aki & Richards 2002; Lawrence et al. 2013; Cheng et al. 2016):

\[
u(x, \omega) = \sum_{i=1}^{N_s} F(s_i, \omega) e^{i(\omega t_i - k_0(\omega) r_{si} x)} / \sqrt{r_{si} x},
\]

where \(u(x, \omega)\) is the vertical component of displacement of the noise wavefield obtained at receiver \(x\) (here we consider the even-sampled linear array case, where the interstation distance is defined as \(dx\)), \(F(s_i, \omega)\) denotes the spectrum of a single point-force source located at \(s_i\), \(\omega\) is the angular frequency, \(j\) is the unit imaginary number, \(N_s\) is the total source number, \(k_0(\omega)\) is the fundamental mode surface wavenumber (multiple modes are not considered in this derivation), \(t_i\) is the source emitting time, \(\alpha(\omega)\) is the frequency-dependent attenuation coefficient, \(r_{si}\) is the source–receiver distance. The geometric (cylindrical) damping term, \(\sqrt{r_{si} x}\), is neglected in this work since it is an amplitude-modulation factor, which will be generally cancelled by the spectral whiten operator during the ambient seismic data pre-processing. For convenience, we use the general assumptions, which have been involved in mathematical derivations of many passive seismic methods. In the context of this work, we list them here:

1. The far-field approximation \(r_{si} \gg dx\); Tromp & Dahlen 1993; Nishida 2011; Lawrence et al. 2013).
2. The lossless medium assumption \(\alpha \approx 0\); Wapenaar 2004; Wapenaar & Ruigrok 2011; Draganov et al. 2013).
3. The white-source field assumption \(F(s_i, \omega) \approx F(\omega) = 1\); Lobkis & Weaver 2001; Kimman & Trampert 2010; Lawrence et al. 2013).

Fig. 2 illustrates the geometric relationship between the receiver \(X\) and the source \(S_i\) under the far-field approximation,

\[
r_{si} \approx r_{si} + x \cos \theta_i \approx r_{si} + x \cos \phi_i,
\]

where \(\theta_i\) is the azimuth of source \(S_i\); \(\phi_i\) is the azimuth between receiver \(X\) and source \(S_i\); \(\theta_i\) is approximate to \(\phi_i\), and varies from 0 to \(2\pi\).
2.1 Random ambient source distribution

In order to formulate the general passive surface wave imaging environment, we consider random sources that are uniformly surrounding the linear receiver array. Under the above general assumptions, eq. (1) becomes

$$u(x, \omega) = \sum_{i=1}^{N_s} e^{j(\omega_0 t_i - k_0 r_{si})}$$

$$= \sum_{i=1}^{N_s} e^{j(\omega_0 t_i - k_0 r_{si} - k_0 x \cos \theta_i)}.$$  (3)

We simulate the ambient seismic wavefield based on eq. (1) for explicit dispersion imaging comparisons. To make the simulations comparable, we use the same four-layer earth model parameters (presented in Table 1) and the same receiver recording configurations (100 traces, 1 m spatial interval and 10 ms temporal sampling interval). The total noise duration is 15-min-long; the total source number is 200; all source impulses will be randomly emitted at every 20-s window. As for the source configuration, we consider two typical ambient source distribution models: a random source model and a linear source model. The temporal and spatial distribution of noise source will be further detailed, respectively.

Table 1. Parameters of a four-layer model, modified from Luo et al. (2011).

<table>
<thead>
<tr>
<th>Layer number</th>
<th>$V_s$ (m s$^{-1}$)</th>
<th>$V_p$ (m s$^{-1}$)</th>
<th>$\rho$ (g cm$^{-3}$)</th>
<th>$h$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>800</td>
<td>2.0</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>400</td>
<td>2.0</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
<td>1200</td>
<td>2.0</td>
<td>10</td>
</tr>
<tr>
<td>Half-space</td>
<td>800</td>
<td>1600</td>
<td>2.0</td>
<td>Infinite</td>
</tr>
</tbody>
</table>

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2.1.1 Non-interferometric passive surface wave methods

Cheng et al. (2018b) presented a review of the PMASW method (Park et al. 2004; Park & Miller 2008) and the ReMi method (Louie 2001; Xu et al. 2017) and indicated the equivalence of the dispersion imaging scheme between two techniques. The PMASW method uses a slant-stacking algorithm in the frequency–offset ($f-x$) domain and transfers the wavefield into the frequency–velocity ($f-v$) domain (Park et al. 1998, 2004). The ReMi approach first uses slant-stacking algorithm to transfer the offset–time ($x-t$) domain wavefield into the intercept–time–slowness ($\tau-p$) domain (where $\tau$ indicates the intercept time and $p$ indicates the slowness), then applies a Fourier transform to obtain an $f-p$ domain dispersion image (e.g. Thorson & Claerbout 1985; Louie 2001; Xia et al. 2007). Shen et al. (2015) also demonstrated the resolution equivalence of these dispersion imaging methods for noise-free surface wave data. Therefore, we focus on the PMASW method to understand the non-interferometric dispersion imaging scheme, including the underlying physics and the approximation.

According to Park et al. (2004) and Park & Miller (2008), we apply the phase-shift method (Park et al. 1998) to transform the seismic wavefield ($u(x, \omega)$) into dispersion spectra. We first calculate the phase spectrum of the ambient seismic wavefield as

$$\Phi(x, \omega) = \frac{u(x, \omega)}{|u(x, \omega)|} = \sum_{i=1}^{N_s} e^{j(\omega_0 t_i - k_0 r_{si} - k_0 x \cos \theta_i)}$$

$$= \sum_{i=1}^{N_s} \Phi_{x_i} e^{-j k_0 x \cos \theta_i},$$  (4)

where $\Phi_{x_i} = e^{j(\omega_0 t_i - k_0 r_{si})}$ is referred to as the source distribution term, associated with the time and spatial distribution of noise sources. For consistency, we present the dispersion spectra in the $f-k$ domain as

$$E(k, \omega) = \sum_{n=1}^{N_s} e^{j k x_n} \frac{|u(x_n, \omega)|}{|u(x_n, \omega)|}$$

$$= \sum_{n=1}^{N_s} e^{j k x_n} \sum_{i=1}^{N_s} \Phi_{x_i} e^{-j k_0 x_n \cos \theta_i}$$

$$= \sum_{i=1}^{N_s} \Phi_{x_i} \sum_{n=1}^{N_s} e^{j(k - k_0) \cos \theta_i x_n},$$  (5)

where $E(k, \omega)$ is dispersion spectra for scanning wavenumber ($k$) and scanning angular frequency ($\omega$); $N_s$ is the total receiver number; $x_n$ denotes the offset information of the $n$th receiver, relative to the first receiver or some pre-defined origin point.

We know that the summation of $e^{j(k - k_0) \cos \theta_i}$ has a maximum when $k$ is approaching $k_0 \cos \theta_i$. Given we could ignore the source distribution term $\Phi_{x_i}$, in extreme cases all noise sources are uniformly distributed in both spatial and temporal domain (Figs S1a–c), we can further formulate the dispersion...
spectra as

$$E(k, \omega) \approx \sum_{i=1}^{N_t} \sum_{n=1}^{N_s} e^{i(k\cdot r_{si} - \omega \cdot t_{si})} \phi_i(r_{si}) \psi_i(t_{si})$$

$$= \sum_{i=1}^{N_t} \sum_{n=1}^{N_s} e^{i(k\cdot r_{sn} - \omega \cdot t_{sn})} \phi_i(r_{sn}) \psi_i(t_{sn})$$

which is a positive function of wavenumber and has a peak when $k = 0$. Eq. (6) produces a complex dispersion energy image which consists of multiple peaks $k_0(w)\cos(\theta)$, at each frequency. Each peak can be taken as an apparent wavenumber associated with the source located at azimuth $\theta_i$, and the energy at each pixel of the dispersion image represents the probabilities of various peaks considering the modal superposition property of surface waves (Aki & Richards 2002). Given $\theta$ is uniformly distributed from 0 to $2\pi$, it is known that the probability density function (PDF) of a cosine function ($x = \cos(\theta)$) is an arcsine function ($f_0(x) = \frac{1}{\sqrt{1-x^2}}$) (Fig. S2) which possesses the highest probability when $|x| = \cos(\theta)$ is approaching 1 (Ushakov 2001; Stirzaker 2003). Therefore, the dispersion energy image obtained from eq. (6) will show the highest probability/peak when $k$ is approaching $k_0\cos(\theta) = k_0$ (the true wavenumber) at each frequency.

Considering the source distribution term was omitted, eq. (6) presents the approximate solution of eq. (5) and illuminates the underlying physics of non-interferometric passive surface wave methods. Unfortunately, the condition to fulfill the approximation is rarely achievable in practice, and so is eq. (6). Instead, the source distribution term $\Phi_S$ will smear the dispersion energy image and prevents the energy peaks function $P_i(k)$ from converging at $k_0$. Fig. 3 presents two numerical examples to exhibit comparisons between the original (eq. 5) and approximate (eq. 6) solutions of non-interferometric method on random source distribution models. In the individual sources case (Fig. 3a), both dispersion measurements (Figs 3b and c) show 5 energy peaks at each frequency which is associated with the defined 5 individual sources. Note that, with the existence of the source distribution term, the observed multiple energy peaks on the original measurement (Fig. 3b) are messy and a bit off the predicted apparent wavenumber $k_0(w)\cos(\theta)$. In the total 200 sources case (Fig. 3d), the original measurement (Fig. 3e) shows a section of messy energy below the theoretical dispersion curve (the blue dash line) instead of converging at the theoretical dispersion curve as the approximate measurement (Fig. 3f).

In order to enhance the dispersion spectra, the time ensemble averaging, a typical data processing technique used in seismic interferometry (Bensen et al. 2007; Weemstra et al. 2014; Cheng et al. 2018a), is usually applied to the non-interferometric passive surface wave methods (Fig. 1). By applying time ensemble averaging, the continuous and large-duration ambient seismic records will be divided into many short-window time segments and dispersion spectra will be measured and stacked for all segments to attenuate effects from the source distribution term $\Phi_S$. We indicate this ensemble averaging using the notation (…), then rewrite eq. (5) as

$$\langle E(k, \omega) \rangle = \left\{ \sum_{i=1}^{N_t} \sum_{n=1}^{N_s} \phi_i(r_{si}) \psi_i(t_{si}) \right\}$$

where $\langle E(k, \omega) \rangle$ indicates the ensemble stacked dispersion spectra.

According to the numerical observations on Fig. 3, eq. (5) will not converge at the theoretical wavenumber $k_0(w)$ as eq. (6), but presents as multiple apparent wavenumbers $k_0(w)\cos(\theta)$, where $\cos(\theta)$ is an arbitrary coefficient that is not equal to $\cos(\theta)$ due to the existence of the non-negligible source distribution term [see the biases between the predicted apparent dispersion curves $(k_0(w)\cos(\theta))$ and the multiple energy peaks in Fig. 3b]. With the ensemble stacking of multiple dispersion spectra from each short-window time segment, the highest probability/peak of the ensemble stacked dispersion spectra can be tracked with the PDF of these apparent wavenumbers $k_0(w)\cos(\theta)$. Since $\cos(\theta)$ will not reach 1, $(E(k, \omega))$ from the original measurements will eventually converge at somewhere close to the theoretical wavenumber $k_0(w)$ due to the monenity property of the PDF function (Fig. S2). Based on above analysis, we can implicitly express the final peaks of $(E(k, \omega))$ as

$$\arg\max_{(k, \omega)} \langle E(k, \omega) \rangle \approx k_0(\omega) \cdot C,$$

where $C$ is a constant with a value smaller than but close to 1, and varies with site and receiver geometry. Note that the directional ambient source effects might introduce frequency-dependent $C$ values due to the source characteristics, which is not examined in this work due to the complexity; more details will be further discussed later. In some degree, $k_0(\omega) \cdot C$ can be taken as the ‘dominant’ apparent wavenumber over the time ensemble period. Eq. (8) indicates the real ability of non-interferometric methods, that the dispersion measurements will be usually biased from the accurate dispersion information in practice. The biased measurements usually present with the lower wavenumber peaks or higher apparent velocity peaks in terms of the accurate values. The ensemble stacked dispersion spectra from eq. (6) would converge at the theoretical wavenumber only if the source distribution term can be omitted. A special case might fulfill this condition that the noise sources are homogeneously distributed along a single radius ($r_i = 3$ km) and emitted at a uniform time ($t_s = 0$; Fig. S1).

We presents a numerical example to explicate the comparison between the original (Fig. 4a) and approximate (Fig. 4b) solutions of non-interferometric method after 15-min time-averaged ensemble. The energy peaks of the original measurement are close to but still a bit off the theoretical dispersion curve, which stands at the top boundary of the apparent wavenumber energy; while the energy peaks of the approximate measurement appear much cleaner and closely follow the theoretical dispersion curve. These numerical observations are consistent with the analyses above.

2.1.2 Interferometric passive surface wave methods

Several studies have argued on the advantages of interferometric passive surface wave methods in contrast with non-interferometric methods (O’Connell & Turner 2011; Le Feuvre et al. 2015; Hayashi et al. 2015; Cheng et al. 2016, 2018b; Xu et al. 2017). In order to reveal the underlying differences, we follow the same assumptions to build the Green’s function approximations and the dispersion imaging schemes for the MAPS and SPAC methods.

We follow the conventions in Sánchez-Sesma & Campillo (2006) with two records $[u(x_m, \omega) \text{ and } u(x_n, \omega)]$ as described in eq. (3), and present the cross-correlation spectrum $C_{x_m-x_n}$ as

$$C_{x_m-x_n} = u(x_m, \omega) u^*(x_n, \omega)$$

$$= \sum_{i=1}^{N_t} e^{-j\theta r_{si} x_m - \omega t_{si}} C_{x_m-x_n}$$

where $^*$ denotes the complex conjugate; $C_{x_m-x_n}$ is the cross term,

$$C_{x_m-x_n} = \sum_{m=1}^{N_t} e^{j\omega r_{si} x_m - \omega t_{si}} C_{x_m-x_n} e^{-j\omega r_{si} x_n} (m \neq n).$$
Comparisons of passive surface wave methods

Figure 3. Comparisons between original and approximate solutions of non-interferometric method (PMASW) over one segment (20 s) on random source distribution models, including the individual sources case (left-hand panels a, b, c) and the total sources case (right-hand panel d, e, f). (a) Source–receiver configuration of the individual sources case (the colour coded by the random source impulse time); panels (b) and (c) dispersion spectra measured by the original and approximate solutions of non-interferometric method (PMASW); (d) source–receiver configuration with the total sources case (the colour coded by the random source impulse time); panels (e) and (f) dispersion spectra measured by the original and approximate solutions of non-interferometric method (PMASW). The dashed lines display the theoretical dispersion curves $k_0(\omega)$ calculated by Knopoff’s method (Schwab & Knopoff 1972); the thin solid lines show the apparent dispersion curves $k_0(\omega)\cos \theta$, where $\theta$ is associated with the azimuth of the corresponding source. The individual sources case includes only 5 random sources as labelled with $s_i$, $i=1.5$; the total sources case includes the total 200 random sources.

In most analytical derivations of the noise cross-correlation function, noise sources are assumed to be uncorrelated in time and space, and the contribution of each source to the cross-correlation function could be determined independently (Tromp et al. 2010; Lawrence et al. 2013). It suggests that the cross term $(C_{xm, xn})$ could be negligible with a sufficiently time-averaged ensemble. According to the geometric relationship in eq. (2) with the far-field approximation, we get the relationship, $r_{x_{m}, \theta} \cong x_{m} \cos \theta$. Substituting this into eq. (9) and applying ensemble averaging yields

$$\langle C_{xm, xn} \rangle \approx \left\{ \sum_{i=1}^{N_i} e^{-j k_0 \cos \theta_i x_{mn}} \right\} \approx J_0(k_0 x_{mn}),$$

where $J_0$ is the order zero Bessel function of the first kind; $x_{mn}$ denotes the inter-station distance between two receivers located at $x_m$ and $x_n$, respectively. The derivation of $J_0$ was found by Aki (1957) in the framework of microtremors with an integral representation over azimuthal from 0 to $2\pi$. The transition from an integral to the discrete summation limits this equation under the scope of uniform ambient noise source distribution cases. Eq. (11) simplifies the cross-correlation function into the analytical solution of Sánchez-Sesma & Campillo (2006) which expresses the linear relationship between the real part of the cross-correlation spectrum $\langle C_{xm, xn} \rangle$ and the imaginary part of the Green’s function (the Bessel function $J_0$) between two receivers. Therefore, we can easily derive the SPAC representation with the whitened cross-correlation function, which is also called cross-coherence function in ambient noise.

We use the slant-stacking algorithm (Park et al. 1998) by replacing the wavefield \( u(x, \omega) \) in eq. (5) with ensemble averaged cross-correlation wavefield \( \langle C_{xu, xu} \rangle \) to obtain the MAPS representation

\[
\langle E(k, \omega) \rangle = \left\langle \frac{\sum_{m=1}^{N_x} \sum_{n=1}^{N_u} e^{jkx_{mn}} \langle C_{xu, xu}(x_{mn}, \omega) \rangle}{\sum_{m=1}^{N_x} \sum_{n=1}^{N_u} e^{jkx_{mn}} \langle C_{xu, xu}(x_{mn}, \omega) \rangle} \right\rangle
\approx \sum_{m=1}^{N_x} \sum_{n=1}^{N_u} e^{jkx_{mn}} J_0(k_0 x_{mn}),
\]

(12)

With the random noise source distribution, the interferometric method (eq. 12) only requires sufficient time-averaged ensemble to cancel out the cross term and produce the accurate dispersion curve information, instead of depending on the ideal homogeneous source distribution hypothesis as the non-interferometric method (eq. 6).

In order to present the effects from the cross term on dispersion imaging, we use similar numerical tests on interferometric method to compare the original and approximate solutions before time-averaged ensemble (Fig. 5). For either the individual sources case or the total sources case, both original and approximate dispersion measurements present the accurate energy peaks as predicted by the theoretical wavenumbers (highlighted by the blue dashed lines) and the apparent wavenumbers (highlighted by the black solid lines), but the cross term existing in the original dispersion measurements produces lots of artefacts and pollutes the dispersion energy images. Whereas, after 15 min time-averaged ensemble, the original dispersion image turns to be rather cleaner with the cross term significantly cancelled out as shown on Fig. 6. We also present two zoomed-in windows on the original and approximate dispersion images to highlight the accuracy of the interferometric method.

2.2 Roadside (in-line) ambient source distribution

Realistic source fields can be far from uniform (Stehly et al. 2006; Kedar et al. 2008; Landès et al. 2010; Hillers et al. 2012). With the increasing necessity of surveys within urban areas, passive surface wave methods utilizing the traffic-induced ambient sources along roadside profiles provide an ideal geometry from a logistics perspective as discussed in past studies (Louie 2001; Okada & Suto 2003; Park et al. 2004; Cheng et al. 2015, 2018a). In the case of a survey line alongside a road, the ambient sources are usually generated by moving vehicles on the surface of the road. If the road is fairly straight and of reasonable extension (in subsequent examples, 10 times or more the array length), the ambient seismic records observed by the linear array alongside the road can be typically considered as in-line plane waves.

Given that the near-field effect is negligible, the geometrical relationship of the far-field approximation for the in-line source distribution becomes \( r_{i} = r_{0} + x \). Replacing \( r_{i} \) in eq. (3), the ambient seismic wavefield for the in-line source distribution can be written as

\[
u(x, \omega) = \sum_{i=1}^{N_i} e^{i(\omega t_{0} - k_0 r_{i} - k_{0} x)}
\]

\[
= e^{-j k_0 x} \sum_{i=1}^{N_i} e^{i(\omega t_{0} - k_{0} r_{i})}.
\]

(13)

2.2.1 Non-interferometric passive surface wave methods

We can modify the dispersion spectra of eq. (5) for the non-interferometric methods as,

\[
E(k, \omega) = \sum_{i=1}^{N_i} \frac{\nu(x_{i}, \omega)}{\nu(x_{i}, \omega)} = \sum_{i=1}^{N_i} e^{jkx_{i} \nu(x_{i}, \omega)} = \sum_{i=1}^{N_i} e^{jkx_{i} \nu(x_{i}, \omega)} \nu(x_{i}, \omega),
\]

(14)
Comparisons of passive surface wave methods

Figure 5. Comparisons between original and approximate solutions of interferometric method (MAPS) over one segment (20 s) on random source distribution models, including the individual sources case (left-hand panel, a and b) and the total sources case (right-hand panel, c and d). Panels (a) and (b): dispersion spectra measured by the original and approximate solutions of interferometric method (MAPS) under the individual sources case; panels (c) and (d): dispersion spectra measured by the original and approximate solutions of interferometric method (MAPS) under the total sources case. Source-receiver configurations of both cases are the same as that on Fig. 3.

Figure 6. Comparison between original (a) and approximate (b) solutions of interferometric method (MAPS) over 15-min time-averaged ensemble on the total 200 random sources distribution model. The highlighted windows on (a) and (b) are zoomed-in displayed at bottom for better comparison.

where $\Theta = \sum_{i=1}^{N_s} e^{i(o_{\text{lin}} - k_0 \Delta s_{\text{lin}})}$ indicates the ensemble source distribution term, which contributes as random noise on dispersion spectra and will converge to a constant value with sufficient time-averaged ensemble. Therefore, we can further simplify eq. (14) as

$$\langle E(k, \omega) \rangle \approx \sum_{n=1}^{N_s} e^{i(k - k_0)\Delta s_{\text{lin}}}.$$  (15)

Eq. (15) explains the underlying physics for roadside non-interferometric passive surface wave methods. Under the in-line source distribution hypothesis, non-interferometric passive surface wave methods finally derive out an ensemble-averaged dispersion measurement though a series of MASW measurements with the corresponding sources located around the first Fresnel zone. However, due to the existence of the off-line sources, the ensemble-averaged dispersion measurement will usually appear smeared compared to a real MASW measurement considering the same frequency target.
2.2.2 Interferometric passive surface wave methods

Substituting eq. (13) into eq. (9), and applying ensemble averaging, yields the updated cross-correlation spectrum \( \langle C_{x_m,x_n} \rangle \) under the in-line source distribution

\[
\langle C_{x_m,x_n} \rangle = \langle (a(x_m, \omega))^*(x_n, \omega) \rangle = \left( \sum_{i=1}^{N_s} e^{-j k x_m x_n} + C_{x_m,x_n} \right)
\]

\[
\approx e^{-j k x_m x_n}.
\]

(16)

Therefore, we can update the MAPS representation for the roadside passive surface wave survey as

\[
E(k, \omega) = \sum_{m=1}^{N-1} \sum_{n=m+1}^{N} e^{i k x_m} \frac{\langle C_{x_m,x_n}(x_{m,n}, \omega) \rangle}{\langle C_{x_m,x_n}(x_{m,n}, \omega) \rangle} = \sum_{m=1}^{N-1} \sum_{n=m+1}^{N} e^{i(k-k_0)x_m}.
\]

(17)

Eq. (17) is able to produce accurate dispersion curve. It also indicates the potential to apply interferometric passive surface wave methods to active surface wave data to improve coherent signal quality (Li & Li 2018), but the impact of potential spurious higher order terms will break, resulting in incorrect dispersion curve.

Similar to the slant-stacking-based dispersion imaging methods, for a fixed receiver array, the measured dispersion spectra from eq. (5) at individual time segment will varies with the individual source distribution, however, we know that there is no possibility to underestimate apparent velocity. It means the accurate dispersion curve will always stand at the top boundary of the apparent wavenumber energy for each individual dispersion spectral image (Fig. S3). This argument about there is no lower apparent velocities is also the underlying rules to pick the spectrum boundary or the steepest gradient of the dispersion curve envelope given by Louie (2001). Therefore, it is possible to estimate the bias of the measurement using a kind of attribute for the boundary indicator, which is similar to imaging resolution; a higher imaging resolution means a thinner boundary zone, as well as smaller bias, and vice versa.

Here we use the array response function (ARF), which is a property of the acquisition array and the spectral estimator (Birtill et al. 1965; Johnson & Dudgeon 1993; Boiero & Socco 2011; Bergamo et al. 2012; Ruigrok et al. 2017), to present the imaging resolution. Similar to the slant-stacking-based dispersion imaging methods, for example PMASW and ReMi, we define the ARF as:

\[
W(k) = \sum_{n=1}^{N} e^{i(k-k_0)x_n}.
\]

(18)

Fig. 8(a) shows normalized ARFs for two linear arrays with the same trace number \( N = 50 \), but different array lengths, \( L = 50 \) m (the deep pink line) and \( L = 250 \) m (the dark slate grey line). The main lobes of the ARF determines the imaging resolution, which is controlled by the array length \( L \). In general, the longer array length captures higher resolution spectral imaging, and vice versa.
Comparisons of passive surface wave methods

Figure 7. Comparisons between non-interferometric and interferometric methods on linear source distribution models with different off-line distances $d h = 10$ m (left-hand panel, a, b, c) and $d h = 100$ m (right-hand panel, d, e, f). (a) Source–receiver configuration with the linear source distribution at off-line distance $d h = 10$ m; panels (b) and (c): dispersion spectra measured by non-interferometric method (PMASW) and interferometric method (MAPS). (d) Source–receiver configuration with the linear source distribution at off-line distance $d h = 100$ m; panels (e) and (f): dispersion spectra measured by non-interferometric method (PMASW) and interferometric method (MAPS). The highlighted windows on (b), (c), (e) and (f) are zoomed-in displayed at bottom for better comparison.

Note that the dispersion images are measured over 15 min time-averaged ensemble as signed by $\langle E(k, \omega) \rangle$ on the caption equation of each subfigure.

The imaging resolution or the width of the main lobe also indicates the bias between accurate wavenumber and the measured spectral peaks. Figs 8(b) and (c) present two model tests with the same source configuration defined in Fig. 7(d) and two different receiver arrays defined in Fig. 8(a). With different receiver arrays and different ARFs, PMASW measurements present different biases from the theoretical values (the dashed lines in Figs 8b and c). Note that the accurate wavenumber at 17 Hz (the thin black lines in b and c) do not intersect the peaks of the vertical resolution curves due to the biases. In detail, for frequency samples at 17 Hz, the shorter array observes wavenumber bias $\varepsilon_s = 3.3 e - 3$ m$^{-1}$ which is located at about 96.5 per cent ARF peak (as indicated by the thin black line in Fig. 8b); the longer array observes wavenumber bias $\varepsilon_l = 1.0 e - 3$ m$^{-1}$ which is located at about 90.5 per cent ARF peak (as indicated by the thin black line in Fig. 8c). In fact, picking the spectrum boundary or the steepest gradient can not get rid of these biases (Fig. S4). Therefore, it is significant to estimate the bias of the measurements, and we offer a simple alternative in this work.

According to the Rayleigh resolution criterion (Johnson & Dudgeon 1993; Boiero & Socco 2011), we can use half of the ARF peak...
Fig. 8. (a) Array smoothing functions for two linear arrays with different array lengths, $L = 50$ m (the pink line) and $L = 250$ m (the grey line). The dashed windows indicate the imaging resolutions for the longer array ($k_h$) and the shorter array ($k_s$). Panels (b) and (c): dispersion spectra (PMASW) for two linear arrays with different array lengths $L = 50$ m (b) and $L = 250$ m (c). The black dashed lines in (b) and (c) are the theoretical dispersion curves. The thick solid lines in (b) and (c) are the corresponding array smoothing functions which have been gained with fourth power ($ARF^4$) to be comparable with the gained dispersion spectra ($E(k, \omega)^4$) for better display. The thin black lines in (b) ($L = 50$ m) and (c) ($L = 250$ m) indicate the corresponding wavenumber locations at frequency 17 Hz.

($k_h$) to quantify the imaging resolution as indicated by the dashed windows in Fig. 8(a). The measured biases of non-interferometric methods should be within the imaging resolution range ($0 < \varepsilon < k_h$). Therefore, we suggest $k_h$ as a bias indicator during the interpretation of non-interferometric passive surface wave methods. For examples, we can calculate $k_h$ values for all frequencies and use them as weights for dispersion curve inversion to normalize the observe dispersion curve data alternative to the standard deviation. Besides, people might also approximate the image resolution by directly using the geometry/offset information and the interested reader is referred to the works of Ruigrok et al. (2017).

For comparison, we also define the corresponding ARFs for interferometric methods. The MAPS method applies a slant-stack algorithm for dispersion imaging but uses $C_N^2$ cross-correlation pairs, so the ARF can be expressed as:

$$W(k) = \sum_{m=1}^{N-1} \sum_{n=m+1}^{N} e^{i(d-k_0)\omega_{mn}}. \quad (19)$$

Fig. 9 displays a comparison of array smoothing functions between different passive surface wave methods. Compared with the ARF of non-interferometric method, the ARF of interferometric method shows smoother side lobes due to the increased $C_N^2$ interstation pairs. The smoother side lobes decrease the possibility of the interference between the wiggles on ARF [or the array response artefacts (Wu et al. 2017; Cheng et al. 2021c)] and the incoherent noise. This might be one reason for the higher quality dispersion measurement produced by the interferometric methods compared to that by non-interferometric methods. Note that interferometric methods based on a single virtual source gather ($N$ or $N - 1$ interstation pairs) will not have this advantage.

3 FIELD EXAMPLES

We present three typical field examples with different observation systems (e.g. instrument, receiver configuration, site environment) to further demonstrate comparisons between different passive surface wave methods. We applied the same data processing workflow, as illustrated in Fig. 1, on all three data sets. First, we split the continuous and long-duration ambient seismic records, usually around 10 min for urban passive surface wave survey, into short overlapped time segments. Based on our experiences, a 20-s window is good to ensure sufficient noise sources propagation range, as well as the efficiency of the ensemble averaging/stacking; too large stack count will increase computing costs. We apply 75 per cent overlap on each segment. To remove potential near-field interferences and whiten noise spectrum, we apply basic data pre-processing, including removing the mean, linear trend, dead traces, as well as instrument response as necessary, temporal normalization, spectral whitening and tapering two ends, for each individual time segment. Next, we directly apply non-interferometric dispersion measurements, for example PMASW and ReMi, on preprocessed time segments, while for interferometric methods we calculate empirical Green’s function between each interstation pair.

In this work, we choose to use the whitened cross-correlation algorithm (Prieto et al. 2009; Weemstra et al. 2014) for MAPS for better comparison with SPAC method. Prior to picking the dispersion curve, non-interferometric methods stack all dispersion spectra together to improve the dispersion image quality, while interferometric methods stack all interferograms together before dispersion measurement to ensure the in-coherent noise, as well as cross terms (eq. 9), cancelled with sufficient ensemble averaging. Note that for interferometric methods, ensemble averaging/stacking needs to be done before the dispersion measurement, otherwise biases will be...
Comparisons of passive surface wave methods

Figure 9. Array smoothing functions for different passive surface wave methods, non-interferometric method (PMASW, the black solid line), interferometric method (MAPS, the red dashed-dotted line).

introduced due to the existence of cross terms. All the dispersion spectral images measured with different passive surface wave methods are raised to the power of fourth \( [E(k, \omega)]^4 \) for better display. Besides the steps mentioned, we do not apply any special data processing approaches on individual measurements in order to allow better comparison between the techniques. However, improvement of the results would likely be possible through utilization of other techniques such as spectral de-spiking (Girard & Shragge 2019), time segment data-selection (Cheng et al. 2018b; Pang et al. 2019; Cheng et al. 2019), and related approaches. Considering either non-interferometric methods or interferometric methods are not sensitive to specific surface wave inversion techniques once dispersion curves are picked from the measured dispersion spectra, we only focus on surface wave dispersion measurement and do not include discussions or interpretations on surface wave inversion in this study. Besides, the SPAC method discussed in this study use the standard SPAC procedure, with the exception of the azimuthal average, to derive the dispersion curve by the inversion of spatial autocorrelation curves; other SPAC methods, like the direct-fitting method by Asten & Roberts (2006) and Asten & Hayashi (2018), might not fit the discussions in this study.

3.1 Field data set #1

Fig. 10 displays the photograph of the layout of data set #1, which is located inside the campus of Zhejiang university. A linear array of 38 Zland nodes (5 Hz) was deployed on a grassy plot; data were acquired at a 500 Hz sampling rate and 1 m spatial-interval. Five-min-long continuous ambient seismic records were acquired, and processed according to the previously described workflow and parameters. We only focus on the Z-component, which is relevant to Rayleigh waves, in the present analysis. We compare the dispersion measurements generated by different passive surface wave methods, PMASW (Fig. 11a), ReMi (Fig. 11b), MAPS (Fig. 11c) and SPAC (Fig. 11d). The black crosses indicate the picked dispersion curve based on the MAPS measurement.

In general, the interferometric methods provide much clearer spectral images, and their dispersion measurements are consistent with each other. However, the SPAC measurement provides lower sensitivity to higher modes. This is due to the fact that the classical SPAC method, as described by Aki (1957), is based on the assumption of a single-valued wavenumber component per frequency contained in the wavefield, which results in superposed modes rather than individual modes when multiple modes exist (Cornou et al. 2006). To analyse the higher modes from SPAC method, one can calculate the theoretical effective phase velocities corresponding to the observed one (Asten & Roberts 1997; Ohori et al. 2002; O’Neill & Matsuoka 2005; Ikeda et al. 2012) or directly fit the theoretical SPAC coefficients with the observed SPAC coefficients (Asten et al. 2004, 2022; Asten 2006; Ikeda et al. 2012; Asten & Hayashi 2018; Hayashi et al. 2022). Besides, it is worth to mention that a similar technique, the frequency–Bessel transform, might be able to improve higher modes with an appropriate spectral decomposition on the frequency–Bessel spectrogram (Forbriger 2003; Wang et al. 2019; Hu et al. 2020; Xi et al. 2021).

In terms of computing cost, the MAPS method is more efficient than the SPAC method without the costs of Bessel functions evaluation. Compared with MAPS, both PMASW and ReMi measurements contain biased spectral peaks which is consistent with eq. (8), and noisy spectra which are likely caused by near-field incoherent noise from the road perpendicular to the observation line. Compared with the PMASW measurement, ReMi performs poorly in this case with a discontinuous higher mode. This results from the lower sensitivity of the time-domain slant-stack algorithm for frequency variations at extremely high apparent velocities. Generally, the \( \tau - p \) transform breaks down in resolution along the frequency axes as well as velocity axes when surface waves with different
Figure 11. Measured dispersion spectra based on data set #1 with different passive surface wave methods, PMASW method (a), ReMi method (b), MAPS method (c) and SPAC method (d). The black crosses indicate the picked dispersion curve from MAPS measurement. Note that, we limited wavenumber (0–0.4 m\(^{-1}\)) below the Nyquist wavenumber (1/dx/2 = 0.5 m\(^{-1}\)) to better display the performances of different methods considering the measurements from the non-interferometric methods (PMASW and ReMi) are noisy.

Figure 12. Site map of data set #2 with the receiver array along the Zhaoxia Road in Nantong city.

3.2 Field data set #2

Fig. 12 displays the site map of data set #2. A linear array of 48 RefTek 125A digitizers connected to 2.5 Hz vertical-component geophones was deployed along the Zhaoxia Road in Nantong city. Continuous ambient seismic data (10-min records) were recorded with a 2 ms sampling interval and 5 m spatial interval. The previously described workflow and parameters were then applied for data analysis. An active surface wave (MASW) survey was simultaneously performed along the observation line with a Geometrics Geodes equipped with 48 2.5 Hz vertical-component geophones. The sampling interval for active surface wave survey is 0.25 ms and the receiver interval is 1 m. A 6.3 kg (~14 lbs) sledge hammer was utilized as an active source with a nearest offset of 10 m. Fig. 13 presents a comparison between the different passive surface wave methods, PMASW (Fig. 13a), ReMi (Fig. 13b), MAPS (Fig. 13c) and SPAC (Fig. 13d).

Except for the higher imaging resolution, all four methods generally present similar response for this data set as they do in data set #1. Compared with data set #1, the receiver array is longer (240 m versus 38 m) which results in higher imaging resolution (kh = 2.3e − 3 m\(^{-1}\)) based on eq. (18). We observe smaller biases between interferometric measurements and non-interferometric measurements. Here we can take the imaging resolution, kh, as an error factor for the assessment of non-interferometric measurements, especially when interferometric measurements do not exist as a reference. For example, we can estimate the maximum relative error for PMASW measurement at a frequency of 8.5 Hz with ε < kh/k\(_{\text{measure}}\) = 0.0023/0.05 = 4.6 per cent. Besides, it is worth to point out a weak extension of the fundamental mode from 12 Hz to around 15 Hz in non-interferometric measurements (Figs 13a and b), particularly the ReMi measurement; the corresponding extension can also be observed on the MAPS measurement as light shadow energy (Fig. 13c) because of the relative stronger first higher frequencies and phase velocities are superimposed (Park et al. 1998). In this way, PMASW is superior to ReMi due to its algorithmic advantage from the frequency-domain slant-stacking, which is also the reason why the phase-shift method (Park et al. 1998) is more popular and computationally efficient than the conventional T−p transform method (McMechan & Yedlin 1981) for MASW imaging.
Comparisons of passive surface wave methods

Figure 13. Measured dispersion spectra based on data set #2 with different passive surface wave methods, PMASW method (a), ReMi method (b), MAPS method (c) and SPAC method (d). The black crosses indicate the picked dispersion curve from MAPS measurement.

According to eq. (8), the true dispersion curve always stand at the top boundary of the apparent wavenumber energy for each individual dispersion spectral image (Fig. S3); with sufficient spectral stacking of the individual dispersion spectral images, non-interferometric methods have the potential advantage to highlight the lowest apparent dispersion energy boundary over interferometric methods which only use single-time dispersion measurement after temporal ensemble averaging/stacking. In this context, it is useful to use non-interferometric measurements as reference for interferometric measurement for dispersion curve picking, particularly at the high frequency band.

In order to verify the accuracy of the passive surface wave methods, we provide a comparison to the active surface wave measurement. We convert the active surface wave record (Fig. 14a) from the $x-t$ domain into the $f-v$ domain (Fig. 14b) using the high resolution linear radon transform (HLRT) developed by Luo et al. (2008). Fig. 14(c) displays the comparison between picked dispersion curves from the active surface wave measurement (MASW) and the passive surface wave measurement (MAPS). We observe great agreement between two fundamental mode curves and a similar trend for two first overtone curves. The consistency convinces us of the accuracy of the interferometric methods, as well as their advantages in lower frequency dispersion measurements relevant to deeper $V_s$ estimation. For example, MAPS extends the lower frequency end of active measurement from 10 Hz to almost 1 Hz, and the maximum wavelength from 16 m to about 240 m.

### 3.3 Field data set #3

Fig. 15 shows the site map for field data set #3. A 240 m linear array recorded by a HX-DZ-02A 24-channel digital seismograph was connected to 4.5 Hz vertical-component geophones, deployed along Dongting Avenue in Yueyang city. Continuous ambient seismic data (10-min records) were recorded with a 500 Hz sampling rate and a 10 m receiver spacing, and processed according to the previously described workflow and parameters. Fig. 16 presents a comparison between different passive surface wave methods, PMASW (Fig. 16a), ReMi (Fig. 16b), MAPS (Fig. 16c) and SPAC (Fig. 16d).

The goal of this comparison is to show how these passive surface wave methods behave with sparse spatial sampling. The symmetric spectra along the wavenumber direction (Figs 16a and b) is caused by the bidirectional velocity scanning scheme in non-interferometric surface wave methods (Cheng et al. 2018b). For example, the slant-stacking algorithm is scanning a reverse (backward) propagating surface wave train instead of the expected forward propagating one. Under the sparse spatial sampling configuration, these artefacts interfere with the true dispersion energy. We can observe the spectral energy around the crossed points becomes smeared for the non-interferometric measurement techniques; while MAPS method produces a clean dispersion image (Fig. 16c or Fig. 13c) because the direction of the scanning velocity has been defined as from virtual sources to virtual receivers. The existence of weak ‘crossed’ alias in SPAC measurement (Fig. 16d) is related to the periodicity and symmetry characteristic of Bessel function or Hankel function (Forbriger 2003; Cho et al. 2008; Xi 2021), which is beyond the scope of current work. The interested reader is referred to the works of Asten & Roberts (2006), Cho et al. (2006), Tada et al. (2007) and Cho et al. (2008) for discussions on the spatial aliasing of SPAC measurements due to the finite number of seismic sensors. Cheng et al. (2018b) first demonstrated this phenomenon as ‘crossed’ artefacts in $f-v$ domain and proposed an effective technique with data-selection to attenuate them. Xi et al. (2020) proposed to use the SVD-based Wiener filter to attenuate the
‘crossed’ artefacts. The more fundamental solution is to use a dense array; for example, we can get rid of the crossed spectral feature with a dense array, for example using a spatial interval of \( dx = 5 \) m and the corresponding Nyquist wavenumber limitation increasing to \( k_{Nyquist} = 1/(2* dx) = 0.1 \) m\(^{-1}\).

It is worth noticing the different aliasing features present on Fig. 11 (\( dx = 1 \) m), Fig. 13 (\( dx = 5 \) m) and Fig. 16 (\( dx = 10 \) m) under different spatial intervals and array lengths. It indicates two key parameters for the linear receiver array deployment: first, we need long array length to ensure imaging resolution, as well as propagation depth; second, we need fine spatial sampling intervals to ensure sufficient Nyquist wavenumber. Considering the deployment cost for cabled dense arrays, it might not be easy to fulfil both constraints. However, recent advances in nodal large-N deployments, as well as DAS acquisition, provide routes to solve these problems; DAS in particular allows for acquisition over tens of kilometres.

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**Figure 14.** (a) Active-source surface wave shot gather (Z-component) along the receiver line. (b) Measured MASW dispersion spectra using HLRT method. (c) Comparison of the picked dispersion curves between MAPS measurement (black crosses) and MASW measurement (red dots).

**Figure 15.** Site map of data set #3 with the receiver array along the Dongting Avenue in Yueyang city.
Comparisons of passive surface wave methods

4 DISCUSSION

As the first comprehensive comparison work between non-interferometric and interferometric passive surface wave methods, we admit that further works are required to check the performances of these various techniques under the non-uniform noise source case. Whereas, two typical noise source distribution cases, random sources and linear sources, in this work are doubtlessly significant to illuminate the underlying physics of these various passive surface wave dispersion imaging techniques and lay a foundation for the further work. Besides, results in this work can also be extended to multicomponent passive surface waves (Xu et al. 2019, 2020).

The non-uniform noise source or the directional noise sources could produce biased cross-correlations, as well as biased dispersion measurements, particularly for linear receiver arrays. In order to attenuate the directional sources effect on dispersion measurements, Cheng et al. (2016) proposed to apply azimuthal adjustment to the slant-stacking algorithm. Our numerical derivations (eqs 12 and 17) exactly throw light on the ability of interferometric methods to reveal accurate dispersion measurements after azimuth adjustment. Liu et al. (2020) adapted a linear receiver array into a pseudo-linear array by adding two more off-line receivers to increase the array response to off-line signals, in order to address a similar problem with a linear array. On the other hand, data-selection is an effective tool for data quality control, and might be an alternative to mitigate influences from time segments in which non-uniform noise sources dominate. Studies have successfully used data-selection techniques on passive-source surface wave imaging for dispersion spectra enhancement (e.g. Cheng et al. 2018b, 2019; Zhou et al. 2018; Pang et al. 2019; Xie et al. 2020a, b; Dangwal & Behm 2020).

5 CONCLUSIONS

We first derive numerical solutions for dispersion analysis of both non-interferometric and interferometric passive surface wave methods under two typical source distribution cases, random sources and linear sources. We prove the strength of interferometric methods for accurate dispersion imaging over the non-interferometric methods under two common noise source environments including a homogeneous source distribution (e.g. the quiet rural environment) and a dominant in-line source distribution (e.g. the urban traffic line environment). We present an approximate solution for non-interferometric methods when the required assumptions on the ideal noise source distribution fail, and provide a way to estimate the biases in non-interferometric measurements. We finally make comprehensive comparisons between different passive surface wave methods with three typical field examples, and conclude the following summaries.

1. In general, dispersion measurements of interferometric methods are cleaner and more accurate than those of non-interferometric methods.
2. Without considering the directional ambient source effects, non-interferometric methods are able to provide dispersion measurements with estimable biases using imaging resolution.
3. Compared to the MAPS method, the SPAC method has less sensitivity to the spectral peaks generated by higher order modes.
4. In terms of computing cost, the MAPS method is more efficient than the SPAC method without the costs of Bessel functions evaluation; the PMASW method is also more efficient than the ReMi method due to the frequency-domain slant-stacking.

ACKNOWLEDGMENTS

This study was supported by the National Natural Science Foundation of China under grant no. 41830103. We thank SISL crews, as well as AoCheng Tech crews, for the field-data acquisitions. Finally, we thank Editor Sidao Ni, E. Bozdag, Assistant Editor Fern Storey, Yong Zhou, Dylan Mikesell and other anonymous reviewers for their constructive and detailed comments that significantly improve our paper.

DATA AVAILABILITY

The real-world data underlying this paper will be shared on reasonable request to the corresponding author.

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**SUPPORTING INFORMATION**

Supplementary data are available at *GJI* online.

**Figure S1.** Comparisons of non-interferometric and interferometric methods on uniform source distribution models, including the homogeneous distribution case (left-hand panel) and the random distribution case (right-hand panel). (a) Source–receiver configuration with the homogeneous source distribution (the colour coded by the random source impulse time) and the linear receiver array. Panels (b) and (c): Dispersion spectra measured by non-interferometric method (PMASW) and interferometric method (MAPS). (d) Source–receiver configuration with the random source distribution (the colour coded by the random source impulse time) and the linear receiver array. Panels (e) and (f): Dispersion spectra measured by non-interferometric method (PMASW) and interferometric method (MAPS). (g) Zoomed-in view of the highlighted window in (e). The black dashed lines show the theoretical dispersion curves. Note that we set a uniform source emitting time ($t_0 = 0$) in a to fulfil the ideal imaging environment.

**Figure S2.** Probability density function (PDF) of cosine of a random $\theta$ on $(0, 2\pi)$. The grey bars show the histogram of $\cos \theta$, which has been normalized to estimate PDF; the red curve displays the theoretical PDF distribution of $\cos \theta$. 1000 variables of $\theta$ are randomly sampled between 0 and $2\pi$.

**Figure S3.** PMASW dispersion measurements of a series of individual noise segments before stacking with source–receiver configuration presented on Fig. 7(d). No power spectra gain is applied here. The synthetic dispersion curve acts as a fixed boundary between the apparent wavenumber energy and weak random noise energy.

**Figure S4.** Biases of dispersion curve picking for non-interferometric dispersion measurements. The black dashed line shows the normalized dispersion energy envelope at 17 Hz, which is extracted from the dispersion spectra in Fig. 8(b); the blue dotted line shows the absolute gradient of the dispersion envelope curve; the red vertical line indicates the theoretical wavenumber value at 17 Hz. The black square at the bottom denotes the picked wavenumber value at the dispersion energy peak; the blue square denotes the picked wavenumber value at the steepest gradient of the dispersion energy envelope; the red square denotes the true theoretical wavenumber value. Both picking methods, using the energy peak and using the steepest gradient, introduce biases into measured phase velocities.

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